Chapter - 7

ALTERNATING CURRENT

A simple type of ac is one which varies with time is simple harmonic manner- Represented by sine curve.

ac voltage $V = V_0 \sin \omega t$ - Where $\mathbf{V_0} = NAB\omega$ Amplitude and $\omega = 2\pi\upsilon$

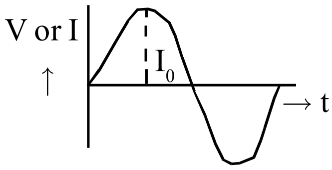
ac current - I - $I_0 \sin\omega$ Where $I_o = \dfrac{NAB\omega}{R}$, Amplitude

ac - Time depend emf current

dc - Time independend emf current

What are the advantages of AC

1) Easily stepped up or stepped down using transformer
2) Can be regulated using choke coil without loss of energy
3) Easily converted in to dc using rectifier (Pn - diode)
4) Can be transmitted over distant places
5) Production of ac is more economical



VI relation for Reisitor, Inductor and capacitor

For resistor V=IR Inductor $V = L\dfrac{dI}{dt}$ capacitor $I = C\dfrac{dv}{dt}$

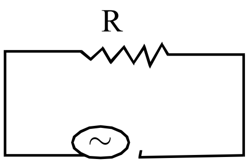
Note :

$Q = CV$

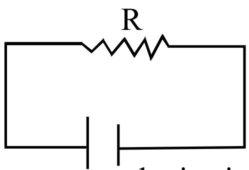
$\dfrac{dQ}{dt} = C\dfrac{dv}{dt}$

$I = C\dfrac{dv}{dt}$

What is AC circuit Electrical circuit to which ac voltage is applied



$V = V_0 \sin \omega t$

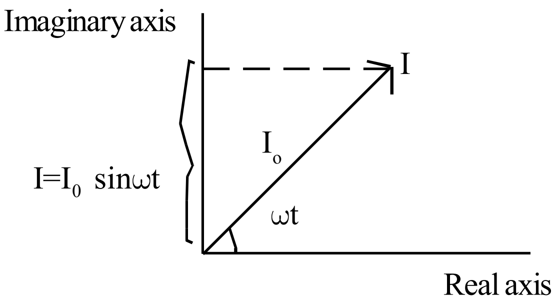


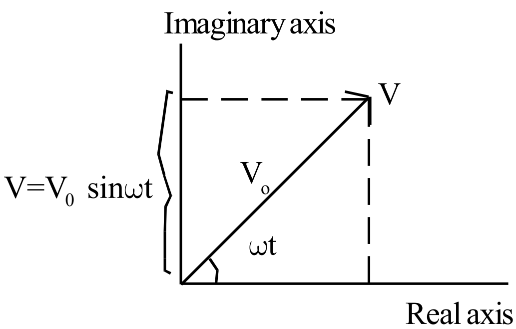
dc circuit

What is Phasor (Rotating Vector)

To study ac circuit alternating voltage and current in a circuit can be treated as phaser.

(Note : Voltage and current are scalars)





Length of the Phasor is amplitude

Projection of Phasor along the imaginery axis - Instantaneous value of voltage or current

What is RMS value or virtual value of AC (Since Average value of ac for a cycle is Zero)

$$V_{rms} = \sqrt{V_{ms}^2}$$

$$V = V_0 \sin \omega t$$

$$V^2 = V_0^2 \sin^2 \omega t$$

$$V_{ms}^2 = \frac{V_0^2}{2}$$ (since average value of $\sin^2 \omega t$ for a complete cycle of ac is ½)

$$V_{rms} = \frac{V_0}{\sqrt{2}}, \quad I_{rms} = \frac{I_o}{\sqrt{2}}$$

- **Importance of RMS value**

  1) To express ac power in the same form as dc power
  2) It is used to construct hot wire instrument used for the measurement of ac

> **Note :**
>
> dc - Power P = VI
>
> ac power $P_{av} = V_{rms} \times I_{rms}$

- **Ordinary MCG cannot used for measuring AC**

  It indicates average value, The average value of ac is O. Hence is it shows no deflection

- Hot wire instrument is used for measuring ac. Principle of hotwire instrument is Heating effect

  1) It is common to both ac and dc
  2) It is independent of direction of current

- Graduation is the Galvanometer used for the measurement of ac is not equi distant.

  It works on the basis of Heating effect.

  Since $H = I^2 R$. Deflection in the galvanometer is directly α to I^2 But in MCG Deflection is α I

Disadvantages of ac -

1. Cannot used for electroplating - can't fix cathode and anode (Polarity of ac changes)
2. ac is more dangerous

   $V_{rms} = 230V$ (line voltage)

   $$V_o = \sqrt{2} V_{rms}$$

   $$\sqrt{2} \times 230 = 325v$$

3. It can't store for longer time.

- **Number of thin wires are used for flowing ac - why**

  ac shows skin effect - ac is flowing on outer layer of a wire.

> **Note - Thick Cu wire** isused for flowing dc - It has low resistance - It is used as connecting wire in the lab.

- **Electric main in a house is marked as 230V, 50Hz, write down the equation for instentaneous ac voltage.**

  Instentaneous ac voltage $V = V_0 \sin \omega t$

$$V_0 = \sqrt{2}V_{rms} = \sqrt{2} \times 230 = 325\,\text{volt}$$

$$\omega = 2\pi\upsilon = 2\pi \times 50 = 100\pi$$

$$\therefore V = 325 \sin 100\pi t$$

AC circuit Containing resistor

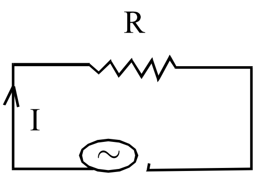
Applied voltage, $V = \sin \omega t$ ....(1)

By Ω 's law, $-\ I = \frac{V}{R} = \frac{V_o \sin \omega t}{R}$

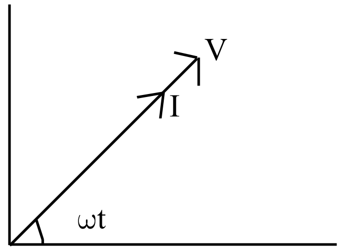
$I = I_0 \sin \omega t$ .....(2) where $I_0 = \frac{V_0}{R}$

From equation (1) and (2)

V and I are in the same phase



$V = V_0 \sin \omega t$



Power dissipation

$$P_{av} = \langle VI \rangle$$

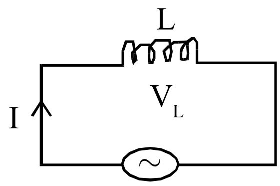
$$= V_0 \sin \omega t\ I_o \sin \omega t \qquad 2\sin^2 \omega t = 1 - \cos 2\omega t$$

$$= V_o I_o \left(\frac{1 - \cos 2\omega t}{2} \right)$$

But $\frac{V_o I_o}{2}$ $\langle \cos 2\ \omega t \rangle = 0$ for 1 cycle of ac, $Pav = \frac{V_0 I_0}{2}$

$Pav = V_{rms} \cdot I_{rms}$

AC circuit containg inductor (L)



$V = V_0 \sin \omega t$

$V = V_0 \sin \omega t$

$V_L = \frac{LdI}{dt}$ (since $V_L = V$)

$-\,LdI = V_o \sin \omega t\, dt$

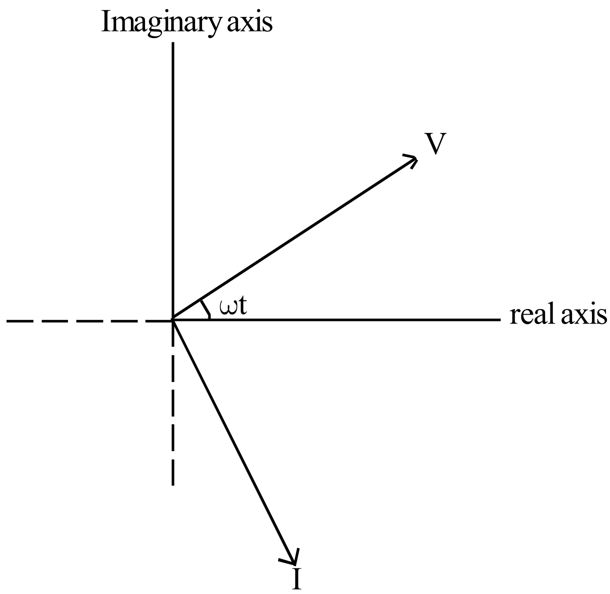
$LI = \frac{V_o \cos \omega t}{\omega}$

$I = \frac{V_o}{\omega L} \sin\left(\omega t - \pi/2\right)$

$I = I_0 \sin\left(\omega t - \pi/2\right)$

I lags V or V_L by $\pi/2$

Where $I_o = \frac{V_0}{\omega L}$



Hence ωL is the opossition affered by the inductor to ac called inductive reactance.

Inductive Reactance

$X_L = \omega L$ Note : for dc $\upsilon = 0$, $X_L = 0$

$= 2\pi \upsilon L$

$\boxed{X_L \alpha \upsilon}$

Power Dissipation Pav $= \langle VI \rangle$

$$Pav = V_o \sin \omega t \cdot I_0 \cos\left(\omega t - \pi/2\right)$$

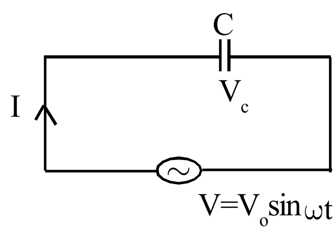
$$Pav = \frac{V_o I_o}{2} 2 \sin\omega t \cos\omega t$$

$$Pav = = \frac{VoIo}{2} \sin 2\omega t$$

Average value $\langle \sin 2\omega t \rangle = O$ for a cycle of ac

$Pav = O$ (For ideal inductor)

AC circuit containing Capaciotr C



$V = V_0 \sin \omega t$

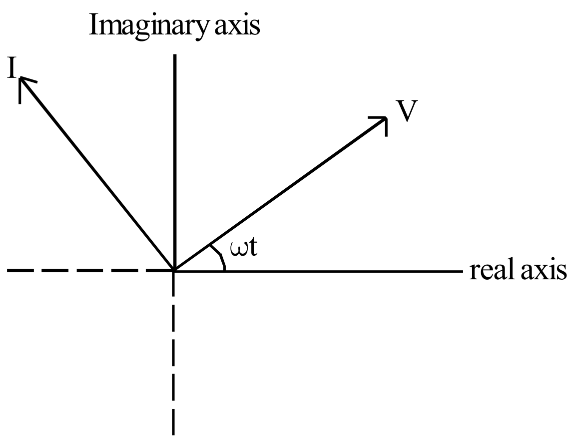
$$I = C\frac{dV_C}{dt} \quad \text{But } V_c = V, \; I = C\frac{dv}{dt}$$

$$I = C\frac{d}{dt} V_0 \sin\omega t$$

$$I = CW\, V_0 \cos \omega t$$

$$\frac{V_0}{1/\omega C} \sin\left(\omega t + \pi/2\right)$$

$$I = I_0 \sin\left(\omega t + \pi/2\right)$$



I leads V or V_C by $\pi/2$

Where $I_0 = \dfrac{V_0}{1/wc}$ Amplitude of current

Here $\dfrac{1}{\omega C}$ is the opposition offered by capacitor to ac - capacitve reactance

$$X_C = \frac{1}{\omega C}$$

$$= \frac{1}{2\pi\upsilon C}$$

$$\boxed{X_C \alpha \frac{1}{\upsilon}}$$

Note : for dc $\upsilon = 0$

$$X_C = \frac{1}{O} \Rightarrow \text{infinity}$$

Capacitor blocks dc

Power desipation Pav = $\langle VI \rangle$

$$P_{av} = V_o \sin\omega t \; I_o \sin(\omega t + \pi/2)$$

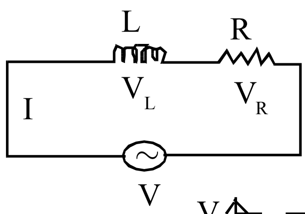
$$= \frac{V_O I_O}{2} \sin 2\omega t$$

Average value of sine 2 ωt =0 for a complete cycle

$\therefore P_{av} = 0$ (Ideal)

- In a purely resistive circuit power dissipation never be zero - Because V and I are always either +ve or -ve. Hence the product always +ve.
- In a purely inductive or capacitive circuit Pav = O what it shows - In the a circuit Inductor or capacitor offers opposition to ac with out loss of energy ie, current in the circuit does not perform any work. The current is called Idle or watt less current.

Explain AC circuit conatining LR

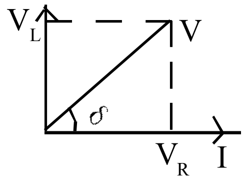


Amplitude of $V_R = I_o R$, which is in phase with current

$V_L = I_O X_L$ Which leads I by π_2

If V is the resultant of V_L and V_R, by vector algebra.

$$V = \sqrt{V_R^2 + V_L^2 + 2V_R V_L \cos \pi/2} \qquad (\cos \pi/2 = O)$$



$$V = \sqrt{(I_0 R)^2 + (I_0 X_L)^2}$$

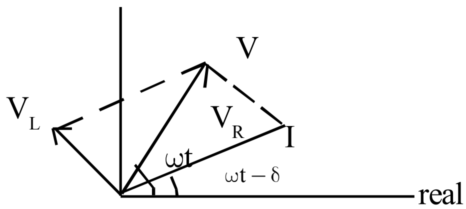
$\frac{V}{I_O} = \sqrt{R^2 + X_L^2} = Z$, Impedence of LR circuit - Resistance offered by combination of L and R

δ is the phase angle between V and I

$$\text{Tan}\,\delta = \frac{V_L}{V_R} \qquad \text{Tan}\,\delta = \frac{I_O X_L}{I_O R}$$

$$\therefore \delta = \text{Tan}^{-1}\left(\frac{X_L}{R}\right)$$

Phaser diagram



Hence $V = V_O \sin \omega t$ 1

$I = I_0 \sin(\omega t - \delta)$ 2

$V_R = R_O \sin(\omega t - \delta)$ 3

$V_L = X_L I_O \sin(\omega t - \delta + \pi/2)$ 4

$\therefore |V|^2 = |V_R|^2 + |V_C|^2$

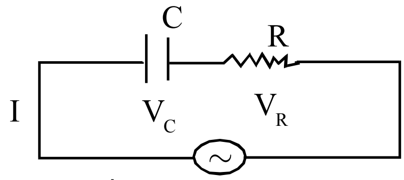
AC circuit cantaining C and R

Amplitude of

$V_R = I_o R$, Which is phase with I

$V_C = I_o X_C$ Which lags I by $\pi/2$

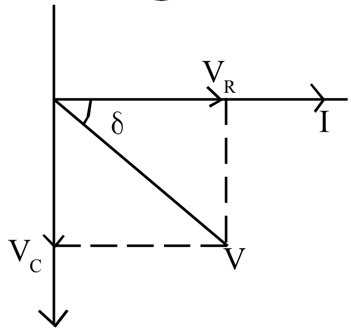
If V is the resultant voltage by vector algebra.



$$V = \sqrt{V_R^2 + V_C^2 + 2V_R V_C \cos \pi/2}$$

$$\frac{V}{I_O} = \sqrt{R^2 + X_C^2} = Z, \text{ Impedence of R C circuit}$$

δ - phase angle between V and I



$$\tan\delta = \frac{V_C}{V_R}$$

$$= \frac{I_0 X_C}{I_0 R} \text{ .... } \therefore \text{ .... } \delta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

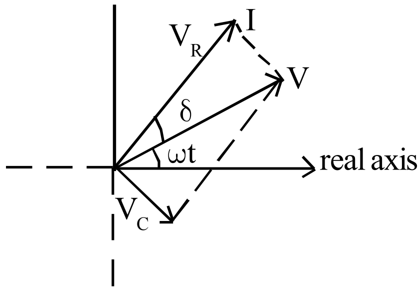
Hence $V = V_0 \sin \omega t$ 1

$I = I_0 \sin(\omega t + \delta)$ 2

$V_R = R\, I_O \sin(\omega t + \delta)$ 3

$V_C = X_C I_O \sin ?(\omega t + \delta - \pi/2)$ 4

$|V|^2 = |V_R|^2 + |V_C|^2$

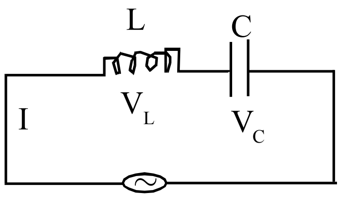


AC circuit Containing L and C

Amplitude of $V_L = I_O X_L$ leads I by $\pi/2$

$V_L = I_O X_C$ lags I by $\pi/2$

$\therefore$ Phase angle between V_L and V_C is π



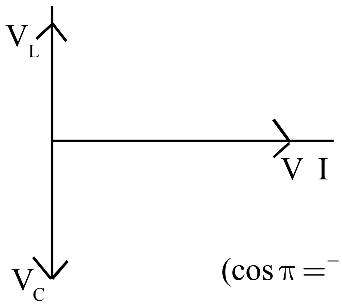
If V is the resultant voltage, by vector algebra.

$$V = \sqrt{V_L^2 + V_L^2 + 2V_L V_L \cos \pi}$$

$$V = V_L - V_C$$

$$V = I_O X_L - I_O X_C$$

$$\frac{V}{I_O} = X_L - X_L = Z, \text{ Impedence of LC circuit}$$



$(\cos \pi = ^{-}1)$

Amplitude of current $I_o = \dfrac{V}{z}$

$X_L > X_C$ (At high frequency) Circuit is inductive $\therefore X_L \alpha\, \upsilon$

$X_L > X_L$ (At law frequency circuit is capacitve $\therefore X_C \alpha\, 1/\upsilon$

If $X_L = X_L$, Z=0, $I_0 \Rightarrow \alpha$ (max). The circuit exhibits electrical resonence.

- Difference b/w resistance reactance and Impedence

Resistance - Opposition offered by a resisiter - same for both dc and ac

Reactance - Opposition offered by inductor and capacitor to ac.

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Impedences

Combined opposition affered by L, C & R to ac

- In heating coil heat produced is greater in dc than in ac

  Impedenc of heating coil is greater for ac

  Since In dc $P=\frac{V^2}{R}$ In ac $P=\frac{V^2}{Z}$ Where $Z=\sqrt{R^2+(WL)^2}$

- A coil of inductance $\frac{4}{\pi}H$ is joined in series with a resistance of 30Ω calculate the current in the circuit when it connected to an ac main of 200v and frequency 50Hz

$$I_{rms}=\frac{V_{rms}}{Z} \qquad \text{Where } Z=\sqrt{R^2+(\omega L)^2}=\sqrt{R^2+(2\pi\upsilon L)^2}=\sqrt{30^2+\left(2\times3.14x50\times\frac{4}{\pi}\right)^2}$$

$$=401.1\Omega$$

$$\therefore I_{rms}=\frac{200}{4011}$$

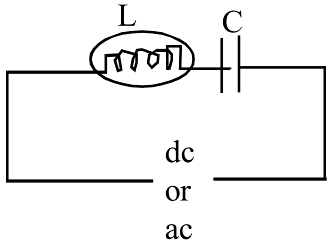
$$=0.499A$$

- A lamp L is connected in series with the capacitor C. Predict your observations for dc and ac

  For pure dc, bulb will not glow - capacitor blocks dc

  For ac bulb glows - when c is low, X_C, $\frac{1}{WC}$ large .

  Brightness reduces



Explain ac circuit containing L, C and R - Series L C R circuit

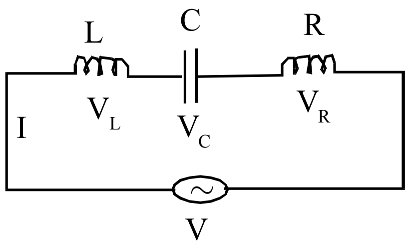
Amplitude of $V_R=I_0R$ which is in phase with I

$V_L=I_0X_L$ which leads I by $\pi/2$

$V_C=I_0X_L$ which lags I by $\pi/2$

Resultent of V_L and V_C is V_L-V_C if $V_L>V_C$

If V is the resultant of V_C, V_L and V_R

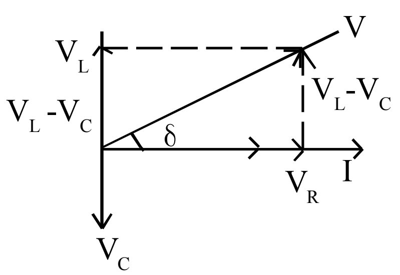


$$V=\sqrt{V_R^2+(V_L-V_L)+2V_R(V_L-V_C)\cos\pi/2}$$

$$V=\sqrt{V_R^2+(V_L-V_C)^2}$$

$$V=\sqrt{(I_0R)^2+I_0^2(x_L-X_L)^2}$$

$$\frac{V}{I_o}=\sqrt{R^2+(X_L-X_C)^2}=Z,$$ impedence of LCR circuit



δ in the phase angle b/w V and I

$$Tan\delta = \frac{V_L - V_C}{V_R} = \frac{I_o X_L - I_o X_C}{I_o R} = \frac{X_L - X_C}{R}$$

$$\therefore \delta = Tan^{-1}\left(\frac{X_L - X_C}{R}\right) \quad \therefore I = I_0\ Sin(\omega t - \delta), where\ I_0 = \frac{V_0}{Z}$$

Hence $V = V_o \sin(\omega t)$ applied voltage

$I = I_0 \sin(\omega t - \delta)$ current in the circuit.

I lags V by δ.

$V_L = I X_L = X_L \sin(\omega t - \delta + \pi/2)$ leads I by $\pi/2$

$V_c = IX_C = X_C \sin(\omega t - \delta - \pi/2)$ lags I by $\pi/2$

$VR = IR = RI_0 \text{ Sin}(\omega 1^- - \delta)$ Phase with in current.

If,

$X_L > X_C, \delta\ is\ +ve\ I\ lags\ V$

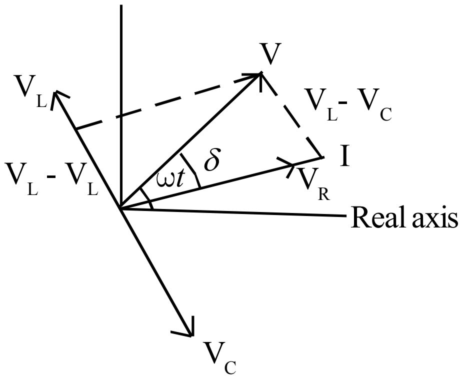
$X_L < X_C, \delta\ is\ -ve\ I\ leads\ V$

$X_L < X_C, \delta\ is\ zero\ I\ and\ V$

in the same phase

Assignment :
Draw Phaser diagram of LCR circuit with $X_C > X_L$

Phaser diagram ($X_L > X_C$) of LCR Circuit



$$\therefore \therefore |V|^2 = |VR|^2 + |V_L|^2 + |V_C|^2$$

Electrical resonance in LCR

At resonance Amplitude of current

$I_0 \Rightarrow$ max

But $I_o = \dfrac{V_o}{\sqrt{R^2 + (X_L - X_C)^2}}$

It is maximum only when $X_L = X_C$ or $V_L = V_C$ or $\delta = O$

$\therefore$ Impedence of resonant LCR circuit Z = R

* Resonant current in the circuit $I_0 = \dfrac{V_0}{R}$

* The frequency at which LCR circuit exhibits resonance is called resonant frequency

Since $X_L = X_L$ $\omega L = \dfrac{1}{\omega C}$ $\therefore \omega_r = \dfrac{1}{\sqrt{LC}}$ Hence freequency $\upsilon_r = \dfrac{1}{2\pi\sqrt{LC}}$ Note : $\omega = 2\pi\upsilon$

Resonance depends on L and C

What are the uses of LCR ciruict

1. Used in the tuning mechanism of Radio, TV
2. Metal detector

What is Q factor in LCR circuit - Shows sharpnes of resonance. If I_0 is max sharpness is greater

At resonance $Q = \dfrac{X_L}{R} or \dfrac{X_C}{R}$ i e, $Q = \dfrac{\omega_r L}{R}$ or $\dfrac{1}{\omega_r CR}$

Selectivity of LCR circuit - Depends on Q - factor

I_o is max when R in low since at resonance $I_0 = \frac{V_o}{R}$

* In parallel LCR circuit Current vanishes for a certain frequency only such a circuit is filter circuit.

Power dissipation in LCR circuit

Average power(True power) consumed during one cycle of ac.

$$Pav = \frac{\int_0^T VIdt}{\int_0^T dt}$$

$$Pav = \frac{\int_0^T V_0 \sin \omega t I_0 \sin(\omega t - \delta)dt}{\int_0^T dt}$$

$$\frac{\int_0^T V_0 \sin \omega t I_0 (\sin \omega t \cos \delta - \cos \omega t \sin \delta)dt}{\int_0^T dt}$$

$$Pav = \frac{\int_0^T V_0 I_0 \sin^2 \omega t - \cos \delta dt}{\int_0^T dt} - \frac{\int_0^T V_0 I_0 \sin \omega t - \cos \omega t \sin \delta dt}{\int_0^T dt}$$

$$= V_o I_0 \cos \delta \frac{\int_0^T \sin^2 \omega t dt}{\int_0^T dt} - \frac{V_o I_o}{2} \sin \delta \frac{\int_0^T \sin 2\omega t - dt}{\int_0^T dt}$$

For a complete cycle $\left\langle \frac{\int_0^T \sin^2 \omega t dt}{\int_0^T dt} \right\rangle = \frac{1}{2}, \left\langle \frac{\int_0^T \sin 2\omega t dt}{\int_0^T dt} \right\rangle = 0$

$$Pav = \frac{V_0 I_0}{2} \cos\delta$$

$$Pav = V_{rms} I_{rms} \cos\delta$$

True Power =Apparent power x Power factor

- Explain power factor - It signifies power loss

$$Cos\delta = \frac{R}{Z}$$

At resonance Z = R, $\cos\delta = 1$ Pav = $I_{rms} V_{rms,}$ maximum

- If the circuit is pure inductive or capacitive $\delta = \pi/2$, $\cos \pi/2 = 0$, pav = 0
- What is power factor in Resistive circuit.

  In ac resistive circuit $\delta = 0$, V and I are is same phase

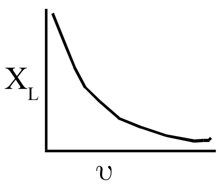
  cos $\delta = 1$, Pav = I_{rms} $\mathbf{V}_{rms}$
- What is the min and max value of power factor - O and I
- Total impedance of circuit decreases when capacitor is added in series with the given impedence - Explain -

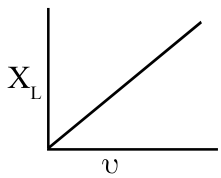
  The capacitance reduces the net reactance and hence the imepedance decreases
- What is the disadvantage in supplying a given power to a circuit having low power factor. To supply a given power in a circuit (Transmission line) having low power factor a large current is required. This produces large heat loss.

Evaluation

1) What is meant by ac, How can you represent ac mathematically
2) What is the mean value of ac for one complete cycle
3) An ac of 220 V is more dangerous than a DC of 220V -
4) In a DC circuit what is the reactance of

   a) Inductor b) Capacitor
5) Why voltages across and Land C in series are π^o out of phase
6) What is the nature of impedence of an LCR circuit if the applied frequeny $(\upsilon)(i)$, $\upsilon = \upsilon_r (2)$, $\upsilon > \upsilon_r (3)$ $\upsilon < \upsilon_r$
7) Draw graphics showing variation of reactance 1) A capacitor 2) an inductor with frequency of applied voltage



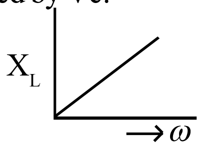


8) Properties of resonant LCR circuit
9) If the frequency of ac is doubled how do R, X_L and X_C get affected
10) What do you mean by amplitude of AC, How it related to RMS value

* There is no electrical resonance in LR or RC circuit - Reasonance takes place only if L and C. Present, Because V_L cancelled by Vc.

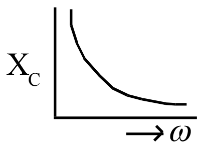
* Variation of X_L with ω.

$X_L = \omega L$



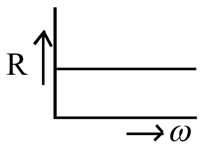
* Variation of X_C with ω

$$X_C = \frac{1}{\omega c}$$



* Variation of R with ω

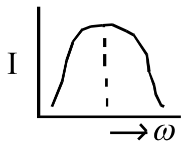
R independent of ω



* Variation of I with ω

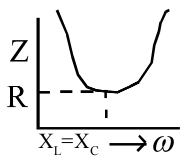
$I = I_0 \text{ Sin } \omega t$

(Sine curve)



* Variation of Z with ω

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}$$



* Resonant LCR circuit is acceptor circuit - Admits maxi current at resonance.

* Importance of acceptor circuit in tuner of Radio, TV receiver by tunning (varying) the capacitance of vairiable capacitor in the LCR circuit the natural frequenct of LCR circuit is made equal to the frequency of the signal (EM wave) to be detected.

* Can a capacitor of suitable capacitance replace a choke coil in an AC circuit.

Yes. AC voltage lags behind the current in capacitor circuit and Pav = 0.

Transformers

- Used to change the alternating voltage and current without changing its frequency
- Working Principle Mutual induction. (Electro magnetic induction)
- Transformers works in only ac not in dc. Because its working principle is Electromagnetic Induction.

Tranformer law of voltages

N_p, N_s, Number of turns in the primary and secondary coils

ϕ_B Magnetic flux in the ironcore linked with Primary and Secondary coils.

Total flux linked with the Primary coil $\phi_P = N_P \phi_B$

$\therefore$ Emf induced in the primary coil $\varepsilon_p = -N_P \dfrac{d\phi_B}{dt}$

$$\text{similarly, } \varepsilon_s - Ns \frac{d\phi_B}{dt}$$

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_S}{N_P}$$

$\varepsilon_P = V_{P-}$, applied voltage, $\varepsilon_S = V_S$, Terminal voltage $\frac{V_S}{V_P} = \frac{N_S}{N_P} = K$ is a constant called turns ratio or Transformer ratio.

Types of transformers

Step up transformer

If $N_s > N_p$, $V_s > V_p$ primary voltage is increased

so, $I_s < I_p$ then $R_s > R_p$, secondary coil is thinner than primary coil.

Step down transformer

If $N_s < N_p$, $V_s < V_p$ primary voltage is reduced.

SO, $I_s > I_p$ then $R_s < R_p$ secondary coil in thicker than primary coil.

- For a transformer if there is no power loss (I deal case)

  ac input power = ac out put power

$$V_p I_p = V_s I_s$$

- Efficiency of transformer $= \frac{\text{output power}}{\text{input power}}$ $\qquad \eta = \frac{V_s I_s}{V_p I_p}$

- In a transformer there is no violation of law of conservation of energy.

  Input ac energy = output ac energy (Ideal case)

* **Application of Transformer - Electrical Power Transmission.**

* Electric power is transmitted in ac not dc - In Electrical power transmission tranformer is used in various stages. It works only in ac.

* Energy losses in a transformer.

(i) Copper loss or Joule loss - Due to resistance of primary and secondary coils.

(ii) Eddy current loss or Iron loss.

(iii) Hysterisis loss

(iv) Flux leakage - Because total flux linked with the primary coil is not

(v) Humming Noise- linked with secondary coil

* How can reduce the flux leakage in a transformer.

By winding secondary coil over primary coil and insulated each other.

* Device which is used to step down dc - Resistor

* Device which is used to step up dc - Induction coil

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