7. ALTERNATING CURRENT

1. Mention the expression for instantaneous, peak and rms values of alternating current and voltage.

Expression for instantaneous emf , $v = v_m \sin\omega t$ ---(1)

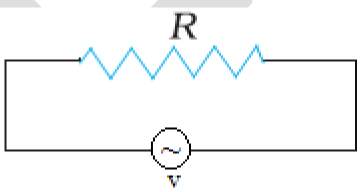
Peak value of induced emf $v_m = NAB\omega$,

Root mean square(rms) or effective current i_{rms} or I. $I = \frac{i_m}{\sqrt{2}} = 0.707\, i_m$

Similarly $v_{rms} = V = \frac{v_m}{\sqrt{2}} = 0.0707 v_m$

2. Derive the expression for current when AC voltage applied to a resistor. What is the phase relation between voltage and current. Represent in phasor diagram.

Consider pure resistor of resistance R connected to sinusoidal AC.



Let $v = v_m \sin\omega t$ ---(1) be the instantaneous voltage.

According to Kirchoff's loop rule,

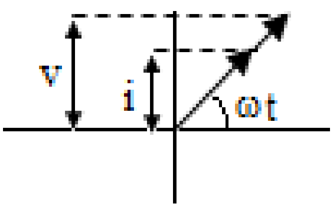
$v_m \sin\omega t = iR$; here 'i' is the AC current.

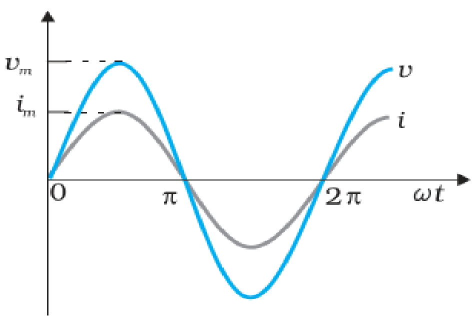
$\therefore\ i = \frac{v_m}{R}\sin\omega t$

$\Rightarrow i = i_m \sin\omega t$ ---(2)

$\Rightarrow i_m = \frac{v_m}{R}$; here i_m is called current amplitude (or peak current)

From (1) and (2), **voltage-current are in phase with each other** and the phasor diagram is as shown.





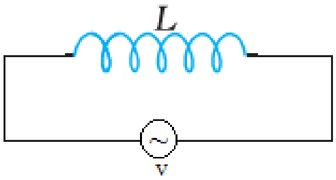
3. **Derive the expression for current when AC voltage applied to a inductor. Mention the expression for inductive reactance.**

Consider an inductor of inductance L is connected across an AC source,

Let $v = v_m \sin\omega t$ --------(1) , here v - the source voltage,

v_m – peak voltage ,

ω - angular frequency of AC.



The self induced emf in the inductor is $\varepsilon = -L\dfrac{di}{dt}$

According to Kirchoff's loop rule, $v - L\dfrac{di}{dt} = 0$

$\Rightarrow v_m \sin\omega t - L\dfrac{di}{dt} = 0$

$\Rightarrow L\dfrac{di}{dt} = v_m \sin\omega t$. This indicates the current in an inductor is a function of time.

$\Rightarrow di = \dfrac{v_m}{L} \sin\omega t \, dt$

To obtain the current at any instant, we integrate the above expression.

i.e $i = \int di = \dfrac{v_m}{L}\int \sin\omega t \, dt \qquad \Rightarrow i = \dfrac{v_m}{L}\left[\dfrac{-\cos\omega t}{\omega} + \text{constant}\right]$

$\Rightarrow i = \dfrac{v_m}{L\omega}[-\cos\omega t]$ [because, we can show the integration constant over a cycle is zero]

If we take $\dfrac{v_m}{L\omega} = i_m$, the amplitude of the current, then $i = i_m[-\cos\omega t]$

$i = i_m \sin\left(\omega t - \dfrac{\pi}{2}\right)$ ------(2)

Inductive reactance is given by $X_L = \omega L = 2\pi\upsilon L$

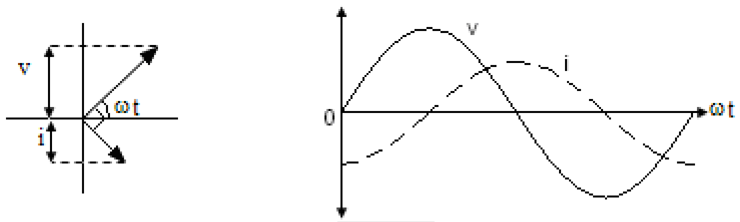
The SI unit of X_L is ohm(Ω)

Definition of $X_L = \dfrac{v_{rms}}{i_{rms}} = \dfrac{\text{RMS value of voltage across inductor}}{\text{RMS value of current through inductor}}$

4. What is the phase relation between voltage and current. Represent in phasor diagram.

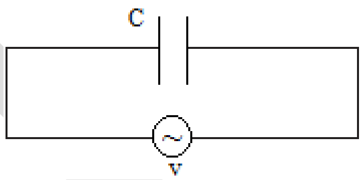
The current is lagging the applied emf by an angle $\frac{\pi}{2}$.

The phasor diagram is as shown.



5. Derive the expression for current when AC voltage applied to a capacitor. mention the expression for capacitive reactance.

Consider a capacitor of capacitance C is connected across an AC source,



Let $v = v_m \sin\omega t$ --------(1) , here v - the source voltage,

v_m – peak voltage ,

ω - angular frequency of AC.

The p.d acorss the capacitor at any instant of time is $v = \frac{q}{C}$.

According to Kirchoff's loop rule, $v_m \sin\omega t - \frac{q}{C} = 0$

$\Rightarrow v_m \sin\omega t = \frac{q}{C}$

$\Rightarrow q = v_m C \sin\omega t$

$\therefore$ Instantaneous current, $i = \frac{dq}{dt} = v_m C \frac{d(\sin\omega t)}{dt}$

$\Rightarrow i = v_m C(\omega \cos\omega t)$

Let $\omega v_m C = i_m$ be the amplitude of the current, then $i = i_m \cos\omega t$

$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$ --------(2)

CAPACITIVE REACTANCE(X_C)

Capacitive reactance is given by. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

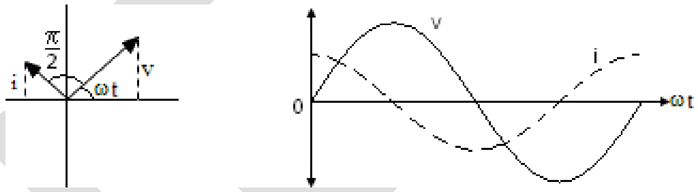
The SI unit of X_C is ohm(Ω)

Definition X_C $X_C = \frac{v_{rms}}{i_{rms}} = \frac{\text{RMS value of voltage across capacitor}}{\text{RMS value of current through capacitor}}$

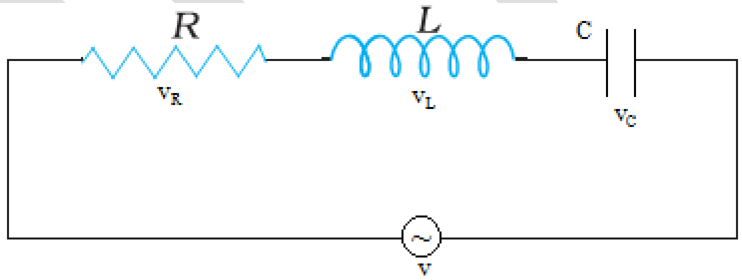
6. What is the phase relation between voltage and current. Represent in phasor diagram.

The current in the circuit is leading the voltage by an angle $\frac{\pi}{2}$.

The phasor diagram is as shown.



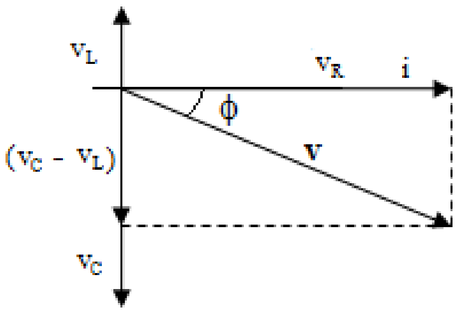
7. Derive the expression for impedance , current and phase angle in a series LCR circuit using phasor diagram.



Consider a series LCR circuit connected to an AC source

$v = v_m \sin\omega t$ ----(1)

Let $i = i_m \sin(\omega t + \phi)$ ----(2) be the instantaneous current through the circuit and ϕ is the phase difference between the appllied voltage and the current.



Voltage equation at any instant

$\vec{v}_R + \vec{v}_L + \vec{v}_C = \vec{v}$

Its magnitude of v is the phasor sum of v_R, v_L and v_C.

And the phasor diagram for the circuit is as shown below.

The symbols in the diagram are having usual meaning.

We know, $v_{Rm} = i_m R$, $v_{Cm} = I_m X_C$ and $v_{Lm} = i_m X_L$

From the diagram, $v_m^2 = v_{Rm}^2 + (v_{Cm} - V_{Lm})^2$

$$\Rightarrow v_m^2 = (i_m R)^2 + (i_m X_C - i_m X_L)^2$$

$$\Rightarrow v_m^2 = i_m^2\left[R^2 + (X_C - X_L)^2\right]$$

$$\Rightarrow i_m^2 = \frac{v_m^2}{R^2 + (X_C - X_L)^2}$$

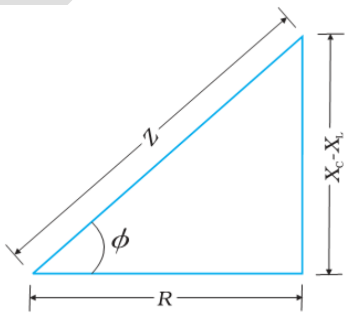
$$\Rightarrow i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

Here, $\sqrt{R^2 + (X_C - X_L)^2}$ is analogous to resistance in DC called impedance, Z.

$$\therefore\ Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\Rightarrow i_m = \frac{v_m}{Z}.$$

If ϕ is the phase angle between i and v,



Impedance Diagram

$$\tan\phi = \frac{v_{Cm} - v_{Lm}}{v_{Rm}}$$

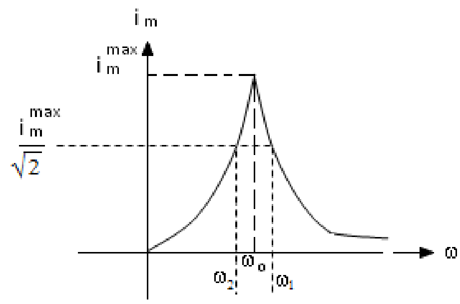
$$\therefore\ \tan\phi = \frac{X_C - X_L}{R}$$

$$\Rightarrow \phi = \tan^{-1}\left[\frac{X_C - X_L}{R}\right]$$

8. What is electrical resonance ? Derive the expression for resonant frequency.

Series LCR circuit is said to be in resonance when current through the circuit is maximum

In an series LCR circuit current amplitude is given by



$$i_m = \frac{v_m}{Z}$$

$$\therefore\quad i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}.$$

Where $X_C = \dfrac{1}{\omega C}$ and $X_L = \omega L$

If frequency is varied, at particular angular frequency ω_o the condition $X_C = X_L$ is achieved, this condition is called resonance, $\frac{1}{\omega_o C} = \omega_o L$

$$\therefore \omega_o = \frac{1}{\sqrt{LC}}$$

$$2\pi\nu_o = \frac{1}{\sqrt{LC}}$$

$\nu_o = \frac{1}{2\pi\sqrt{LC}}$ is called resonant frequency

9. Mention the expressions for bandwidth and sharpness (quality factor)

Let ω_1 and ω_2 are two applied frequencies for which the current amplitude is $\frac{1}{\sqrt{2}}$ times the maximum value, Then $\boldsymbol{\omega_1 - \omega_2 = 2\Delta\omega}$ is called bandwidth of the circuit.

Also, Band width = $2\Delta\omega = \frac{R}{L}$

Sharpness of resonance is denoted by quality factor(Q-factor),

$$Q = \frac{\omega_o}{2\Delta\omega} = \frac{\text{resonacne frequency}}{\text{band width}}$$

Also $Q = \frac{\omega_o L}{R}$ & $Q = \frac{1}{\omega_o CR}$

10. Mention the expression for power and power factor in ac circuit. What are their values in the case of resistive, inductive and capacitive circuit.

In an series LCR circuit, average power over a full cycle of AC, $p = \frac{v_m i_m}{2}\cos\phi$

$$\Rightarrow p = \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi$$

$$\Rightarrow p = VI\cos\phi$$

$$\Rightarrow p = I^2 Z\cos\phi$$

Where V and I are RMS values of voltage and current and the term **cosϕ** is called power factor.

Power factor is given by $\boldsymbol{\cos\phi = \frac{R}{Z}}$

In purely resistive circuit $\phi=0$.

Power factor, $\boldsymbol{\cos\phi = 0}$

Power $p = v\,i = i^2R$.

In purely inductive circuit or capacitive circuit $\phi = \frac{\pi}{2}$.

Power factor, **$\cos\phi = 0$**

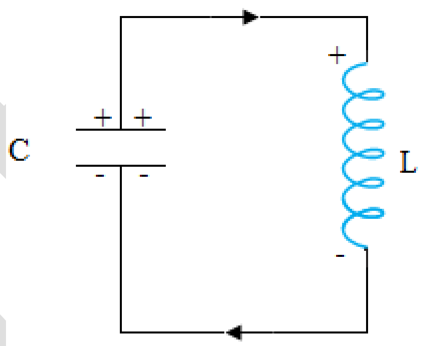
Power p=0.

11. **What is meant by wattless current.**

The AC current through pure L and C circuit is called wattles current.

12. **Explain LC oscillations qualitatively and mention expressions for frequency of LC oscillations and total energy of LC circuit.**

Let a capacitor be charged q_m (at $t = 0$) and connected to an inductor as shown in the figure. The charge oscillates from one plate of capacitor to another plate through the inductor.This results in electric oscillations called LC LC oscillation



The moment the circuit is completed, the charge on the capacitor starts decreasing, giving rise to current in the circuit. As q decreases , energy stored in the capacitor decreases and the energy transferred from capacitor to inductor.

Once the capacitor is fully discharged,magnetic field begin to decrease produces an opposing emf. Now capacitor is begin to but in opposite direction(acc to Lenz's law) .Charge oscillates simple harmonically with natural frequency $\omega_o = \frac{1}{\sqrt{LC}}$.

Charge varies sinusoidally with time as $q = q_m \cos(\omega_o t)$

And current varies sinusoidally with time as $i = \omega_o q_m \sin\omega t = i_m \sin\omega t$

At time t = 0, electrical energy stored in the capacitor, $U_E = \frac{1}{2}\frac{q_m^2}{C}$

and magnetic energy in the inductor $U_B=0$

Similarly ,when $U_B = \frac{1}{2}Li_m^2$, then $U_E = 0$.

$\therefore$ Total energy of the LC circuit at any instant of time, $U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \frac{1}{2}\frac{q_m^2}{C} = \frac{1}{2}Li_m^2$

13. Write a note on transformer with special reference to principle, construction and working.

It is a device used to increase or decrease the AC. It works on the principle of mutual induction.

A transformer consists of two sets of coils, insulated from each other. They are wound on a soft-iron core. One of the coil is called primary (input) with N_P turns and the other is called secondary (output) with N_S turns.

When an alternating voltage v_p is applied to the primary, the induced magnetic flux is linked to the secondary through the core. So an voltage v_s is induced in secondary

If $N_s > N_p$ then transformer is called Step up transformer; where $v_s > v_p$,

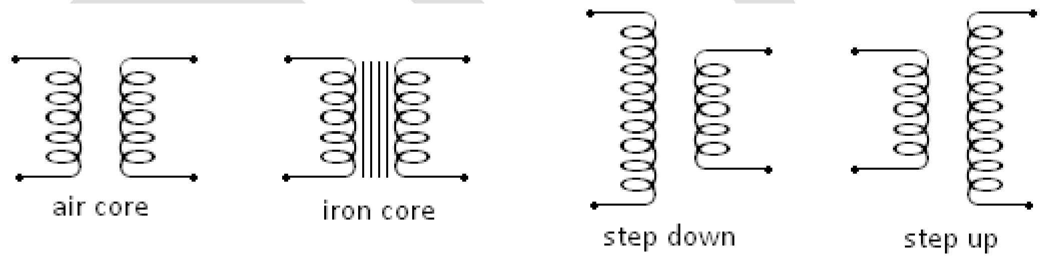
and if $N_p > N_s$, then the transformer is called Step down transformer; where $v_s < v_p$.

If the transformer is ideal, $p_{in} = p_{out}$

i.e $i_p v_p = i_s v_s$.

$$\Rightarrow \frac{i_p}{i_s} = \frac{v_s}{v_p} = \frac{N_s}{N_p}$$

ELECTRICAL SYMBOLS OF TRANSFORMERS;



14. Mention the sources of energy losses in transformer. How they can be minimised?

(i) Magnetic flux leakage - It can be reduced by winding the primary and secondary coils one over the another.

(ii) Ohmic loss due to the resistance of the windings (wires) - It can be reduced by using thick copper wires.

(iii) Eddy current loss- It can be minimised by laminating and insulating the core of the transformer.

(iv) Hysteresis loss - It can be minimised by using material(soft iron) which has a low hysteresis loss.

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