#420012

Topic: Gas Laws

Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made

Temperature	Pressure thermometer A	Pressure thermometer B
Triple-point of water	$1.250 \times 10^5 Pa$	$0.200 \times 10^5 Pa$
Normal melting point of sulphur	$1.797 \times 10^5 Pa$	$0.287 \times 10^5 Pa$

(a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B ?

(b) What do you think is the reason behind the slight difference in answers of thermometers A and B ? (The thermometers are not faulty) What further procedure is needed in th experiment to reduce the discrepancy between the two readings ?

Solution

(a)

Triple point of water, $T = 273.16K$.

At this temperature, pressure in thermometer $A, P_A = 1.25 \times 10^5\ Pa$

Let T_1 be the normal melting point of sulphur.

At this temperature, pressure in thermometer $A, P_1 = 1.797 \times 10^5\ Pa$

According to Charles law, we have the relation:

$$P_A/T = P_1/T_1$$

$$T_1 = \frac{1.797 \times 10^5 \times 273.16}{1.25 \times 10^5} = 392.69\ K$$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer A is 392.69 K.

(b)

At triple point 273.16 K, the pressure in thermometer B, $P_B = 0.2 \times 10^5\ Pa$

At temperature T_1, the pressure in thermometer B, $P_2 = 0.287 \times 10^5\ Pa$

According to Charles law, we can write the relation:

$$P_B/T = P_1/T_1$$

$$0.2 \times 10^5/273.16 = 0.287 \times 10^5/T_1$$

$$T_1 = (\frac{0.287 \times 10^5}{0.2 \times 10^5}) \times 237.16 = 391.98\ K$$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer B is 391.98 K.

#420323

Topic: Introduction to Kinetic Theory

Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic), and t third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?

Solution

All the three vessels have the same capacity, they have the same volume. Hence, each gas has the same pressure, volume, and temperature.

According to Avogadro's law, the three vessels will contain an equal number of the respective molecules. This number is equal to Avogadro's number, $N = 6.023 \times 10^{23}$.

The root mean square speed (V_{rms}) of a gas of mass m, and temperature T, is given by the relation:

$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

where,

k is Boltzmann constant

For the given gases, k and T are constants.

Hence, V_{rms} depends only on the mass of the atoms, i.e., $V_{rms} \propto (1/m)^{1/2}$.

Therefore, the root mean square speed of the molecules in the three cases is not the same. Among neon, chlorine, and uranium hexafluoride, the mass of neon is the smallest. Hence, neon has the largest root mean square speed among the given gases.

#420362

Topic: Mean Free Path

Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17 ^oC. Take the radius of a nitrogen molecule to be roughly 1.0 A. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of N_2 = 28.0 u)

Solution

Mean free path $= 1.11 \times 10^{-7} m$

Collision frequency $= 4.58 \times 10^{9} s^{-1}$

Successive collision time $\simeq 500\times$ (Collision time)

Pressure inside the cylinder containing nitrogen, $P = 2.0\, atm = 2.026 \times 10^{5} Pa$

Temperature inside the cylinder, $T = 17^{o}C = 290K$

Radius of a nitrogen molecule, $r = 1.0\mathring{A} = 1 \times 10^{10} m$

Diameter, $d = 2 \times 1 \times 10^{10} = 2 \times 10^{10} m$

Molecular mass of nitrogen, $M = 28.0g = 28 \times 10^{-3} kg$

The root mean square speed of nitrogen is given by the relation:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where,

R is the universal gas constant $= 8.314\, J\, mol^{-1} K^{-1}$

$$\therefore v_{rms} = \frac{3 \times 8.314 \times 290}{28 \times 10^{-3}} = 508.26\, m/s$$

The mean free path (l) is given by relation:

$$l = \frac{kT}{\sqrt{2} \times d^2 \times P}$$

Where,

k is the boltzmann constant $= 1.38 \times 10^{-23} kg\, m^2 s^{-2} K^{-1}$

$$\therefore l = \frac{1.38 \times 10^{-23} \times 290}{\sqrt{2} \times 3.14 \times (2 \times 10^{-10})^2 \times 2.026 \times 10^5} = 1.11 \times 10^{-7} m$$

Collision frequency $= \dfrac{v_{rms}}{l} = \dfrac{508.26}{1.11 \times 10^{-7}} = 4.58 \times 10^{9} s^{-1}$

Collision time is given as:

$$T = \frac{d}{v_{rms}} = \frac{2 \times 10^{-10}}{508.26} = 3.93 \times 10^{-13} s$$

Time taken between successive collisions:

$$T' = \frac{l}{v_{rms}} = \frac{1.11 \times 10^{-7}}{508.26} = 2.18 \times 10^{-10}$$

$$\therefore \frac{T'}{T} = \frac{2.18 \times 10^{-10}}{3.93 \times 10^{-13}} = 500$$

Hence, the time taken between successive collisions is 500 times the time taken for a collision.