WAVES

Important Points:

1. **Wave Motion:**

   It is a form of disturbance that travels through the medium due to repeated periodic motion of the particles of the medium about their mean positions.

2.Longitudinal Waves:

The particles of the medium vibrate parallel to the direction of propagation of the waves.

3.Transverse Wave:

The particles of the medium vibrate perpendicular to the direction of propagation of wave.

4.Phase(ϕ):

It is the state of vibration of a particle that gives position and direction of motion with respect to time.

5.Wave Length(λ):

It is the distance between two successive particles which are in the same phase of vibration.

6.The general equation of progressive wave is $y = A\sin(\omega t \pm \phi)$ (or) $y = A\cos(\omega t \pm \phi)$

7. Stationary wave equation:

   $y = 2A\cos(\omega t)\sin(kx)$ (or) $y = 2A\sin(\omega t)\cos(kx)$

8. **Vibrating String :**

   a) The waves formed in a string under tension are transverse stationary and polarized waves.

   b) Always nodes are formed at fixed ends and antinodes at plucked points and free ends.

9.Resonance:

When the external periodic frequency is equal to the natural frequency of the body, the amplitude of vibration increases rapidly (Theoretically infinity for no damping). This phenomenon is called resonance. Such vibrations are called resonant vibrations (or) symphethetic vibration.

10. Newton - Laplace formula for speed of sound in gases $V_g = \sqrt{\frac{\gamma P}{d}}$

Where p= pressure of gas

d = density of gas

γ = Adiabatic constant

11. In open pipe, the general formula for frequency-

$n = \frac{pV}{2l}$ Where p = 1, 2, 3..... Number of harmonics

12. In closed pipe the general formula for frequency $n = (2p+1)\frac{V}{4l}$ where p = 0, 1, 2, 3.....

13. Beats:

When two sounds of slightly different frequencies superimpose, the resultant sound consists of alternate waxing and waxing. This phenomenon is called beats.

14. Doppler Effect:

The apparent change in frequency due to relative motion between the source and the listener is called Doppler Effect.

15. Apparent Frequency of sound

$$n^1 = \left(\frac{V \pm V_0}{V \pm V_s}\right) n$$

Where n is actual frequency and V is velocity of sound

+ V_0 (velocity of listener) is used if listener approaches the source

- V_0 is used if listener moves away from the source

+ V_S (velocity of source) is used if the source moves away from the listener

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16. Limitations of Doppler Effect:

a. Doppler Effect fails when the velocity of source or the listener approaches the velocity of sound.

b. Doppler Effect fails when source and listener moves with same velocity in the same direction.

Very Short Answer Questions

1. What does a wave represent?

A. a) Waves represents transport of energy and the pattern of disturbance.

b) The disturbance is handed over from one particle to other particle without any net transport of the medium.

2. Distinguish between transverse and longitudinal waves?

A.

Transverse Waves	Longitudinal Waves
1. Particles of the medium vibrate in a direction perpendicular to the propagation of the wave.	1. Particles of the medium vibrate in a direction parallel to the propagation of the wave.
2. Alternately crests and troughs are formed.	2. Alternately compressions and rarefactions are formed.

3. What are the parameters used to describe a progressive harmonic wave?

A. 1. Frequency, 2. Time Period, 3. Wave Length 4. Wave Velocity and 5. Phase.

4. Establish the relation between the wave velocity (v), frequency (υ), and the wave length (λ) of a progressive wave.

A. If T is the period of vibration then its frequency is $\upsilon = 1/T$

The wave travels a distance of one wavelength (λ) in the time T

The distance travelled in 1 second = $\frac{\lambda}{T}$

The distance travelled by the wave in 1 sec is called wave velocity -

$$V = \frac{\lambda}{T} \Rightarrow V = \upsilon\lambda \left(\frac{1}{T} = \upsilon\right)$$

5. Using dimensional analysis, obtain an expression for the speed of transverse waves in a stretched string?

A. For a transverse wave in a stretched string, speed is proportional to the tension (Restoring force) and inversely proportional to the linear mass density (μ).

Dimensions of tension = $\left[MLT^{-2}\right]$

Dimensions of linear mass density $= \frac{Mass}{Length} = \left[ML^{-1}\right]$

Speed of the wave (v) = $C\sqrt{\frac{T}{M}}$

Where the constant C cannot be determined by dimensional analysis. The constant C is chosen as one -

$$\therefore v = \sqrt{T/M}$$

6. Using dimensional Analysis obtain an expression for the speed of sound waves in a medium?

A. Let $V \alpha\ d^a E^b$ where d is density of medium and E is its elastic constant

$$\Rightarrow V = Kd^a E^b$$

On writing dimensions

$$\Rightarrow \left[M^0 L^1 T^{-1}\right] = [ML^{-3}]^a\ [ML^{-1}T^{-2}]^b$$

$$\Rightarrow [M^0 L^1 T^{-1}] = [M^{a+b} L^{-3a-b} T^{-2b}]$$

$$\Rightarrow a+b=0,\ -3a-b=1 \quad \text{and} \quad -2b=-1$$

$$b = \tfrac{1}{2} \quad \text{Also } a=-b=-1/2$$

$$\Rightarrow V = Kd^{-1/2}E^{1/2} \Rightarrow V = K\sqrt{\frac{E}{d}}$$

Value of 'K' comes out to be '1' $\Rightarrow V = \sqrt{\frac{E}{d}}$

7. What is the principle of superposition of waves?

A. Principle of superposition:

The principle of superposition of waves states that when two or more waves are simultaneously interfere, the resultant displacement of any particle is equal to the algebraic sum of the displacements of all the waves. If $y_1 = f_1(x-vt); y_2 = f_2(x-vt)..... \ y_n = f_n(x-vt)$ Then

$$y = f_1(x-vt) + f_2(x-vt) + ... + f_n(x-vt) = \sum_{i=1}^{n} f_i(x-vt)$$

8. Under what conditions a wave will be reflected?

A. i) Progressive waves get reflected if the density and rigidity of the medium changes at any point.

ii) Waves get reflected partially when they refract into another medium.

9. What is the phase difference between the incident and reflected waves when the wave is reflected by a rigid boundary?

A. Phase difference between the incident and reflected waves when the wave is reflected by a rigid boundary is π.

10. What is stationary or standing wave?

A. A stationary wave is formed, when two progressive waves of same amplitude and frequency moving in opposite directions along the same straight line superpose each other.

11. What do you understand by the terms 'Node' and 'Antinodes'?

A. Nodes:

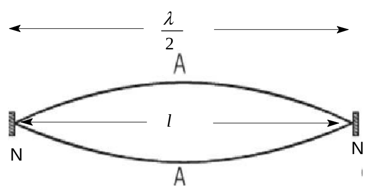
In a stationary wave the points where the amplitude is zero are called nodes.

Antinodes:

In a stationary wave the points where the amplitude is largest are called anti nodes.

12. What is the distance between a Node and an Antinode in a stationary wave?

A. The distance between a node and the adjacent antinode is equal to $\left(\frac{\lambda}{4}\right)$



13. What do you understand by 'natural frequency' or normal mode of vibration?

A. Natural frequency:

If an impulse is given to a body and it is left free to itself, it vibrates with a finite frequency. These vibrations are called natural vibration.

Ex: The air column in a closed (or) open pipe can be characterized by a set of natural frequencies (or) normal mode of oscillations.

14. What are harmonics?

A. Harmonics:

Harmonics are the notes of frequencies which are integral multiple of the fundamental frequencies.

15. A string is stretched between two rigid supports. What frequencies of vibration are possible in such a string?

A. When a string is stretched between two rigid supports, the fundamental frequency $\upsilon_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ where 'T' is tension in the string, 'μ' be linear density of the string.

In general, for 'p' loops $\upsilon = \frac{p}{2l}\sqrt{\frac{T}{\mu}}$

If p = 1 loop $\Rightarrow \upsilon_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow$ fundamental (or) 1st harmonic

p = 2 loops $\Rightarrow \upsilon_1 = \frac{2}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow$ 2nd Harmonic (or) 1st overtone

p = 3 loops $\Rightarrow \upsilon_2 = \frac{3}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow$ 3rd Harmonic (or) 2nd overtone and so on.

16. The air column in a long tube, closed at one end, is set in vibration. What harmonics are possible in the vibrating air column?

A. In the case of closed pipe only odd harmonics forms even harmonics are absent in a closed pipe.

First mode of vibration is 1st harmonic $\upsilon_1 = \frac{V}{4l}$

Second mode of vibration is 3rd Harmonic (or) 1st overtone $\upsilon_2 = 3\upsilon_1 = \frac{3V}{4l}$

Third mode of vibration is 5th Harmonic (or) 2nd overtone $\upsilon_3 = 5\upsilon_1 = \frac{5V}{4l}$

17. If the air column in a tube open at both ends is set in vibration, what harmonics are possible?

A. If the air column in a tube open at both ends is set in vibration**,** all harmonics are formed. Hence the possible frequencies are $\frac{v}{2l}, \frac{2v}{2l}, \frac{3v}{2l}, \frac{4v}{2l}$..... where v is velocity of sound in air column and *l* is the length of the tube.

18. What are "Beats"?

A. Beats:

The phenomenon of waxing and waning of sound due to the interference of two sound waves of slightly different frequencies travelling in the same direction is called **Beats**.

19. Write down an expression for beat frequency and explain the terms therein?

A. Expression:

Number of beats per second $\Delta\upsilon = \upsilon_1 \sim \upsilon_2$.Where n_1 and n_2 are frequencies of two bodies.

20. What is "Doppler Effect"? Given an example?

A. "When a source of sound (or) an observer or both are in motion relative to each other there is an apparent change in frequency of sound as heard by the observer. This phenomenon is called the **Doppler's Effect**"

Ex: The frequency of the whistle of a fast moving train decreases as it recedes away from us where as it increases and appears to be higher pitched as it approaches us.

21. Write down an expression for the observed frequency when both source and observer are moving relative to each other in the same direction?

A. The apparent frequency $\upsilon^1 = \left(\frac{V - V_0}{V - V_s}\right)\upsilon$

$V \rightarrow$ Velocity of sound

$V_0 \rightarrow$ Velocity of observer

$V_s \rightarrow$ Velocity of source

$\upsilon \rightarrow$ Actual frequency of sound

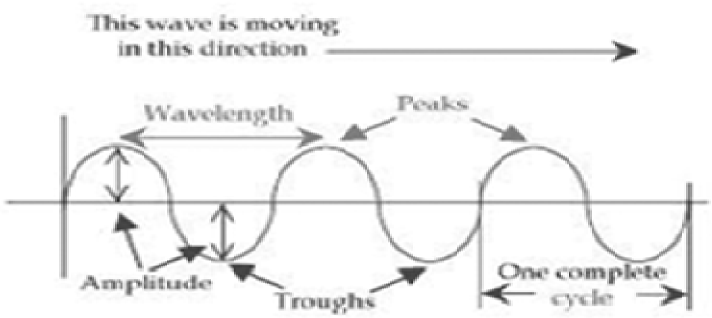
Short Answer Questions

1. What are Transverse Waves? Give illustrative examples of such waves?

A. Transverse Waves:

The wave in which particles of the medium vibrate perpendicular to the direction of propagation of the wave is called a transverse wave.

Example: Let a string tied to a rigid wall and a continuous periodic up and down jerk is given to the other end of the string. The resulting disturbance on the string is then a periodic wave. As the wave passes through the sting, the elements of the string oscillate about their equilibrium position. These oscillations are normal to the direction of wave motion in the string.

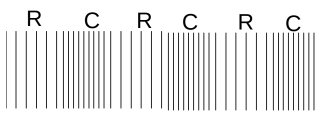


2. What are Longitudinal Waves? Explain with an example?

A. **Longitudinal Waves:**

The wave in which the particles of the medium vibrate parallel to the direction of propagation of the wave is called a longitudinal wave.

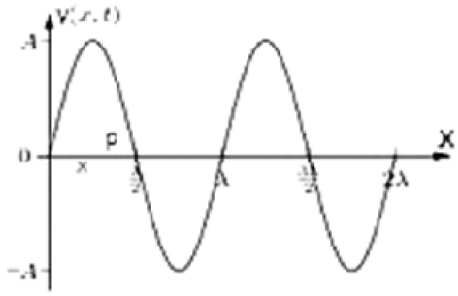
Ex: Sound Waves.



Explanation: Let a light spring is held horizontally and a light push is given to it in the same direction. The spring gets compressed sending a pulse of pressure along the spring. The compressions and rarefactions are formed in the string. These waves can travel in solids, liquids and gases.

3. Write an expression for progressive harmonic wave and explain the various parameters used in the expression?

A. Consider a particle ‘p’ at a distance of ‘x’ from the origin let its amplitude is ‘A’ and angular frequency is 'ω'.



If 'ϕ' is the phase difference between the point ‘p’ and origin ‘O’, then equation of progressive wave is given by $y = A\sin(\omega t - \phi)$

(Or)

$y(x,t) = A\sin(\omega t - kx)$ Along +ve x-axis

If the wave travels in -ve x-direction then

$$y(x,t) = A\sin(\omega t + kx)$$

In the above equations

$y(x,t) \rightarrow$ Displacement of the wave as a function of position and time

A $\rightarrow$ Amplitude

(Maximum displacement of the vibrating particle of the medium from its mean position)

K $\rightarrow$ angular wave number $= \frac{2\pi}{\lambda}$

$Sin(wt - Kx) \rightarrow$ Oscillating term

$(wt - Kx) \rightarrow$ Phase (which describes direction and position of a wave at any given point)

x $\rightarrow$ Position

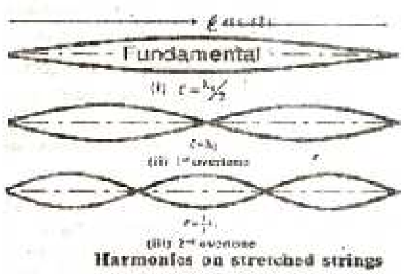
$w \rightarrow$ Angular frequency $\left(2\pi/T \ (or) \ 2\pi n\right)$

$t \rightarrow$ Time

4. Explain the modes of vibration of a stretched string with examples?

A. Let a string of length 'l' stretched between two fixed supports.

1. When the string is plucked at the middle of the string it vibrates in a single loop



$\therefore$ Length of the string $l = \lambda / 2 \ or \ \lambda = 2l.$

Frequency of vibration $\upsilon_1 = \dfrac{v}{\lambda} = \dfrac{v}{2l}$

Here υ_1 is called fundamental frequency or first harmonic.

2. When the string is plucked at a point at the distance $\dfrac{l}{4}$ from fixed end, it then vibrates with two loops.

Frequency of vibration, $\upsilon_2 = \dfrac{2}{2l} v = \dfrac{v}{l} = 2\upsilon_1$

Here υ_2 is called second harmonic.

3. Similarly if three loops are formed $\upsilon_3 = \dfrac{3}{2l} v = 3\upsilon_1$

Here υ_3 is called Second Overtone or Third Harmonic.

In general $\upsilon_n = \dfrac{Pv}{2l}$ Where P is the number of loops .is P then,

Hence the Harmonics are in the ratio 1: 2: 3.... n

5. Explain the modes of vibration of an air column in an open pipe?

A. Open Pipe:

An open pipe means both ends are open. At the open ends of the pipe always antinodes are formed. Let ‘v’ be the velocity of sound.

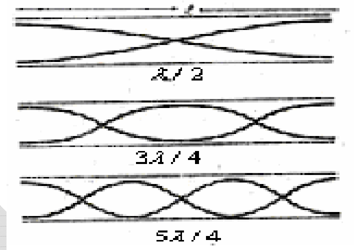
1. When the air column vibrates with two antinodes and one node as shown in figure .Since the distance between two successive antinodes is

Length of the pipe, $l = \dfrac{\lambda_1}{2}$ or $\lambda_1 = 2l$

Frequency of vibration $\upsilon_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2l}$

Where υ_1 is called fundamental frequency or first harmonic.

2. In the second mode of vibration, the air column vibrates with two nodes and three antinodes as shown in the figure.



Length of the pipe, $l = \dfrac{2\lambda_2}{2} or \lambda_2 = l$

Frequency of vibration,

$$\upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{l} = 2\upsilon_1$$

Where υ_2 is called second harmonic or first overtone.

3. In the third mode of vibration the air column vibrates with three nodes and four antinodes as shown in the figure.

Length of the pipe, $l = \dfrac{3\lambda_3}{2} or \lambda_3 = \dfrac{2l}{3}$

Frequency of vibration $\upsilon_3 = \dfrac{v}{\lambda_3} = \dfrac{3v}{2l} = 3\upsilon_1$

Where υ_3 is called third harmonic or second overtone.

Hence for an open pipe harmonics are in the ratio 1: 2: 3: ….

6. What do you understand by 'Resonance'? How would you use resonance to determine the velocity of sound in air?

A. **Resonance:**

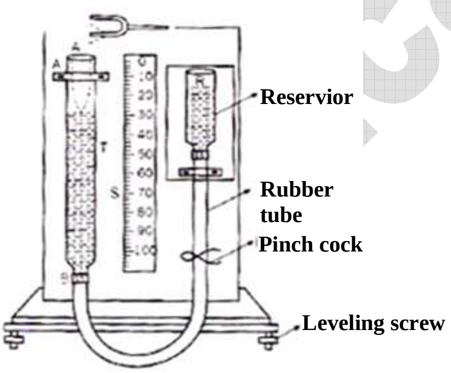
If the driving frequency (applied force frequency) is equal to any natural frequency of the given body, then the body will vibrate with larger amplitude at that frequency. This phenomenon is called 'Resonance'.

At Resonant frequency, the object vibrates with maximum Amplitude.

Ex: - In the case of organ pipes, if the vibrating tuning fork frequency is equal to natural frequency of vibrating air column, resonance occurs.

Determination of velocity of sound in air:

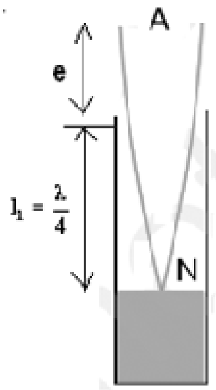
Let us consider a long vertical glass tube (T), of 1m long and 1cm diameter. The lower end of the glass tube is connected to a reservoir 'R' of water through a rubber pipe as shown in figure. The water level in the tube can be adjusted by the adjustable screws attached with the reservoir. The entire apparatus is fixed to a stand arrangement with proper adjustments.



Procedure:

1) By keeping a vibrating tuning fork over the open end of the tube and by lowering the water level, the first resonance is observed at a length 'l_1'.

$$\Rightarrow l_1 + e = \frac{\lambda}{4} = \frac{1}{4}\left(\frac{V}{\upsilon}\right) \text{.............} (i)$$



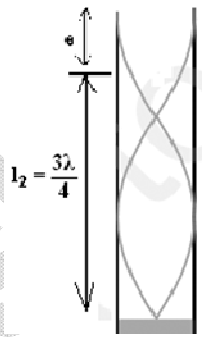
(As we know $V = \upsilon\lambda$)

Here 'e' is end-correction of the pipe.

End Correction: The antinodes do not occur precisely at the open ends of the pipes, due to the pressure from outside the pipe. Their position depends on the diameter of the pipe. Due to this the vibrating length of the air column is extended slightly beyond the open end. This correction 'e' to the vibrating length to be applied at the open end is called end correction

End correction can be empirically written as e = 0.3d, where'd' is the diameter of the pipe.

2) The water level is lowered further and observed the 2nd resonance at a level, with the same vibrating tanning fork. $(l_2 > l_1) \Rightarrow l_2 + e = \frac{3\lambda}{4} = \frac{3}{4}\left(\frac{V}{\upsilon}\right)$.......... (ii)



Subtracting equation (i) from (ii)

$$\Rightarrow l_2 - l_1 = \frac{V}{\upsilon}\left(\frac{3}{4} - \frac{1}{4}\right) \Rightarrow l_2 - l_1 = \frac{V}{2\upsilon}$$

$$\therefore \boxed{V = 2\upsilon(l_2 - l_1)}$$

7. What are standing waves? Explain how standing waves may be formed in a stretched string?

A. Standing Waves:

When two progressive waves with same amplitude and frequency moving in opposite directions along the same straight line superpose each other a standing wave is formed.

Standing waves in a stretched string:

Consider a wave travelling in positive x-direction and its reflected wave of same amplitude and frequency in the negative X-direction.

$y_1(x,t) = a\ \sin(kx - \omega t)$ And $y_2(x,t) = a\ \sin(kx + \omega t)$

The resulting wave in the string is given by $y(x,t) = a\ \sin(kx - \omega t) + a\ \sin(kx + \omega t)$ (or)

$y(x,t) = 2a\ \sin kx \cos \omega t.$

Here A =2a sin k*x* is called the amplitude of the resultant wave.

Nodes:

These are the points where amplitude is zero.

sin k*x* = 0 (or) $kx = n\pi$

Where n = 0, 1, 2, 3...

(Or) $x = \frac{n\lambda}{2}.$

Antinodes:

These are the points where amplitude is maximum $|\sin kx| = 1$ (or) $kx = \left(n + \frac{1}{2}\right)\pi$

Where n = 0, 1, 2, 3........ (Or) $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}.$

For a stretched string of length L, the ends x = 0, and *x* = L are the positions of nodes.

$L = \frac{n\lambda}{2}$ or $\lambda = \frac{2L}{n}.$ Where n = 1, 2, 3........

The harmonics formed are $\upsilon = \frac{nv}{2L}$ where n = 1, 2, 3.....

The fundamental mode of vibration is given when n =1 i.e. the fundamental frequency is given by

$\upsilon = \frac{v}{2L}$. Where v is the speed of the wave

8. Describe a procedure for measuring the velocity of sound in a stretched string?

A. The velocity of a longitudinal wave in any medium is given by $v = \sqrt{\frac{B}{\rho}}$ where B is the bulk modulus of elasticity and ρ is the density of the medium.

In the case of solids velocity of sound is given by $v = \sqrt{\frac{Y}{\rho}}$ where y is the Young's modulus.

The Young's modulus of elasticity of the stretched string is determined from the formula $Y = \frac{Fl}{Ae}$

Where F = load on the wire

L = length of the string

A = Area of cross section

e = elongation

By knowing the density of the material of the wire, velocity of a sound wave can be known.

9. Explain, using suitable diagrams, the formation of standing wave in a closed pipe. How may this be used to determine the frequency of a source of sound?

A. Closed Pipe:

The stationary wave equation is given by $y(x,t) = 2a\ \sin kx\ \cos\omega t$ where $2a\ \sin kx$ is the amplitude of the resultant standing wave.

Nodes:

These are the positions of zero amplitude sin kx = 0 (or) kx = $n\pi$ where n = 0, 1, 2, 3.....

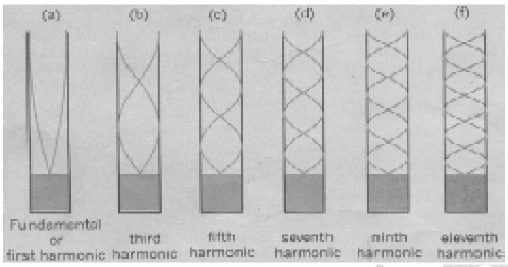
As $x = \frac{n\lambda}{2}$ $\left(\because k = \frac{2\pi}{\lambda}\right)$

Antinodes:

These are the positions of maximum amplitude (or) $kx = \left(n + \frac{1}{2}\right)\pi$

Where n = 0,1,2,3.....

Again $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$



Modes of Vibration:

In a closed pipe one end is closed while the other end is open. At the closed end node and at the open end antinodes are formed.

Taking the closed end as x = 0, and x = L (length of the pipe)

$L = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$ (Or) $\lambda = \frac{2L}{\left(n + \frac{1}{2}\right)}$, where n = 0, 1,2, 3,.......

The natural frequencies are $\upsilon = \left(n + \frac{1}{2}\right)\frac{v}{2L}$ where n = 0, 1, 2, 3...

The fundamental frequency (n = o), $\upsilon_1 = \frac{v}{4L}$

The higher frequencies are odd harmonics $\frac{3v}{4L}, \frac{5v}{4l}$ etc

Hence the harmonics are in the ratio 1: 3: 5...

(b) An open pipe is placed in water as shown and a vibrating fork of unknown frequency 'υ' is kept just above the rim of the pipe. Now the pipe is gradually raised. For certain length of the air column in the pipe, resonance between the fork and air column occurs and a humming

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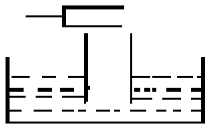
sounds is heard from the pipe. The length of the air column (l) is measured. If 'v' is the velocity of sound air, the fundamental frequency is given by $\upsilon = \frac{v}{4l}$



As the air column is in resonance with the tuning fork, this frequency is equal to that of tuning fork (source).

10. What are 'beats'? When do they occur? Explain their use, if any?

A. Beats:

The phenomenon of waxing and waning of sound due to the superposition of two sound waves of slightly different frequencies travelling in the same direction is called Beats. As the persistence of sound on the human ear is $\frac{1}{10}$ of a second, the frequency difference must not be more than 10,

Uses:

1. To detect the dangerous gases in mines
2. To tune the musical instruments
3. To produce special effect in cinematography.

11. What is 'Doppler effect'? Give illustrative Examples?

A. **Doppler Effect**:

"When sources of sound (or) an observer or both are in motion relative to each other there is an apparent change in frequency of sound as heard by the observer. This phenomenon is called the Doppler's effect".

Examples:

1) SONAR (Sound Navigation and Ranging)

1) It is a device to locate the position and speed of an enemy submarine and also to locate some hidden objects such as an iceberg inside the sea. Here the object (submarine) acts like reflector of sound. The echo is picked up. SONAR system records the frequency difference $(n^1 - n)$ (which is called Doppler shift) are so calibrated as to read the position and speed of an approaching object.

2) Like radar (or) sonar, bats also estimate the velocities of moving objects around them by observing the change in frequency of the ultrasonic waves emitted by them.

3) Doppler Effect is used in police radar systems to measure the speed of motor vehicles.

4) Meteorologists use a similar technique to measure wind speed of potentially damaging storms.

5) Doppler Effect is used to find whether a particular star or a galaxy is approaching the Earth (or) moving away from Earth.

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Long Answer Questions

1. Explain the formation of stationary waves in stretched strings and hence deduce the laws of transverse waves in stretched strings?

A. Consider a string of length '*l*' and linear density 'μ' be fixed between two supports under a tension 'T'. A stationary wave is formed in the string due to the superposition of the waves. At the points (ends) where the string was fixed rigidly nodes are formed. The velocity of transverse vibration in a stretched string is given by $V = \sqrt{\frac{Tension}{Linear\ density}} = \sqrt{\frac{T}{\mu}}$

1. When the string is plucked at the middle of the string, it vibrates in a single loop.

Length of the string $l = \lambda/2 \ \ or \ \ \lambda = 2l$

If υ_1 is the fundamental frequency of string, then

$$\upsilon_1 = \frac{V}{\lambda} = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

2. When the string is plucked at a point at the distance *l*/4 from fixed end, it then vibrates in two loops.

The frequency of vibration, $\upsilon_2 = \frac{2}{2l}\sqrt{\frac{T}{\mu}} = 2\left[\frac{1}{2l}\sqrt{\frac{T}{\mu}}\right] = 2\upsilon_1$

Here υ_2 is called 1st overtone or second harmonic. Similarly if three loops are formed

$$\upsilon_3 = \frac{3}{2l}\sqrt{\frac{T}{\mu}} = 3\upsilon_1$$

Here υ_3 is called 2nd overtone or third harmonic. If the number of loops formed in the string is P then,

Frequency of vibration, $\upsilon_p = \frac{P}{2l}\sqrt{\frac{T}{\mu}}$.

Hence $\upsilon_1 : \upsilon_2 : \upsilon_3 .... \upsilon_p = 1:2:3....P$.

Laws of Transverse Vibrations:

First Law (Law of Lengths): Fundamental frequency (υ) of a stretched vibrating string is inversely proportional to its length (*l*), its tension T and linear density μ being constant.

$\upsilon \ \alpha \ \frac{1}{l}$ When (μ and m are constant.)

Second Law (Law of Tensions): Fundamental frequency (υ) of a stretched vibrating string is proportional to the square root of tension ($\sqrt{T}$) in the string, its length and linear density μ being constant.

$\upsilon \propto \sqrt{T}$ When (*l* and μ are constant).

Third Law (Law of Linear Densities): Fundamental frequency of (υ) of a stretched string vibrating string is inversely proportional to the square root of linear density of wire, its tension and length being constant $\upsilon \propto \frac{1}{\sqrt{\mu}}$ when (T and *l* are constant).

2. **Explain the formation of stationary waves in an air column enclosed in open pipe. Derive the equations for the frequencies of harmonics produced.**

A. **Open pipe:** A tube open at both ends is called open pipe.

a) When a compressional wave (sound wave) is sent through an organ pipe, the wave gets reflected at the ends of the pipe. These incident and reflected waves which are of the same frequency, travelling in opposite directions, were superimposed along the length of the pipe and forms longitudinal stationary waves.

b) At open end of the pipe, air is free to move and hence an anti-node (AN) is produced at this end.

c) In an open pipe antinodes are formed at both the open ends nodes (N) are formed at its middle.

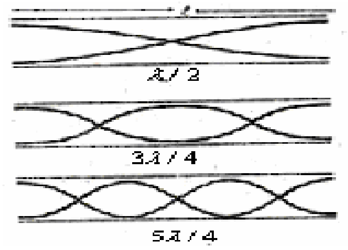
1. In the first mode of vibration, the air column vibrates with two antinodes and one node as shown. Since the distance between two successive antinodes is

Length of the pipe, $l = \frac{\lambda_1}{2}$ or $\lambda_1 = 2l$

Frequency of vibration, $\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{2l}$

Where υ_1 is called fundamental frequency or first harmonic.

2. In the second mode of vibration, the air column vibrates with two nodes and three antinodes as shown in the figure.



Length of the pipe, $l = \frac{2\lambda_2}{2} = \lambda_2 \Rightarrow \lambda_2 = l$

Frequency of vibration,

$$\upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{l} = 2\upsilon_1$$

Where is called second harmonic or first overtone.

3. In the third mode of vibration the air column vibrates with three nodes and four antinodes as shown in the figure.

Length of the pipe, $l = \frac{3\lambda_3}{2} = \lambda_3 = \frac{2l}{3}$

Frequency of vibration is given by, $\upsilon_3 = \frac{v}{\lambda_3} = \frac{3v}{2l} = 3\upsilon_1$

Where υ_3 is called third harmonic or second overtone.

Hence for a open pipe harmonics are in the ratio 1: 2 : 3 : ….

3. How are stationary waves formed in a closed pipe? Explain the various modes of vibrations in a closed pipe and establish the relation between their frequencies?

A. **Closed Pipe**: A tube closed at one end is called a closed pipe.

a) If a vibrating tuning fork is kept at open end of this pipe, compressions and rarefactions start moving inside the pipe they get reflected at the closed end. The reflected wave and incidence waves are interfere with each other produce stationary waves.

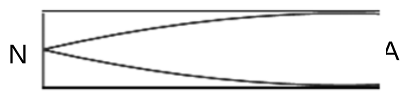
b) At closed end the air particles are in contact with fixed wall and they cannot move freely, hence a node is formed at this end.

c) At open end an Antinode is formed.

1. If the air column is vibrating like as shown in fig. then one anti node and node is formed.

The distance between node and anti node is $\frac{\lambda}{4}$

$\therefore l = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4l$



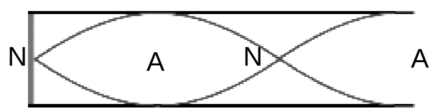
Frequency of vibration $\upsilon_1 = \frac{V}{\lambda}$

$$\therefore \quad \upsilon_1 = \frac{V}{4l} \qquad \text{............ (1)}$$

This is called fundamental frequency (or) first harmonic.

2. If the air column is vibrating with one and half loop then two anti nodes and two nodes are formed.

The distance between node and antinode is $\frac{\lambda}{4}$ and node and node is $\frac{\lambda}{2}$.



$\therefore l = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} = \frac{3\lambda_2}{4}$

$\lambda_2 = \frac{4l}{3}$

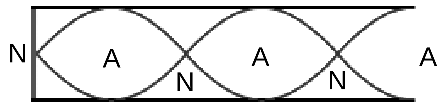
Frequency of vibration $\upsilon_2 = \frac{V}{\lambda} = \frac{3V}{4l}$

$\Rightarrow \upsilon_2 = 3\upsilon_1 \qquad$ .... (2)

This is called 3rd harmonic (or) first over tone.

3. The air column is vibrating with two and half loops then three nodes and three anti nodes are formed.

The distance between node and antinode is $\frac{\lambda}{4}$ and node and node is $\frac{\lambda}{2}$



$$\therefore l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{4} = \frac{5\lambda_3}{4} \Rightarrow \lambda_3 = \frac{4l}{5}$$

Frequency of vibration $\upsilon_3 = \frac{V}{\lambda}$

$$\therefore \quad \upsilon_3 = \frac{5V}{4l}$$

$$\upsilon_3 = 5\upsilon_1 \qquad \text{............ (3)}$$

This is called 5th harmonic (or) 2nd overtone.

From equations 1, 2 and 3

The ratio between fundamental frequency and its overtones is

$$\upsilon_1 : \upsilon_2 : \upsilon_3 : .... = 1:3:5: ....$$

4. What are beats? Obtain an expression for the beat frequency. Where and how beats are made use of?

A. Beats:

The phenomenon of waxing and waning of sound due to the superposition of two sound waves of slightly different frequencies travelling in the same direction is called beats.

Expression:

Consider two progressive waves $S_1 = a \cos \omega_1 t$ and $S_2 = a \cos \omega_2 t$

Using the principle of super position, $S = S_1 + S_2 = a(\cos \omega_1 t + \cos \omega_2 t)$

$$= 2a \cos \frac{(\omega_1 - \omega_2)t}{2} \quad \cos \frac{(\omega_1 + \omega_2)t}{2} \text{ (Or) } S = (2a \ \cos \omega_b t] \ \cos \omega_a t$$

Where $\omega_b = \frac{\omega_1 - \omega_2}{2}$ and $\omega_a = \frac{\omega_1 + \omega_2}{2}$ this is the resultant wave with average angular frequency and amplitude varies with time.

Condition for maximum (waxing)

$Cos\left(\frac{\omega_1 - \omega_2}{2}\right)t = \pm 1$ Or $2\pi\left(\frac{\upsilon_1 - \upsilon_2}{2}\right)t = n\lambda$ where n = 0,1,2,3.....

$$t = \frac{n}{v_1 - v_2}; o, \frac{1}{(\upsilon_1 - \upsilon_2)}, \frac{2}{(\upsilon_1 - \upsilon_2)}; \frac{3}{(\upsilon_1 - \upsilon_2)}...$$

Hence time internal between two maxima is $\left(\frac{1}{\upsilon_1 - \upsilon_2}\right)$

Condition for minimum (waning)

$\cos\frac{2\pi(\upsilon_1 - \upsilon_2)}{2}t = 0$ Or $\frac{2\pi(\upsilon_1 - \upsilon_2)}{2}t = (2n+1)\frac{\pi}{2}$

$$t = \frac{(2n+1)}{2(\upsilon_1 - \upsilon_2)}; \frac{1}{2(\upsilon_1 - \upsilon_2)}, \frac{3}{2(\upsilon_1 - \upsilon_2)}, \frac{5}{2(\upsilon_1 - \upsilon_2)}....$$

The time interval between two successive minima is $\left(\frac{1}{\upsilon_1 - \upsilon_2}\right)$

Hence the beat frequency $x = (\upsilon_1 - \upsilon_2)$

Uses of Beats:

The phenomenon of beats can be used

1. To detect the presence of dangerous gases in mines.
2. To determine an un-known frequency of sound note.
3. To determine the frequency of a given tuning fork.
4. To tune a musical instrument.

5. **What is Doppler Effect? Obtain expression for the apparent frequency heard when the source is in motion with respect to an observer at rest?**

A. **i) Doppler Effect:**

The apparent change in the frequency of sound heard due to relative motion between the source and observer is defined as Doppler Effect.

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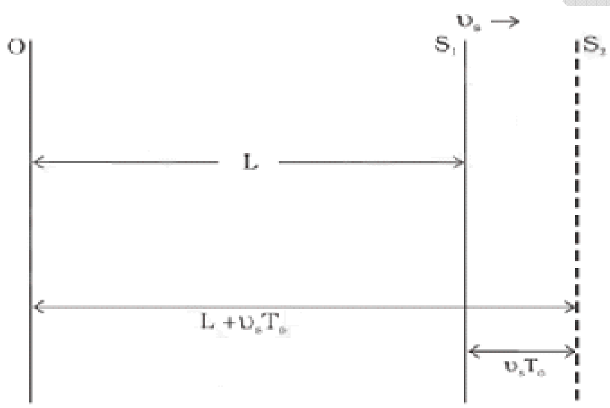
Case 1: Apparent frequency when the source is in motion and observer is at rest:

Let us choose the convention to take the direction from the observer to the source as the positive direction of velocity. Consider a source S moving with velocity V_s and an observer who is stationary in a frame in which the medium is also at rest. Let the speed of a wave of angular frequency ω and period T_0, both measured by an observer at rest with respect to the medium, be V. We assume that the observer has a detector that counts every time a wave crest reaches it. As shown in Figure, at time t = 0 the source is at point S_1 , located at a distance L from the observer, and emits a crest. This reaches the observer at time $t_1 = \frac{L}{V}$.

At time t $= T_0$ the source has moved a distance and is at point, located at a distance $\left(L + V_s T_0\right)$ from the observer. At point S_2, the source emits a second crest. This reaches the observer at time $t_2 = T_0 + \frac{\left(L + V_s T_0\right)}{V}$



Doppler effect (change in frequency of wave) detected when the source is moving and the observer is at rest in the medium.

At time nT_0, the source emits its (n+1)th crest and this reaches the observer at time

$$t_{n+1} = nT_0 + \frac{\left(L + nV_s T_0\right)}{V}$$

Time interval between 1st crest and (n+1)th crest to reach the observer

$$\Delta t = t_{n+1} - t_1 = \left[nT_0 + \frac{\left(L + nV_s T_0\right)}{V} - \frac{L}{V}\right]$$

$$= nT_0 \left[\frac{V + V_s}{V}\right]$$

The observer's detector n crests in time interval Δt then the observer records the period of the wave as T given by $T = \frac{\Delta t}{n}$

$$T = T_0\left(1+\frac{V_s}{V}\right) \text{.........................} (1)$$

Equation (1) may be rewritten in terms of the frequency υ_0 that would be measured if the source and observer were stationary, and the frequency υ observed when the source is moving, as

$$\upsilon = \upsilon_0\left(1+\frac{V_s}{V}\right)^{-1} \text{.............} (2) \qquad \left(T = \frac{1}{\upsilon}\right)$$

If V_s is small compared with the wave speed V, taking binomial expansion to terms in first order in V_s / V and neglecting higher power, Eq.(2) may be written as

$$\upsilon = \upsilon_0\left(1-\frac{V_s}{V}\right) \text{.............} (3)$$

For source approaching the observer, we replace V_s by $-V_s$ and we get,

$$\upsilon = \upsilon_0\left(1+\frac{V_s}{V}\right) \text{.............} (4)$$

By equation (3), the observer thus measures a lower frequency when the source recedes from him as compared to when he is at rest.

By equation (4), the observer measures higher frequency when the source approaches him.

6. What is Doppler's shift? Obtain an expression for the apparent frequency of sound heard when the observer is in motion with respect to a source at rest?

A. If the relative motion between the source and observer is to separate them more with time, the apparent frequency is lower than actual frequency.

Doppler's shift: The change in the frequency (υ) and apparent frequency of sound heard (υ^1) is called **Doppler's** shift.

Doppler shift $\Delta\upsilon = \upsilon - \upsilon^1$.

Observer moving towards a source at rest:

Let the source be at rest and the observer be moving with a velocity V_0 towards the source.

In the reference frame of moving observer, the source and the medium are approaching at a speed V_0.

The speed with which the wave approaches is $V+V_0$.

At t=0, let 'L' be the distance between the source and the observer. The time taken by the crest emitted by the source to reach the observer is

$$t_1 = \frac{L}{V+V_0}$$

At $t=T_0$, the observer has moved a distance V_0 T_0 and it is located at $(L-V_0T_0)$ from the source.

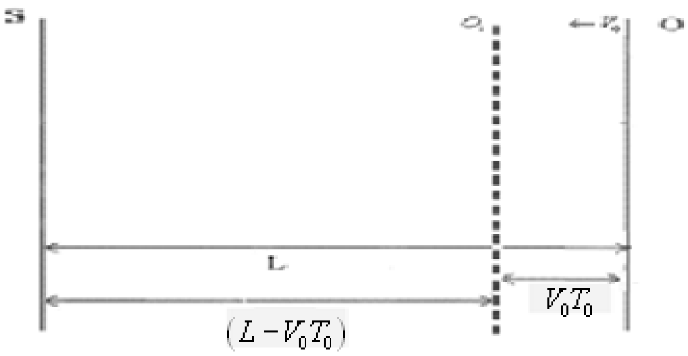
At $t=nT_0$, the source emits $(n+1)^{th}$ crest and this reaches the observer in a time

$$t_{n+1} = nT_0 + \frac{L-nV_0T_0}{V+V_0}$$

The time interval between the arrival of first and $(n+1)^{th}$ crest is

$$\Delta t = t_{n+1} - t_1 = nT_0 + \left(\frac{L-nV_0T_0}{V+V_0}\right) - \frac{L}{V+V_0}$$

$$\Delta t = nT_0 - \frac{nV_0T_0}{V+V_0}$$



The observer measure the time period as $T = \frac{\Delta t}{n}$

$$T = T_0\left(1 - \frac{V_0}{V+V_0}\right)$$

$$\mathrm{T} = T_0\left(1+\frac{V_0}{V}\right)^{-1}$$

$$\therefore \upsilon = \upsilon_0\left(1+\frac{V_0}{V}\right)$$

Hence if the observer moves towards the source, the apparent frequency increases.

Similarly it the observer moves away from the source at rest.

$$\upsilon = \upsilon_0\left(1-\frac{V_0}{V}\right)$$

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PROBLEMS

1. A stretched wire of length 0.6m is observed to vibrate with a frequency of 30Hz in the fundamental mode. If the string has a linear mass of 0.05 kg/m find

(a) The velocity of propagation of transverse waves in the string

(b) The tension in the string?

Sol: $l = 0.6m,\ n_1 = 30Hz$ **;** $m = 0.05\ kg\ m^{-1}$

a) From $n_1 = \frac{v}{2l} \Rightarrow 30 = \frac{v}{2(0.6)}$

$v = 30 \times 1.2 = 36\ ms^{-1}$

b) $v = \sqrt{\frac{T}{m}} \Rightarrow 36 = \sqrt{\frac{T}{5\times10^{-12}}}$

$\Rightarrow 36\times36 = \frac{T}{5\times10^{-2}}$

$\Rightarrow$T= 68.4 N

2. A steel cable of diameter 3 cm is kept under a tension of 10KN. The density of steel is 7.8 g/cm^3. With what speed would transverse waves propagate along the cable?

Sol: $T = 10KN = 10\times10^3 = 10^4\ N$

$\rho = 7.8\ gm\ cm^{-3} = 7800\ kgm^{-3}$

$D = 3cm,\ r = \frac{D}{2} = \frac{3}{2}\times10^{-2}\ m$

$v = \sqrt{\frac{T}{A\rho}} = \sqrt{\frac{10^4}{\frac{22}{7}\times\frac{9}{4}\times10^{-4}\times7800}} = 42.6\ ms^{-1}$

3. Two progressive transverse waves given by $y_1 = 0.07\sin\pi(12x - 500t)$ and $y_2 = 0.07\sin\pi(12x + 500t)$ travelling along a stretched string form nodes and antinodes. What is displacement at the (a) Nodes (b) Antinodes? What is the wavelength of standing wave?

Sol: $y_1 = 0.07\sin(12\pi x - 500\pi t)$ **and** $y_2 = 0.07\sin(12\pi x + 500\pi t)$

Resultant displacement, $= y = y_1 + y_2$.

$y = 2(0.07)\sin(12\pi x)\cos(500\pi t)$ ..... (1)

Comparing (1) with $y = 2a \sin kx \cos \omega t$

2a = 2 (0.07) = 0.14

$$k = \frac{2\pi}{\lambda} = 12\pi$$

$$\Rightarrow \lambda = \frac{2}{12} = \frac{1}{6}$$

a) Displacement at nodes = 0

b) Displacement at antinodes is maximum and given by '2a'

i.e., 2a = 2 (0.07) = 0.14m.

c) $\lambda = \frac{1}{6} = 0.16m.$

4. A string has a length of 0.4m and a mass of 0.16g. If the tension in the string is 70N, what are the three lowest frequencies it produces when plucked?

Sol: l = 0.4m, mass = 0.16 gram = 16×10^{-5}kg.

$$m = \frac{mass}{l} = \frac{16\times10^{-5}}{0.4} = 4\times10^{-4}\ kgm^{-1}$$

T = 70 N

$$n_1 = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2(0.4)}\sqrt{\frac{70}{4\times10^{-4}}} = 523 \text{ Hz}$$

$$n_2 = 2n_1 = 2\times523 = 1046\ Hz$$

$$n_3 = 3n_1 = 3\times523 = 1569\ Hz.$$

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5. A metal bar when clamped at its centre, resonates in its fundamental frequency with longitudinal waves of frequency 4 kHz. If the clamp is moved to one end, what will be its fundamental resonance frequency?

A. When the clamping positions has been changed from centre to one of its ends.

$$\upsilon_2 = \frac{\upsilon_1}{2} = \frac{4KHz}{2} = 2KHz$$

6. A closed organ pipe 70 cm long is sounded. If the velocity of sound is 331 m/s, what is the fundamental frequency of vibration of the air column?

Sol: $l_c = 70\ cm = 70 \times 10^{-2} m.$

$v = 331\ ms^{-1}$

$$n_1 = \frac{v}{4lc} = \frac{331}{4\left(70 \times 10^{-2}\right)} = 118.2 \text{ Hz.}$$

7. A vertical tube is made to stand in water so that the water level can be adjusted. Sound waves of frequency 320 Hz are sent into the top the tube. If standing waves are produced at two successive water levels of 20cm and 73 cm. what is the speed of sound waves in the air in the tube?

Sol: $n = 320\ Hz.$

$l_1 = 20\ cm,\ \ l_2 = 73\ cm$

$v = 2n\left(l_2 - l_1\right)$

$= 2\left(320\right)\left(53 \times 10^{-2}\right)$

$= 339\ ms^{-1}$

8. Two organ pipes of length 65 cm and 70 cm respectively, are sounded simultaneously. How many beats per second will be produced between the fundamental frequencies of the two pipes? (Velocity of sound = 330 m/s).

Sol: $l_1 = 65\ cm = 65 \times 10^{-2} m$

$l_2 = 70\ cm = 70 \times 10^{-2} m$

$v = 330\ ms^{-1}$

Number of beats $= n_1 - n_2$

$$= \frac{v}{2l_1} - \frac{v}{2l_2} = \frac{330}{2}\left(\frac{1}{(65\times10^{-2})} - \frac{1}{(70\times10^{-2})}\right) = 18$$

9. A train sounds its whistle as it approaches and crosses a level-crossing. An observer at the crossing measures a frequency of 219 Hz as the train approaches and a frequency of 184Hz as it leaves. If the speed of sound is taken to be 340 m/s, find the speed of the train and the frequency of its whistle?

A For the apparent frequency

a) $\upsilon^1 = \left(\frac{V}{V - V_s}\right)\upsilon = 219Hz$ .... (1)

For the receding train

$$\upsilon^{11} = \left(\frac{V}{V + V_s}\right)\upsilon = 184Hz \text{ .... } (2)$$

$$\Rightarrow \frac{\upsilon^1}{\upsilon^{11}} = \left(\frac{V + V_s}{V - V_s}\right)$$

By applying componendo and dividendo

$$\frac{V_s}{V} = \left(\frac{\upsilon^1 - \upsilon^{11}}{\upsilon^1 + \upsilon^{11}}\right) \Rightarrow V_S = \left(\frac{\upsilon^1 - \upsilon^{11}}{\upsilon^1 + \upsilon^{11}}\right)V$$

$$\Rightarrow V_s = \left(\frac{219 - 184}{219 + 184}\right)(340) = 29.5\, m/\sec.$$

b) Frequency of the whistle

$$\upsilon^1 = \left(\frac{V}{V - V_s}\right)\upsilon$$

$$\Rightarrow \quad 219Hz = \left(\frac{340}{340 - 29.5}\right)\upsilon$$

$$\Rightarrow \quad (219) \text{ x } 310.5 = (340)\upsilon$$

$$\Rightarrow \quad \upsilon = \frac{219 \text{ x } 310.5}{340}\ Hz$$

$$\Rightarrow \quad \upsilon = 199.99 \simeq 200Hz$$

10. Two trucks heading in opposite direction with speeds of 60kmph and 70kmph respectively approach each other. The driver of the first truck sounds his horn of frequency 400Hz. What frequency does the driver of the second truck hear? (Velocity of sound = 330 m/s). After the two trucks have passed each other, what frequency does the driver of the second truck hear?

Sol: $v_1 = v_s = 60\ kmph = 60 \times \frac{5}{18} = \frac{50}{3}\ ms^{-1}$

$v_2 = v_o = 70\ kmph = 70 \times \frac{5}{18} = \frac{175}{9}\ ms^{-1}.$

$n = 400\ Hz,\ v = 330\ ms^{-1}$

When they approach each other

$$n_1 = \left(\frac{v + v_o}{v - v_s}\right) n = \left(\frac{330 + \frac{175}{9}}{330 - \frac{50}{3}}\right) 400 = 445.94\ Hz.$$

When they move away from each other

$$n_2 = \left(\frac{v - v_o}{v + v_s}\right) n = \left(\frac{330 - 19.4}{330 + 16.6}\right)(400) = 358.45\ Hz.$$

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