#419297

Topic: Order of Magnitude

Explain this statement clearly:

"To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison." In view of this, which of the following statements are complete :

(a) atoms are very small objects

(b) a jet plane moves with great speed

(c) the mass of Jupiter is very large

(d) the air inside this room contains a large number of molecules

(e) a proton is much more massive than an electron

(f) the speed of sound is much smaller than the speed of light.

Solution

When any dimensional quantity is to be specified as large or small, it should always be relative. For example, a person weighing 150 kg might be heavy for us, but compared to weight of earth, it is negligible.

The sentences which require re frame are as follows:

(a) Atoms are very small objects compared to normal objects like duster, watch etc.

(b) A jet moves with great speed compared to a train.

(c) The mass of Jupiter is much more than the mass of earth.

(d) The air inside the room contains more molecules than entire human population of earth.

(e), (f) are complete

#419304

Topic: Measuring Instruments

Which of the following is the most precise device for measuring length?

A A vernier callipers with 20 divisions on the sliding scale.

B A screw gauge of pitch 1 mm and 100 divisions on the circular scale.

C An optical instrument that can measure length to within a wavelength of light.

D All instruments have same precision.

Solution

Most precise device is the one with the minimum least count

1. Vernier Calliper

20 divisions on the sliding scale

$$L.C = \frac{1}{20} mm = 0.05 mm$$

2. Screw Gauge

pitch 1 mm and 100 divisions on the circular scale

$$L.C = \frac{1mm}{100} = 0.01mm$$

3.optical instrument

within a wavelength of light

Wavelenght of light $\approx 589nm$

Hence Optical instrument is the most precise

#419319

Topic: Significant Figures and Rounding Off

State the number of significant figures in the following:

$(a) 0.007 m^2$

$(b) 2.64 \times 10^{24} kg$

$(c) 0.2370 gcm^{-3}$

$(d) 6.320 J$

$(e) 6.032 Nm^{-2}$

$(f) 0.0006032 m^2$

Solution

The significant figures of a number are those digits that carry meaning contributing to its measurement resolution. This includes all digits except:

1. All leading zeros;

2. Trailing zeros when they are merely placeholders to indicate the scale of the number (exact rules are explained at identifying significant figures);

3. Spurious digits introduced, for example, by calculations carried out to greater precision than that of the original data, or measurements reported to a greater precision than the equipment supports.

(a)

$0.007 = 7 \times 10^{-3}$

Significant digit $= 7$

Only one significant digit.

(b) Number of significant digit $= 3$

(c)

All non-zero numbers are always significant.All zeroes before a non zero number are insignificant. All zeroes which are simultaneously to the right of the decimal point and at tl end of the number are significant. The only non significant digit here is the zero before the decimal point.

Hence, number of significant digits is 4.

(d)

All non-zero numbers are always significant. All zeroes which are simultaneously to the right of the decimal point and at the end of the number are significant.

Hence, number of significant digits is 4.

(e)

All the digits are significant. (no leading and trailing zero)

No. of significant digits $= 4$

(f)

Significant digits $= 6032$

No. of sig. digits $= 4$

#419322

Topic: Dimensions

Write the dimensional formula of Force.

A MLT

$\boxed{\text{B}}$ MLT^{-2}

C MLT^2

D None of the above

Solution

Force $=$ mass $\times$ acceleration

$[F] = [M] \times [LT^{-2}] = MLT^{-2}$

#419323

Topic: Significant Figures and Rounding Off

Write the number of significant digits in 0.6464 :

A 4

B 3

C 5

D None of the above

Solution

All non-zero numbers are always significant.All zeroes before a non zero number are insignificant. All zeroes which are simultaneously to the right of the decimal point and at tl end of the number are significant. The only non significant digit here is the zero before the decimal point.

Hence, number of significant digits is 4.

#419329

Topic: Significant Figures and Rounding Off

Write the number of significant digits in 6.032 ?

A 4

B 3

C 2

D none of the above

Solution

The significant figures of a number are those digits that carry meaning contributing to its measurement resolution. This includes all digits except:

1. All leading zeros;

2.Trailing zeros when they are merely placeholders to indicate the scale of the number (exact rules are explained at identifying significant figures);

3. Spurious digits introduced, for example, by calculations carried out to greater precision than that of the original data, or measurements reported to a greater precision than the equipment supports.

According to rules,

6.032

All the digits are significant. (no leading and trailing zero)

No. of significant digits = 4

#419333

Topic: Significant Figures and Rounding Off

The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Solution

$A = 2 \times (L \times B + B \times T + T \times L)$

$\therefore A = 2 \times (4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)$

$\therefore A = 2 \times (4.2552 + 0.0202 + 0.0851)$

$\therefore A = 8.721 m^2$

$\therefore A = 8.721 m^2$ to correct significant digits

$V = L \times B \times T$

$\therefore V = 4.234 \times 1.005 \times 0.0201$

$\therefore V = 0.0855 m^3$ to correct significant digits

#419337

Topic: Significant Figures and Rounding Off

The total mass of a box measured by grocer's balance is $2.300kg$. Two gold pieces of masses $20.15g$ and $20.17g$ are added to the box. What is:

(a) the total mass of the box?

(b) the difference in the masses of the pieces to correct significant figures?

Solution

$m_i = 2.300kg$(2 significant digits)

$m_1 = 20.15g = 0.02015kg$(4 significant digits)

$m_2 = 20.175g = 0.02017kg$(4 significant digits)

(a)

Adding

$M = \Sigma m = 2.3 + 0.02015 + 0.02017kg = 2.34032kg$

But since the initial mass has least 2 signifincant digits, the final mass should not have more than 2 significant digits

Hence,

$M = 2.3Kg$

(b)

Subtracting

$m_2 - m_1 = 0.02g$

Since the initial mass has 4 significant digits, the result can have 4 or less significant digits

Hence here the no. of significant digit $= 2$

#419343

Topic: Significant Figures and Rounding Off

A physical quantity P is related to four observables a, b, c and d as follows :

P a^3 b^2 / $\sqrt{c}$ d

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P ? If the value of P calculated using th above relation turns out to be 3.763, to what value should you round off the result ?

Solution

$$P = \frac{a^3 b^2}{\sqrt{c}d}$$

Maximum fractional error in P is given by

$$\frac{\Delta P}{P} = \pm[3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}]$$
$$= \pm\frac{13}{100} = 0.13$$

Percentage error in $P = 13\,\%$

Value of P is given as 3.763.

By rounding off the given value to the first decimal place, we get $P = 3.8$

#419350

Topic: Dimesnional Analysis

A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion :

(a) y = a sin 2π t/T

(b) y = a sin v t

(c) y = (a/T) sin t/a

(d) y = (a/$\sqrt{2}$) (sin 2π / T + cos 2π / T)

(a = maximum displacement of the particle, v = speed of the particle. T = time-period of motion). Rule out the wrong formulas on dimensional grounds.

Solution

The displacement y has the dimension of length, therefore, the formula for it should also have the dimension of length. Trigonometric functions are dimensionless and their arguments are also dimensionless. Based on these considerations now check each formula dimensionally.

$(a)\dfrac{2\pi t}{T} = \dfrac{T}{T} = 1 = [M^0L^0T^0]$
$(b)vt = (LT^{-1})(T) = L = [M^0L^1T^0]$
$(c)\dfrac{t}{a} = \dfrac{T}{L} = [M^0L^{-1}T^1]$
$(d)\dfrac{2\pi t}{T} = \dfrac{T}{T} = 1 = [M^0L^0T^0]$

The formulas in (b) and (c) are dimensionally wrong.

#419354

Topic: Dimesnional Analysis

A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light, c. (This relation first arose asa consequen of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes :

$m = \frac{m_o}{(1-v^2)^{1/2}}$

Guess where to put the missing c.

Solution

Quantities inside functions (here square root) should be dimensionless.

Thus, the correct formula is $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

#419514

Topic: Measurement

1 parsec = ___________ metres

Solution

Diameter of Earth's orbit $= 3 \times 10^{11} m$

Radius of Earth's orbit, $r = 1.5 \times 10^{11} m$

Let the distance parallax angle be $1\ '' = 4.847 \times 10^{-6} rad$

Let the distance of the star be D.

Parsec is defined as the distance at which the average radius of the Earth's orbit subtends an angle of $1\ ''$.

$\theta = \dfrac{r}{D}$

$D = 3.09 \times 10^{16} m$

Hence, $1 parsec \approx 3.09 \times 10^{16} m$

#419515

Topic: Measurement

The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun ?

Solution

Distance of the star from the solar system = 4.29 ly

1 light year is the distance travelled by light in one year.

1 light year = Speed of light 1 year

$= 3 \times 10^8 \times 365 \times 24 \times 60 \times 60$

$= 94608 \times 10^{11}$ m

4.29 ly = 405868.32×10^{11} m

1 parsec = 3.08×10^{16} m

4.29 ly = $405868.32 \times 10^{11} / 3.08 \times 10^{16} = 1.32$ parsec

Diameter of Earth's orbit, d = 3×10^{11} m

Distance of the star from the earth, D = 405868.32×10^{11} m

$= 3 \times 10^{11} / 405868.32 \times 10^{11} = 7.39 \times 10^{-6}$ rad

But, 1 sec = 4.85×10^{-6} rad

7.39×10^{-6} rad = $7.39 \times 10^{-6} / 4.85 \times 10^{-6} = 1.52''$

#419516

Topic: Measurement

Precise measurements of physical quantities are a need of science. For example, to as certain the speed of an aircraft, one must have an accurate method to find its positions a closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Solution

Precise measurement of physical quantity is truly needed.

For example,

1) Ultra-shot laser pulses (time interval 1015 s) are used to measure time intervals in several physical and chemical processes.

2) X-ray spectroscopy is used to determine the inter-atomic separation or inter-planer spacing.

3) The development of mass spectrometer helps to measure mass of atom precisely.

#419519

Topic: Errors, Accuracy and Precision

Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity):

(a) The total mass of rain-bearing clouds over India during the Monsoon.

(b) The mass of an elephant

(c) The wind speed during a storm

(d) The number of strands of hair on your head

(e) The number of air molecules in your classroom

Solution

(a) Rainfall is measured extensively covering the entire nation along with an approximate demarcation of the areas involved. This will give the data for computing the total volume of rainfall in the country. The projection of the trend for future can be forecast in general based on the statistical data collected over years. An average rainfall of about 100 cm during monsoon in India is already recorded by the meteorologist.

(b) Consider a ship of known base area floating in the sea. Measure its depth d_1 in the sea. The volume of water displaced by the ship:

$V_1 = Ad$ Now, move an elephant on the ship and measure the depth of the ship, d_2 in this case. The volume of water displaced by the ship with the elephant on board: $V_2 = Ad_2$. The volume of water displaced by the elephant= $Ad_2 - Ad_1$. The density of water$=D$. Mass of elephant = $D \times (Ad_2 - Ad_1)$

(c) Wind speed during a storm can be measured by an anemometer. As the wind blows, it rotates. The rotation made by the anemometer in one second gives the value of win speed.

or

Wind speed can be estimated by floating a gas-filled balloon at a known height h. At the onset of wind, the angle drift of balloon in one second can be estimated. Using simple trigonometry, we can estimate the wind speed.

(d) Area of the head surface carrying hair $= A$

With the help of a screw gauge, the diameter and hence, the radius of a hair can be determined. Let it be r.

Area of one hair = r^2

Number of strands of hair = Total surface area / Area of one hair =$\frac{A}{r^2}$

(e) Let the volume of the room be V

One mole of air at NTP occupies $22.4\ l$ i.e., $22.4 \times 10^{-3}\ m^3$ volume.

Number of molecules in one mole =6.023×10^{23}

Number of molecules in room of volume V

=$6.023 \times 10^{23}\ V/22.4 \times 10^{-3}$ = $134.915 \times 10^{26}\ V = 1.35 \times 10^{28}\ V$

#419526
Topic: Measurement

When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth,its angular diameter is measured to be 35.72 of arc. Calculate the diameter of Jupiter.

Solution

Distance of Jupiter from the Earth, $D = 824.7 \times 10^6 km$

Angular diameter $35.72'' = 35.72 \times 4.874 \times 10^{-6} rad$

Diameter of Jupiter = d

Using the relation,

θ = d/ D

d = θ D = 824.7 x 10^9 x 35.72 x 4.872 x 10^{-6} = 143520.76 x 10^3 = 1.435 x 10^5 Km

#419527
Topic: Dimesnional Analysis

A man walking briskly in rain with speed v must slant his umbrella forward making an angle with the vertical. A student derives the following relation between θ and $v : tan\theta =$ and checks that the relation has a correct limit: as $v \to 0, \theta \to 0$ as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). D you think this relation can be correct ? If not, guess the correct relation.

Solution

Incorrect; on dimensional ground

The relation is $tan\theta = v$

Dimension of R.H.S = $M^0L^1T^{-1}$

Dimension of L.H.S = $M^0L^0T^0$

(The trigonometric function is considered to be a dimensionless quantity)

Dimension of R.H.S is not equal to the dimension of L.H.S. Hence, the given relation is not correct dimensionally.

To make the given relation correct, the R.H.S should also be dimensionless. One way to achieve this is by dividing the R.H.S by the speed of rainfall v'

Therefore, the relation reduces to

$tan\theta = v/v'$

This relation is dimension ally correct.

#419528

Topic: Measurement

It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about $0.02s$. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of $1s$?

Solution

Errror in 100 years $= 0.02s$

Error in $1s = \dfrac{0.02}{100 \times 365\frac{1}{4} \times 24 \times 60 \times 60}$

$= 7.9 \times 10^{-13} \approx 10^{-12}$

Hence, the accuracy of standard caesium clock in measuring a time interval of $1s$ is 10^{-12}

#419530

Topic: Order of Magnitude

Estimate the average mass density of a sodium atom assuming its size to be about 2.5. (Use the known values of Avogadro's number and the atomic mass of sodium). Compar it with the density of sodium in its crystalline phase : $970\ kgm^3$. Are the two densities of the same order of magnitude? If so, why?

Solution

Diameter of sodium atom = Size of sodium atom = 2.5 Å

Radius of sodium atom, r = (1/2) × 2.5 Å = 1.25 Å = 1.25 × 10^{-10}m

Volume of sodium atom, V = (4/3) π r^3= (4/3) × 3.14 × $(1.25 \times 10^{-10})^3$ = V_{Sodium}

According to the Avogadro hypothesis, one mole of sodium contains 6.023 × 10^{23} atoms and has a mass of 23 g or 23 × 10^{-3} kg.

∴ Mass of one atom = 23 × 10^{-3} / 6.023 × 10^{23} Kg = m_1

Density of sodium atom, ρ = m_1 / V_{Sodium}

Substituting the value from above, we get

Density of sodium atom, ρ =4.67 × 10^{-3} Kg m^{-3}

It is given that the density of sodium in crystalline phase is 970 kg m^{-3}.

Hence, the density of sodium atom and the density of sodium in its crystalline phase are not in the same order. This is because in solid phase, atoms are closely packed. Thus, the inter-atomic separation is very small in the crystalline phase.

#419533

Topic: Measurement

A LASER is a source of very intense, monochromatic, and unidirectional beam oflight. These properties of a laser light can be exploited to measure long distances.The distance of the Moon from the Earth has been already determined very preciselyusing a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflectio at the Moons surface. How much is the radius of the lunar orbit around the Earth ?

Solution

Time taken by the laser beam to return to Earth after reflection from the Moon $= 2.56\ s$

Speed of light $= 3 \times 10^8 m/s$

Time taken by the laser beam to reach Moon $= 1/2 \times 2.56 = 1.28\ s$

Radius of the lunar orbit $=$ Distance between the Earth and the Moon

$$= 1.28 \times 3 \times 10^8$$
$$= 3.84 \times 10^8 m$$
$$= 3.84 \times 10^5 km$$

#419535

Topic: Measurement

The farthest objects in our universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (know as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

Solution

Time taken by quasar light to reach Earth = 3 billion years

$= 3 \times 10^9\ years = 3 \times 109 \times 365 \times 24 \times 60 \times 60\ s$

Speed of light $= 3 \times 10^8\ m/s$

Distance between the Earth and quasar $= (3 \times 10^8) \times (3 \times 10^9 \times 365 \times 24 \times 60 \times 60)$

$= 283824 \times 10^{20}\ m = 2.8 \times 10^{22}\ km$