
#419764

Topic: Newton's Law of Gravitation

Answer the following:

(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?

(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. However, the tidal effect of the moon's pull is greater than the tidal effect of the sun. Why?

Solution

(a) No, gravitational force is a long range force and we cannot escape from it.

(b) Due to the small space ship, the force of attraction between the earth and the ship is too less, the astronaut cannot detect gravity. If the size of the space station is very large, the magnitude of the gravity will also become appreciable and hence he can hope to detect it.

(c) Generally, the gravitational force of attraction between two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them. The gravitational force between the sun and the earth or between the moon and the earth varies inversely to the square of the distance. But due to the very large mass of the sun, the gravitational force on the earth due to the sun is about 7x10 times the force due to the moon on it.

The tidal effect varies inversely as the cube of the distance. Therefore, though the mass of the moon is quite small as compared to the mass of the sun, yet the tidal effect of moon's pull is very large (as compared to that due to the sun) due to small distance between moon and earth.

#419771

Topic: Acceleration due to Gravity

Choose the correct alternative :

(a) Acceleration due to gravity increases/decreases with increasing altitude.

(b) Acceleration due to gravity increases/decreases with increasing depth (assume the earth to be a sphere of uniform density).

(c) Acceleration due to gravity is independent of mass of the earth/mass of the body.

(d) The formula $-G\,Mm(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the center of the earth.

Solution

(a) Acceleration decreases according to formula $g' = g(1 - \frac{2h}{R})$,

where h is height and R is radius of earth.

(b) Acceleration decreases according to formula $g' = g(1 - \frac{d}{R})$,

where d is depth and R is radius of earth.

(c) Acceleration due to gravity is given by the formula: $g = \dfrac{GM_e}{R_e^2}$

Hence, it is independent of mass of body, but is dependent on mass of earth.

(d) Gravitational potential energy is given by the formula: $U = -\dfrac{Gm_1m_2}{r}$

$g = \dfrac{GM_e}{R_e^2}$ assuming distance between object and earth is nearly equal to the radius of the earth.

Substituting the second equation in first, we get $U \backsimeq mgh$

Hence, the first formula is more accurate.

#419789

Topic: Kepler's Laws

Suppose there existed a planet that went around the sun twice as fast as the earth.What would be its orbital size as compared to that of the earth ?

Solution

Time taken by the Earth to complete one revolution around the Sun,

T_e = 1 year

Time taken by the planet to complete one revolution around the Sun, $T_P = \frac{1}{2}T_e = \frac{1}{2}\ year$

Orbital radius of the planet = R_p

From Kepler's third law of planetary motion, we can write:

$$(\frac{R_P}{R_e})^3 = (\frac{T_P}{T_e})^2$$

$$(\frac{R_P}{R_e}) = (\frac{T_P}{T_e})^{2/3}$$

$$= 0.5^{(2/3)} = 0.63$$

Hence, the orbital radius of the planet will be 0.63 times of the Earth.

#419792

Topic: Kepler's Laws

Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is $4.22 \times 10^8\ m$. Show that the mass of Jupiter is about one-thousandth that of the sun.

Solution

Gravitational force = Centripetal Force

$$\frac{GM_pM_J}{R_p^2} = \frac{M_pv_p^2}{R_p}\ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{Also, } v_p = \frac{2\pi R_p}{T_p}\ldots\ldots\ldots\ldots\ldots(2)$$

Substituting (2) in (1),

$$M_J = \frac{4\pi^2R_P^3}{GT_P^2}$$

$$= 1.9 \times 10^{27}\ kg$$

$$M_J/M_S = \frac{1.9 \times 10^{27}}{2 \times 10^{30}} \approxeq 10^{-3}$$

#419793

Topic: Kepler's Laws

Let us assume that our galaxy consists of $2.5\ \times\ 10^{11}$ stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution ? Take the diameter of the Milky Way to be 10^5 ly.

Solution

Mass of our galaxy Milky Way, $M = 2.5 \times 10^{11}$ *solar mass*

1 Solar mass = Mass of Sun $= 2 \times 10^{30}$ *kg*

Mass of our galaxy, $M = 2.5 \times 10^{11} \times 2 \times 10^{30} = 5 \times 10^{41}$ *kg*

Diameter of Milky Way, $d = 10^5$ *ly*

Radius of Milky Way, $r = 5 \times 10^4$ *ly*

1 *ly* $= 9.46 \times 10^{15}$ *m*

$\therefore\ r = 5 \times 10^4 \times 9.46 \times 10^{15}$

$= 4.73 \times 10^{20}$

Since a star revolves around the galactic centre of the Milky Way, its time period is given by the relation:

$$T = (4\pi^2 r^3 / GM)^{1/2}$$

$$= \sqrt{\frac{4 \times 3.14^2 \times 4.73^3 \times 10^{60}}{6.67 \times 10^{-11} \times 5 \times 10^{41}}}$$

$$= 1.12 \times 10^{16}\ s$$

$$= \frac{1.12 \times 10^{16}}{365 \times 24 \times 60 \times 60}\ years$$

$$= 3.55 \times 10^{8}\ years$$

#419796

Topic: Satellites

Choose the correct alternative:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic / potential energy.

(b) The energy required to launch an orbiting satellite out of earths gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earths influence.

Solution

Let Height of satellite+Radius of earth=R

(a)

For a satellite,

Gravitational Force = Centripetal Force

$GMm/R^2 = mv^2/R..........(1)$

$U = -GMm/R$

$K.E. = \frac{1}{2}mv^2$

$T.E. = U + K.E.$

$T.E. = -GMm/R + \frac{1}{2}mv^2............(2)$

From (1) and (2),

$T.E. = -\frac{1}{2}mv^2 = -K.E.$

Hence, if the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic energy.

(b)

For an object to escape, energy required should be such that total energy is zero.

For a satellite, $T.E. = -\frac{GMm}{2R}$ from part (a)

Extra energy for satellite, $E_1 = \frac{GMm}{2R}...........(1)$

For a stationary object, $T.E. = U = -\frac{GMm}{R}$

Extra energy for object, $E_2 = \frac{GMm}{R}...........(2)$

From (1) and (2),

$\frac{E_1}{E_2} = \frac{1}{2}$

Hence, the energy required to launch an orbiting satellite out of earth's gravitational influence is **less** than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

#419801

Topic: Escape velocity

Does the escape speed of a body from the earth depend on (a) The mass of the body, (b) The location from where it is projected, (c) The direction of projection, (d) The height of the location from where the body is launched?

Solution

For a body placed on the surface of the earth, Escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$

The escape velocity is independent of mass of the body and the direction of projection. It depends upon the gravitational potential at the point from where the body is launched. Since this potential depends on the height of the point of projection, the escape velocity depends on it.

#419808

Topic: Gravitational Potential Energy

A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) Linear speed, (b) angular speed, (c) Angular momentum, (d) Kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

Solution

A comet while going on elliptical orbit around the Sun has constant angular momentum due to no external torque and total energy due to Law of conservation of energy at all locations but other quantities vary with locations.

(a)

Since angular momentum is constant, $L = \frac{mv^2}{r}$ is constant. As r changes in elliptical orbit, speed is not constant.

(b)

Since angular momentum is constant, $L = m\omega^2 r$ is constant. As r changes in elliptical orbit, angular speed is not constant.

(c)

Angular momentum is constant because there is no external torque.

(d)

Since speed is not constant, Kinetic Energy is not constant.

(e)

Total energy is constant by law of conservation of energy.

$T.E. = P.E. + K.E.$ = constant

Since $K.E.$ is not constant, $P.E.$ is not constant.

(f) Total energy is constant by law of conservation of energy.

#419847

Topic: Satellites

Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.

Solution

(a) Legs hold the entire mass of a body in standing position due to gravitational pull. In space, an astronaut feels weightlessness because of the absence of gravity. Therefore, swollen feet of an astronaut do not affect him/her in space.

(b) A swollen face is caused generally because of apparent weightlessness in space. Sense organs such as eyes, ears nose, and mouth constitute a persons face. This symptom can affect an astronaut in space.

(c) Headaches are caused because of mental strain. It can affect the working of an astronaut in space.

(d) Space has different orientations. Therefore, orientational problem can affect an astronaut in space.

#419852

Topic: Escape velocity

A rocket is fired from the earth towards the sun. At what distance from the earths centre is the gravitational force on the rocket zero? Mass of the sun = $2 \times 10^{30}\,kg$, mass of the earth = $6 \times 10^{24}\,kg$. Neglect the effect of other planets etc. (orbital radius = $1.5 \times 10^{11} m$).

Solution

Mass of the Sun, $M_s = 2 \times 10^{30}\ kg$

Mass of the Earth, $M_e = 6 \times 10^{24}\ kg$

Orbital radius, $r = 1.5 \times 10^{11}\ m$

Mass of the rocket $= m$

Let x be the distance from the centre of the Earth where the gravitational force acting on satellite P becomes zero.

From Newton’s law of gravitation, we can equate gravitational forces acting on satellite P under the influence of the Sun and the Earth as:

$GmM_s/(r-x)^2 = GmM_e/x^2$

$[(r-x)/x]^2 = M_s/M_e$

$(r-x)/x = (\frac{2 \times 10^{30}}{60 \times 10^{24}})^{1/2} = 577.35$

$1.5 \times 10^{11} - x = 577.35x$

$578.35x = 1.5 \times 10^{11}$

$x = 1.5 \times 10^{11}/578.35 = 2.59 \times 10^{8} m.$

#419854

Topic: Kepler's Laws

How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is $1.5\ \times\ 10^8 km$.

Solution

Orbital radius of the Earth around the Sun, $r = 1.5 \times 10^{11}\ m$

Time taken by the Earth to complete one revolution around the Sun,

$T = 1\ year = 365.25\ days$

$= 365.25 \times 24 \times 60 \times 60s$

Universal gravitational constant,$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Thus, mass of the Sun :

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4 \times 3.14^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (365.25 \times 24 \times 60 \times 60)^2}$$

$$\approx 2 \times 10^{30}\ kg$$

Hence, the mass of the Sun is $2 \times 10^{30}\ kg$

#419859

Topic: Kepler's Laws

A saturn year is 29.5 times the earth year. How far is the saturn from the sun if the earth is $1.50\ \times\ 10^8\ km$ away from the sun ?

Solution

Distance of Earth from Sun,$r_e = 1.5 \times 10^{11}\ m$

Time period of Earth $= T_e$

Time period of Saturn,$T_s = 29.5\ T_e$

Distance of Saturn from the Sun $= r_s$

From Kepler's third law of planetary motion,

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$\implies\ r_s^3 / r_e^3 = T_s^2 / T_e^2$$

$$r_s = r_e (T_s / T_e)^{2/3}$$

$$= 1.5 \times 10^{11} \times 29.5^{2/3}$$

$$= 14.32 \times 10^{11}\ m$$

Therefore, the distance between Saturn and the Sun is $1.43 \times 10^{12}\ m$.

#419860

Topic: Acceleration due to Gravity

A body weighs 63N on the surface of the earth. What is the gravitational force (in N) on it due to the earth at a height equal to half the radius of the earth ?

Solution

Weight of the body, $W = 63\ N$

Acceleration due to gravity at height h from the Earths surface is given by the relation:

$$g' = \frac{g}{(1 + \frac{h}{R_e})^2}$$

Substituting $h = R_e/2$,

$g' = 4g/9$

Weight of body of mass m at height h is given as:

$W' = mg'$

$= 4/9\ mg$

$= 4/9 \times 63 = 28\ N$

#419861

Topic: Acceleration due to Gravity

Assuming the earth to be a sphere of uniform mass density, how much would a body weigh (in N) half way down to the center of the earth if it weighed 250 N on the surface ?

Solution

Weight of a body of mass m at the Earth's surface, $W = mg = 250\ N$

Body of mass m is located at depth, $d = R_e/2$

Acceleration due to gravity at depth $g(d)$ is given by the relation:

$$g' = (1 - \frac{d}{R_e})g$$

$g' = (1 - R_e/2R_e)g$

$= g/2$

Weight of the body at depth d,

$W' = mg'$

$= mg/2 = W/2$

$= 125\ N$

#419866

Topic: Escape velocity

A rocket is fired vertically with a speed of $5\ kms^{-1}$ from the earth's surface. How far from the earth does the rocket go before returning to the earth ? Mass of the earth $= 6.0 \times 10^{24}\ kg$; mean radius of the earth $= 6.4 \times 10^6\ m$; $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$.

Solution

Velocity of the rocket, $v = 5 \times 10^3\ m/s$

Mass of the Earth, $M_e = 6 \times 10^{24}\ kg$

Radius of the Earth, $R_e = 6.4 \times 10^6$

Height reached by rocket mass $m = h$

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 + (-\frac{GM_e m}{R_e})$$

At the highest point h,

$v = 0 \implies$ Kinetic energy = 0

Potential energy = $-\dfrac{GM_e m}{R_e + h}$

Total energy of the rocket $= -\dfrac{GM_e m}{R_e + h}$

By law of conservation of energy, total energy at surface = total energy at height h

$$\frac{1}{2}mv^2 - \frac{GM_e m}{R_e} = -\frac{GM_e m}{R_e + h}$$

$$v^2/2 = \frac{GM_e h}{R_e(R_e + h)}$$

Substituting $g = GM_e/R_e^2$ and rearranging the terms,

$$h = \frac{R_e v^2}{2gR_e - v^2}$$

$$= \frac{6.4 \times 10^6 \times (5 \times 10^3)^2}{2 \times 9.8 \times 6.4 \times 10^6 - (5 \times 10^3)^2}$$

$$= 1.6 \times 10^6\ m$$

Height achieved by the rocket with respect to the earth is

$R_e + h = 6.4 \times 10^6 + 1.6 \times 10^6 = 8.010^6 m$

#419870

Topic: Escape velocity

A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite $= 200\ kg$; mass of the earth $= 6.0 \times 10^{24}\ kg$; radius of the earth $= 6.4 \times 10^6\ m$; G = $6.67 \times 10^{-11}\ Nm^2 kg^{-2}$.

Solution

Mass of the Earth, $M = 6.0 \times 10^{24}\ kg$

$m = 200\ kg$

$R_e = 6.4 \times 10^6\ m$

$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Height of the satellite,$h = 400\ km = 4 \times 10^5\ m$

Total energy of the satellite at height h $= (1/2)mv^2 + (-GM_em/(R_e + h))$

Orbital velocity of the satellite, $v = \sqrt{\dfrac{GM_e}{R_e + h}}$

Total energy at height h $= \dfrac{1}{2}\dfrac{GM_em}{R_e + h} - \dfrac{GM_em}{R_e + h}$

$Total\ Energy = -\dfrac{1}{2}\dfrac{GM_em}{R_e + h}$

The negative sign indicates that the satellite is bound to the Earth.

Energy required to send the satellite out of its orbit = – (Bound energy)

$= \dfrac{GM_em}{2(R_e + h)}$

$= \dfrac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 200}{2(6.4 \times 10^6 + 4 \times 10^5)}$

$= 5.9 \times 10^9\ J$

If the satellite just escapes from the gravitational field, then total energy of the satellite is zero. Therefore, we have to supply $5.9 \times 10^9 J$ of energy to just escape it.

#419873

Topic: Gravitational Potential Energy

Two stars each of one solar mass $(= 2 \times 10^{30}\ kg)$ are approaching each other for a head on collision. When they are a distance $10^9\ km$, their speeds are negligible. What is the speed with which they collide ? The radius of each star is $10^4\ km$. Assume the stars to remain undistorted until they collide. (Use the known value of G).

Solution

Mass of each star, $M = 2 \times 10^{30}\ kg$

Radius of each star, $R = 10^4\ km = 10^7\ m$

Distance between the stars,$r = 10^9\ km = 10^{12}\ m$

Total energy of two stars separated at distance r $= -GMM/r + (1/2)Mv^2$

$= -GMM/r. \ldots\ldots\ldots . (1)$

Let velocity of the stars when they are about to collide be v

Distance between the centers of the stars $= 2R$

Total kinetic energy $= (1/2)Mv^2 + (1/2)Mv^2 = Mv^2$

Total potential energy $= -GMM/2R$

Total energy of the two stars just before collision $= Mv^2 - GMM/2R. \ldots\ldots . (2)$

Using the law of conservation of energy, we can write:

$Mv^2 - GMM/2R = -GMM/r$

$v^2 = GM/2R - GM/r$

$= 6.67 \times 10^{-11} \times 2 \times 10^{30}(\dfrac{1}{2 \times 10^7} - \dfrac{1}{10^{12}})$

$\approx 6.67 \times 10^{12}$

$v = 2.58 \times 10^6\ m/s$

#419874

Topic: Newton's Law of Gravitation

Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres ? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable ?

Solution

Gravitational field at the mid-point of the line joining the centres of the two spheres $= \frac{GM}{(r/2)^2} - \frac{GM}{(r/2)^2} = 0\ N$

Gravitational potential at the midpoint of the line joining the centres of the two spheres is $V = -\frac{GM}{r/2} - \frac{GM}{r/2} = -\frac{4GM}{r}$

$$V = -\frac{4 \times 6.67 \times 10^{-11} \times 100}{0.1}$$

$$= -2.68 \times 10^{-7}\ J/kg$$

As the effective force on the body placed at mid-point is zero, so the body is in equilibrium. If the body is displaced a little towards either mass body from its equilibrium position, it will not return back to its initial position of equilibrium. Hence, the body is in unstable equilibrium.

#419879

Topic: Gravitational Potential Energy

As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite ? (Take the potential energy at infinity to be zero). Mass of the earth = $6.0 \times 10^{24}\ kg$, radius = 6400 km.

Solution

Mass of the Earth, $M = 6.0 \times 10^{24}\ kg$

Radius of the Earth, $R = 6400\ km = 6.4 \times 10^6\ m$

Height of a geostationary satellite from the surface of the Earth, $h = 36000\ km = 3.6 \times 10^7\ m$

Gravitational potential due to Earth's gravity at height h

$$V = -\frac{GM}{R+h}$$

$$= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{3.6 \times 10^7 + 0.64 \times 10^7}$$

$$= -9.4 \times 10^6\ J/kg$$

#419881

Topic: Newton's Law of Gravitation

A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity ? (mass of the sun = $2 \times 10^{30}\ kg$).

Solution

A body gets stuck to the surface of a star if the inward gravitational force is greater than the outward centrifugal force caused by the rotation of the star.

Gravitational force, $f_g = -GMm/R^2$

where

Mass of the star, $M = 2.5 \times 2 \times 10^{30} = 5 \times 10^{30}\ kg$

Mass of the body: m

Radius of the star: $R = 12\ km = 1.2 \times 10^4\ m$

$$f_g = \frac{6.67 \times 10^{-11} \times 5 \times 10^{30}}{(1.2 \times 10^4)^2} m$$

$$= 2.31 \times 10^{12} m\ N$$

Centrifugal force, $f_c = mR\omega^2$

$$= m \times 4\pi^2 f^2 R$$

$$= m \times 4 \times 3.14^2 \times 1.2^2 \times 1.2 \times 10^4$$

$$= 6.82 \times 10^5 m\ N$$

Since $f_g > f_c$, the body will remain stuck to the surface of the star.

#419884

Topic: Satellites

A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system ? Mass of the space ship = 1000 kg; mass of the sun = $2 \times 10^{30}\ kg$; mass of mars = $6.4 \times 10^{23}\ kg$;

radius of mars = 3395 km; radius of the orbit of mars = $2.28 \times 10^8\ km$

$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$.

Solution

Mass of the spaceship, $m_s = 1000\ kg$

Mass of the Sun, $M = 2 \times 10^{30}\ kg$

Mass of Mars, $m_m = 6.4 \times 10^{23}\ kg$

Orbital radius of Mars, $R = 2.28 \times 10^{11}\ m$

Radius of Mars, $r = 3.395 \times 10^6\ m$

Universal gravitational constant, $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Potential energy of the spaceship due to the gravitational attraction of the Sun, $U_s = -GMm_s/R$

Potential energy of the spaceship due to the gravitational attraction of Mars, $U_m = -Gm_m m_s/r$

Since the spaceship is stationed on Mars, its velocity and hence, its kinetic energy will be zero.

Total energy of the spaceship, $E = U_m + U_s = -GMm_s/R - Gm_m m_s/r$

The negative sign indicates that satellite is bound to the system.

Energy required for launching the spaceship out of the solar system $= -E$

$$-E = \frac{GMm_s}{R} + \frac{Gm_m m_s}{r}$$

$$= Gm_s(\frac{M}{R} + \frac{m_m}{r})$$

$$= 6.67 \times 10^{-11} \times 10^3 \times (\frac{2 \times 10^{30}}{2.28 \times 10^{11}} + \frac{6.4 \times 10^{23}}{3.395 \times 10^6})$$

$$= 5.91 \times 10^{11}\ J$$

#419885

Topic: Escape velocity

A rocket is fired 'vertically' from the surface of mars with a speed of $2\ kms^{-1}$. If 20% of its initial energy is lost due to martian atmospheric resistance, how far (in km) will the rocket go from the surface of mars before returning to it ?

Mass of mars = $6.4 \times 10^{23}\ kg$; radius of mars = 3395 km; $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$.

Solution

Initial velocity of the rocket, $v = 2\ km/s = 2 \times 10^3 m/s$

Mass of Mars, $M = 6.4 \times 10^{23}\ kg$

Radius of Mars, $R = 3395\ km = 3.395 \times 10^6\ m$

Universal gravitational constant, $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Let mass of the rocket be m

Initial kinetic energy of the rocket $= (1/2)mv^2$

Initial potential energy of the rocket $= -GMm/R$

Total initial energy = $(1/2)mv^2 - GMm/R$

If 20% of the initial kinetic energy is lost due to Martian atmospheric resistance, then only 80% of its kinetic energy helps in reaching a height.

Total initial energy available $= (80/100) \times (\frac{1}{2}mv^2 - \frac{GMm}{R})$

Let maximum height reached by rocket be h

At this height, the velocity and hence the kinetic energy of the rocket becomes zero.

Total energy of the rocket at height h $= \frac{-GMm}{R+h}$

Applying the law of conservation of energy for the rocket, :

$$(80/100) \times (\frac{1}{2}mv^2 - \frac{GMm}{R}) = -\frac{GMm}{R+h}$$

$$0.4v^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$0.4v^2 = \frac{GMh}{R(R+h)}$$

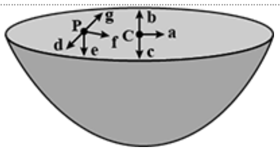
$$\implies h = \frac{0.4R^2v^2}{GM - 0.4v^2R}$$

$$= \frac{0.4 \times (3.395 \times 10^6)^2 \times (2 \times 10^3)^2}{(6.67 \times 10^{-11}) \times (6.4 \times 10^{23}) - 0.4 \times (2 \times 10^3)^2 \times (3.395 \times 10^6)}$$

$$= 495\ km$$

#420254

Topic: Gravitational Potential Energy



Choose the correct answer from among the given ones:

The gravitational intensity at the center of a hemispherical shell of uniform mass density has the direction indicated by the arrow in the above figure.

A a

B b

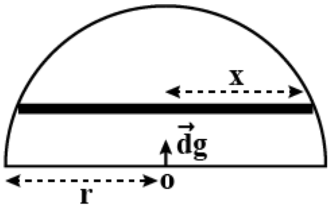
[✓ **C**] c

D 0

Solution

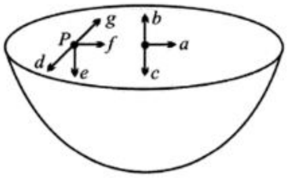
A hemisphere can be considered to be made up of many infinitesimally thin concentric rings. Figure shows cross-section of a hemisphere of a radius r. It is composed of many rings of radius x where 0<x<r. Gravitational intensity due to a small ring of radius x is $\vec{dg}$ as shown in the figure. Gravitational intensity due to all rings $= \int dg\hat{j}$. In other words, total gravitational intensity is the integral of gravitational intensity of all infinitesimally small rings which is in the upward direction.

Therefore for the given figure, the gravitational intensity will be towards (c).



#420257

Topic: Gravitational Potential Energy



The direction of the gravitational intensity at an arbitrary point P for hemispherical shell of uniform mass density is indicated by the arrow:

A *d*

[✓ B] *e*

C *f*

D *g*

Solution

At the all points inside a hollow spherical shell, potential is same. So, gravitational intensity, which is negative of gravitational potential gradient, is zero.

Due to zero gravitational forces acting on any particles at any point inside a spherical shell will be symmetrically placed. It follows from here that if we remove the upper hemispherical shell, the net gravitational force acting on a particle at P will be downwards. Hence geavitational intensity will be along *e*.

#420446

Topic: Acceleration due to Gravity

The acceleration due to gravity on the surface of moon is 1.7 m s^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is 9.8 m s^{-2})

Solution

Acceleration due to gravity on the surface of moon, $g' = 1.7\, m\, s^{-2}$

Acceleration due to gravity on the surface of earth, $g = 9.8 m\, s^{-2}$

Time period of a simple pendulum on earth, $T = 3.5 s$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where,

l is the length of the pendulum

$$\therefore\ l = \frac{T^2}{(2\pi)^2} \times 2$$

$$= \frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8\ m$$

The length of pendulum remains constant

On moon's surface, time period, $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$= 2\pi\sqrt{\frac{\frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8}{1.7}} = 8.4\ s$$

Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

#464717

Topic: Newton's Law of Gravitation

How does the force of gravitation between two objects change when the distance between them is reduced to half?

Solution

We know that According to law of gravitation, the Force between any two object of mass M and m is directly proportional to their masses and inversely proportional to the square of the distance between them.

$F = G\frac{Mm}{r^2}$

So, if Distance is halved, i.e new $r = \frac{r}{2}$

So, New $F = G\frac{Mm}{\frac{r^2}{2^2}}$

$\Rightarrow F = 4G\frac{Mm}{r^2}$

So, Force of gravitation increses by 4 times when distance reduced by half

Gravitational force $F = \frac{Gm_1m_2}{r^2}$ where r is the distance between the bodies

$\therefore\quad F \propto \frac{1}{r^2}$

$r' = \frac{r}{2} \qquad \Longrightarrow F' = 4F$

Hence gravitational force increases by 4 times.

#464718

Topic: Acceleration due to Gravity

Gravitational force acts on all objects in proportion to their masses. Why then, a heavy object does not fall faster than a light object?

Solution

Weight of an object on surface of earth $= mg$

Where , $m =$ mass of the object

$g =$ acceleration due to gravity

Gravitational force on the object ,$F = G\frac{Mm}{r^2}$

Where $G =$ Gravitational constant

$M =$ Mass of Earth

$m =$ mass of object

$r =$ distance between object and earth radius.

Also We know that Weight of Object = Force of gravitation acting on it

$F = G\frac{Mm}{r^2} = mg$

$\Rightarrow g = \frac{GM}{r^2}$

From above we can conclude that the acceleration due to gravity g is same for all the objects, irrespective of the mass and size of the object.

Hence they have same acc. due to gravity and fall with a sthe same speed

Gravitational force acting on a body, $F = \frac{GM_em}{r^2}$

where m and M_e represent the mass of body and earth respectively.

Thus acceleration of the body, $a = \frac{F}{m} = \frac{GM_e}{r^2} = constant$

As the acceleration of all the bodies is same, thus all bodies fall at same rate.

#464721

Topic: Newton's Law of Gravitation

The earth and the moon are attracted to each other by gravitational force. Does the earth attract the moon with a force that is greater or smaller or the same as the force with which the moon attracts the earth? Why?

Solution

Newton's law of universal gravitation states that any two bodies in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Gravitational force on the object ,$F = G\frac{Mm}{r^2}$

Where $G =$ Gravitational constant

$M =$ Mass of Earth

$m =$ mass of object

$r =$ distance between object and earth radius.

The above law applies to all objects anywhere in the universe. This is also true in case of force of attraction between Earth and moon. The magnitude of force (F) of attraction exerted by Earth on the moon, due to gravitation, is the same as that exerted by moon (F) on earth.These forces being equal and opposite, also in accordance with Newton's Third Law of Motion, which states that to every action there is an equal and opposite reaction.

Let m and M be the mass of moon and earth respectively which are separated by a distance r

Force of attraction that earth exert on moon, $F_e = \frac{GMm}{r^2}$

Force of attraction that moon exert on earth, $F_m = \frac{GmM}{r^2}$

Thus earth attracts the moon with same force by which moon attracts the earth.

Also the gravitational force is an internal force of attraction between the two bodies, thus they have to be equal.

#464725

Topic: Newton's Law of Gravitation

What happens to the (gravitational)force between two objects, if

(i) The mass of one object is doubled?

(ii) The distance between the objects is doubled and tripled?

(iii) The masses of both objects are doubled?

Solution

Gravitational force $F = \frac{Gm_1m_2}{r^2}$

Case (i) : Mass of one object is doubled i.e $m_1' = 2m_1$

$\therefore\ F' = \frac{G(2m_1)m_2}{r^2} \implies F' = 2F$

Thus gravitational force gets also doubled

Case (ii) : Distance between the objects is doubled i.e $r' = 2r$

$\therefore\ F' = \frac{Gm_1m_2}{(2r)^2} \implies F' = \frac{F}{4}$

Thus gravitational force gets reduced by 4 times.

Distance between the objects is tripled i.e $r' = 3r$

$\therefore\ F' = \frac{Gm_1m_2}{(3r)^2} \implies F' = \frac{F}{9}$

Thus gravitational force gets reduced by 9 times.

Case (iii) : Mass of both the objects are doubled i.e $m_1' = 2m_1$ and $m_2' = 2m_2$

$\therefore\ F' = \frac{G(2m_1)(2m_2)}{r^2} \implies F' = 4F$

Thus gravitational force becomes 4 times.

#464729

Topic: Acceleration due to Gravity

What is the acceleration of free fall?

Solution

Acceleration of free fall is the acceleration experienced by the freely falling body the effect of gravitation of earth alone. It is also called acceleration due to gravity.

Its value on earth $g = 9.8m/s^2$

In free fall, the body moves under the effect of gravity only, no other external force is applied on it. The acceleration of free fall is equal to acceleration due to gravity.

$\therefore\ a_{freefall} = g = 9.8\ ms^{-2}$

#464731

Topic: Acceleration due to Gravity

What do we call the gravitational force between the earth and an object?

Solution

We call the gravitational force between the earth and the object as the weight of that object.

The gravitation force between the earth and object is called weight.

It is also equal to the product of acceleration due to gravity and mass of the object.

$Weight = m \times g$,

where, $m =$ mass of the object,

$g =$ acceleration due to gravity $= 9.8m/s^2$

#464734

Topic: Acceleration due to Gravity

Amit buys few grams of gold at the poles as per the instruction of one of his friends. He hands over the same when he meets him at the equator. Will the friend agree with the weight of gold bought? If not, why? [*Hint* : The value of g is greater at the poles than at the equator.]

Solution

We know that Weight $= m \times g$

where $m =$ mass of object

$g =$ acceleration due to gravity

We know that acceleration due to gravity is more at the poles than that at the equator.

As we move from Pole to equator, the value of g decreases, and hence weight of the gold also decreases.

So, His friend will not agree with the weight of the gold as it will be less at the equator than what it showed at poles.

Let m be the mass of the gold bought.

Also let the acceleration due to gravity at poles and at equator be g_p and g_e respectively.

Acceleration due to gravity at poles is slightly greater than that at equator.

$\therefore$ Weight of gold at poles $W_p = mg_p$

Weight of gold at equator $W_e = mg_e$

Bus as $g_e < g_p \implies W_p > W_e$

Hence the weight of gold is slightly less at equator.

So the friend will not agree with Amit about the weight of gold bought.

#464736

Topic: Acceleration due to Gravity

Why will a sheet of paper fall slower than one that is crumpled into a ball?

Solution

By crumpling the paper into a ball, the volume of the object decreases but the mass remains the same. Hence its density increases.

As air applies some resistance on falling sheet of paper as well as crumpled ball but the resistance to the motion of crumpled ball through the air decreases that makes it to fall faster than the sheet of paper.

Sheet of paper will occupy larger area as compare to when it is in ball form. So, when both sheet of paper and its ball is dropped at same time, air resistance offered on the surface of sheet of paper will be more. Hence a sheet of paper will fall slowly than one that is crumpled into a ball.

#464741

Topic: Acceleration due to Gravity

A ball is thrown vertically upwards with a velocity of 49 m/s. Calculate

(i) The maximum height to which it rises.

(ii) The total time it takes to return to the surface of the earth.

Solution

(i) By third equation of motion we know,

$v^2 - u^2 = 2 * g * s$

where,

v = final velocity $= 0$

u = initial velocity $= 49 m/s$

$g = -9.8 m/s^2$ as it is against gravity

So, $v^2 - u^2 = 2 * g * s \Rightarrow 0 - 49 * 49 = 2 * 9.8 * s$

$S = \frac{49 * 49}{19.6} = 122.5m$

(ii) Also by 1st equation of motion,

$v = u + gt$

$t = \frac{v - u}{-g} \Rightarrow \frac{0 - 49}{-9.8}$

$\Rightarrow \frac{49}{9.8} = 5s$

So, $5s$ is the time taken to go up.

It will take smae time to come down also.

So, total time taken to return to the surface $= 5 + 5 = 10s$

Given Initial velocity of ball, $u = 49\ m/s$

Let the maximum height reached and time taken to reach that height be H and t respectively.

Assumption: $g = 9.8\ m/s^2$ holds true (maximum height reached is small compared to the radius of earth)

Velocity of the ball at maximum height is zero, $v = 0$

$v^2 - u^2 = 2aH$

$0 - (49)^2 = 2 \times (-9.8) \times H$

$\implies H = 122.5\ m$

$v = u + at$

$0 = 49 - 9.8t$

$\implies t = 5$ s

$\therefore$ Total time taken by ball to return to the surface, $T = 2t = 10\ s$

#464744

Topic: Acceleration due to Gravity

A stone is released from the top of a tower of height 19.6m. Calculate its final velocity just before touching the ground.

Solution

Initial Velocity $u = 0$

Fianl velocity $v = ?$

Height, $s = 19.6m$

by third equation of motion

$v^2 = u^2 + 2gs$

$v^2 = 0 + 2 * 9.8 * 19.6$

$v^2 = 384.16$

$\Rightarrow v = 19.6m/s$

Given : Height of the tower $H = 19.6$ m

Initial velocity of the stone $u = 0$

Acceleration $a = g = 9.8\ ms^{-2}$

Let the final velocity of the stone be v

Using, $v^2 - u^2 = 2aH$

$v^2 - 0 = 2(9.8) \times 19.6 \implies v = 19.6\ \ ms^{-1}$

#464746

Topic: Acceleration due to Gravity

A stone is thrown vertically upward with an initial velocity of $40m/s$. Taking $g = 10m/s^2$, find the maximum height reached by the stone. What is the net displacement and the total distance covered by the stone?

Solution

Initial Velocity $u = 40$

Fianl velocity $v = 0$

Height, $s = ?$

by third equation of motion

$v^2 - u^2 = 2gs$

$0 - 40^2 = -2 * 10 * s$

$s = \frac{160}{20}$

$\Rightarrow s = 80m/s$

Toatl distance travelled by stone = upward distance + downwars distance $= 2 * s = 160m$

Total Diaplacement $= 0$, Since the initial and final point is same.

Given : Initial velocity of stone $u = 40\ m/s$

Let the maximum height reached and time taken to reach that height be H and t respectively.

Velocity of the stone at maximum height is zero i.e $v = 0$

Using, $v^2 - u^2 = 2aH$ where $a = -g = -10\ \ m/s^2$

$0 - (40)^2 = 2(-10)H \implies H = 80$ m

The stone, after reaching the maximum height, starts to fall down and reach the surface again.

$\therefore$ Net displacement of the stone is Zero

Total distance covered by the stone $S = 2H = 2 \times 80 = 160$ m

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#464750

Topic: Acceleration due to Gravity

A stone is allowed to fall from the top of a tower 100m high and at the same time another stone is projected vertically upwards from the ground with a velocity of $25m/s$. Calculate when and where the two stones will meet.

Solution

Relative displacement between the stones $= 100 - 0 = 100\ m$

Relative acceleration of the stones $= 10 - 10 = 0$

Relative initial velocity of the stones $= 25\ m/s$

Time to meet, $t = 100/25 = 4\ m$

For stone moving up,

$s = ut + (1/2)at^2$

$s = 25 \times 4 + (1/2) \times (-10) \times 4^2$

$s = 20\ m$

i.e. They meet 20 m above ground

Let the stones meet at point A after time t.

For upper stone :

$u' = 0$

$x = 0 + \frac{1}{2}gt^2$

$x = \frac{1}{2} \times 10 \times t^2$

$\implies x = 5t^2$ ....(1)

For lower stone :

$u = 25 \ m/s$

$100 - x = ut - \frac{1}{2}gt^2$

$100 - x = (25)t - \frac{1}{2} \times 10 \times t^2$

$\implies 100 - x = 25t - 5t^2$ ....(2)

Adding (1) and (2), we get

$25t = 100$

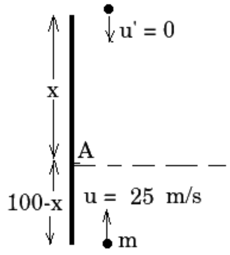
$\implies t = 4 \ s$

From (1),

$x = 5 \times 4^2$

$\implies x = 80 \ m$

Hence the stone meet at a height of 20 m above the ground after 4 seconds.



#464753

Topic: Acceleration due to Gravity

A ball thrown up vertically returns to the thrower after 6s. Find

(a) The velocity with which it was thrown up,

(b) The maximum height it reaches, and

(c) Its position after 4s.

Solution

The ball returns to the ground after 6 seconds.

Thus time taken by the ball to reach to the maximum height (h) is 3 seconds i.e $t = 3$ s

Let the velocity with which it is thrown up be u

For upward motion,

$v = u + at$

$\therefore \quad 0 = u + (-10) \times 3$

$\implies u = 30 \ m/s$

$h = ut + \frac{1}{2}at^2$

$h = 30 \times 3 + \frac{1}{2}(-10) \times 3^2$

$h = 45 \ m$

After 3 second, it starts to fall down.

Let the distance by which it fall in 1 s be d

$d = 0 + \frac{1}{2}at'^2$ where $t' = 1$ s

$d = \frac{1}{2} \times 10 \times (1)^2 = 5 \ m$

$\therefore$ Its height above the ground, $h' = 45 - 5 = 40 \ m$

Hence after $4 \ s$, the ball is at a height of $40 \ m$ above the ground.