NCERT SOLUTIONS
CLASS-XI PHYSICS
CHAPTER-9
MECHANICAL PROPERTIES OF SOLIDS

Q1. A steel wire of cross-sectional area $3 \times 10^{-5} m^2$ and length of 5m stretches by the same length as a copper wire of cross-sectional area $4.0 \times 10^{-5} m^2$ and length 4m under a given load. Find the ratio of Young's modulus of copper to that of steel?

Ans. Given, Length of the steel wire, $L_1 = 5$ m

Cross-sectional area of the steel wire, $A_1 = 3.0 \times 10^{-5}$ m^2
Length of the copper wire, $L_2 = 4$ m

Cross-sectional area of the copper wire, $A_2 = 4.0 \times 10^{-5}$ m^2

Change in length = $\Delta L_1 = \Delta L_2 = \Delta L$

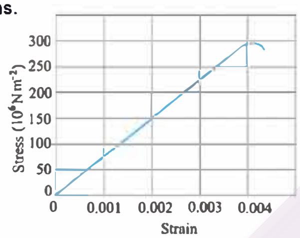
Let the force being applied in both the situations = F

We know, Young's modulus of the steel wire
$Y_1 = (F_1 / A_1) (L_1 / \Delta L_1)$

$= (F / 3 \times 10^{-5}) (5 / \Delta L)$ . . . . (1)
Also, Young's modulus of the copper wire
$Y_2 = (F_2 / A_2) (L_2 / \Delta L_2)$
$= (F / 4 \times 10^{-5}) (3 / \Delta L)$ . . . . (2)
Dividing equation (1) by equation (2), we get
$Y_1 / Y_2 = (5 \times 4 \times 10^{-5}) / (3 \times 10^{-5} \times 4)$
= 1.6 : 1
The ratio of Young's modulus of steel to Young's modulus of copper is 1.6

Q2. The graph given below is the stress-strain curve of a material. Find this material's (i)Approximate yield strength (ii) Young's modulus.

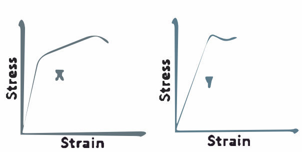
Ans.



(i) It can be seen from the graph that the approximate yield strength of this material is $300 \times 10^6 Nm/^2$ or 3×10^8 N/m^2.

(ii) It is observed from the given graph that for strain 0.001, stress is $75 \times 10^6 N/m^2$.
∴ We know, Young's modulus, Y = Stress / Strain
$= 75 \times 10^6 / 0.001 = 7.5 \times 10^{10}$ Nm^{-2}

Q3. Given below are stress-strain curves for two materials X and Y.



If both the graphs are drawn to the same scale.

(a) Identify the material with a greater Young's Modulus.

(b) Identify the stronger material.

Ans.

(a) Comparing the two graphs we can infer that the stress on X is greater than that on Y for the same values of strain. Therefore, Young's Modulus (stress/strain) is greater for X.

(b) As X's Young's modulus is higher, it is the stronger material among the two. For strength is the measure of stress a material can handle before breaking.

Q4. A state with reasons whether the following statements are true or false.

(a) Shear modulus determines how much a coil can stretch.

(b) Rubber has Young's Modulus greater than that of steel.

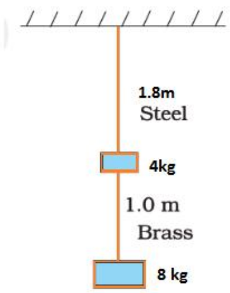
Ans.

(a) True. Stretching a coil does not change its length, only its shape is altered and this involves shear modulus.

(b) False. This is because, for the same value of stress, there is more strain in rubber than in steel. And as Young Modulus is an inverse of strain, it is greater in steel.

Q5. A wire made up of steel and brass and having a diameter of 0.30 cm is loaded as depicted in the diagram below. The unloaded length of the brass wire is 1 m and that of the steel wire is 1.8.Find the length of elongations in the brass and steel wires.

Ans.



Given,

Diameter of the wire, d = 0.25 m
Hence, the radius of the wires, r = d/2 = 0.125 cm
Length of the steel wire, L_1 = 1.8 m
Length of the brass wire, L_2 = 1.0 m

Total force exerted on the steel wire:

$F_1 = (4 + 8) g = 12 \times 9.8 = 117.6$ N
We know, Young's modulus for steel :
$Y_1 = (F_1/A_1) / (\Delta L_1 / L_1)$

Where,
ΔL_1 = Change in the length of the steel wire
A_1 = Area of cross-section of the steel wire = πr_1^2

We know ,Young's modulus of steel, $Y_1 = 2.0 \times 10^{11}$ Pa
$\therefore \Delta L_1 = F_1 \times L_1 / (A_1 \times Y_1)$
$= (117.6 \times 1.8) / [\pi(0.125 \times 10^{-2})^2 \times 2 \times 10^{11}] = 2.15 \times 10^{-4}$m

Total force on the brass wire:
$F_2 = 8 \times 9.8 = 78.4$ N

Young's modulus for brass:

$Y_2 = 0.91 \times 10^{11}$ Pa
Where,
ΔL_2 = Change in the length of the brass wire
A_1 = Area of cross-section of the brass wire = πr_1^2
$\therefore \Delta L_2 = F_2 \times L_2 / (A_2 \times Y_2)$
$= (78.4 X 1) / [\pi \times (0.125 \times 10^{-2})^2 \times (0.91 \times 10^{11})] = 1.75 \times 10^{-4}$ m
Therefore, Elongation of the steel wire = 2.15×10^{-4}m, and
Elongation of the brass wire = 1.75×10^{-4} m

Q6. An aluminium cube has an edge 8cm long, while one face is firmly against the wall the other is pressed against a 50 kg body. If the shear modulus of aluminium is 25 GPa. Find the vertical deflection of this face.

Ans.

Given,

Shear modulus (η) of aluminium = 25 GPa = 25×10^9 Pa

Edge of the cube, L = 8 cm = 0.08 m

The mass attached to the cube, m = 50 kg

Shear modulus, η = Shear stress / Shear strain = (F/A) / (L /ΔL)

Where,
F = Applied force = mg = 50 × 9.8 = 490 N
A = Area of one face of the cube = 0.08 × 0.08 = 0.0064 m^2
ΔL= Vertical deflection of the cube
∴ ΔL = FL / Aη
= 490 × 0.08 / [0.0064× (25×10^9)]
= 2.45×10^{-7} m
Therefore the vertical deflection of this face of the cube is 2.45×10^{-7} m.

Q7. A big building of 25000 kg mass is supported by four follow cylindrical columns made of mild steel. If the outer and inner radii of each column are 60 cm and 40 cm respectively, find the compressional strain of each column. (Consider the load distribution is in uniform)

Ans.

A.Given,

Mass of the building, M = 25,000 kg

Outer radius of the column, R = 60 cm = 0.6 m

Inner radius of the column, r = 40 cm = 0.4 m

Young's modulus of steel, Y = 2×10^{11} Pa

We know,

Total force exerted, F = Mg = 25000 × 9.8 N = 245000 N
Stress = Force exerted on a single column = 245000 / 4 = 61250 N
Also, Young's modulus, Y = Stress / Strain

Strain = (F/A) / Y
Where,
Area, A = $\pi (R^2 - r^2)$

= $\pi ((0.6)^2 - (0.4)^2)$

= 0.628
Strain = 61250 / [0.628× 2×10^{11}] = 4.87×10^{-7}
Therefore, the compressional strain of each column is 4.87×10^{-7}.

Q10. A column having a mass of 20 kg is supported by three wires each of length 2 m. The wires at the end are copper and the one in the middle is iron. If the tension on the three wires in the same, find the ratio of their diameters.

Ans.

As the tension on the wires is the same, the extension of each wire will also be the same. Now, as the length of the wires is the same, the strain on them will also be equal.

Now, we know :

Y = Stress / Strain
= (F/A) / Strain = $(4F/\pi d^2)$ / Strain . . . . (1)
Where,
A = Area of cross-section
F = Tension force
d = Diameter of the wire
We can conclude from equation (1) that $Y \propto (1/d^2)$
We know that Young's modulus for iron, $Y_1 = 190 \times 10^9$ Pa
Let the diameter of the iron wire = d_1
Also ,Young's modulus for copper, $Y_2 = 120 \times 10^9$ Pa
let the diameter of the copper wire = d_2
Thus, the ratio of their diameters can be given as :

$$\frac{d_1}{d_2} = \sqrt{\frac{Y_1}{Y_2}}$$

$= \sqrt{\frac{190\times10^9}{120x10^9}}$ =1 : 25 : 1

Q11. A 15 kg mass is tied to a steel wire of 1m (unstretched length). It is then spun in vertical circles where its angular velocity is 2 rev/s at the lowermost point. If the wire's cross-sectional area is 0.060 cm^2, find the elongation in the wire when the weight is at the lowest point.

Ans.

A.Given,

Mass, m = 15 kg
Length of the wire, l = 1.0 m
Angular velocity, ω = 2 rev/s = 2 × 2π rad/s = 12.56 rad / s
Cross-sectional area of the wire, a = 0.060 cm^2 = 0.06 × 10^{-4}m^2
Let Δl be the increase in the wire's length when the body is at the lower most point.
When the body is at the lowest point of the vertical circle , the force on the body is:
F = mg + mlω^2
= 15 × 9.8 + 15 × 1 × $(12.56)^2$
= 2513.304 N

We know, Young's modulus = Stress / Strain
Y = (F/A) / (Δl / l)
∴ Δl = Fl / AY
Also, young's modulus for steel = 2 × 10^{11} Pa
=> Δl = (2513.304 × 1) / (0.06 × 10^{-4} × 2 × 10^{11}) = 2.09 × 10^{-3} m

Therefore, the increase in the wire is 2.09 × 10^{-3} m.

Q12. Using the data provided, find the bulk modulus of water. Also, compare the bulk modulus of water and air (at constant temperature) and explain why the ratio is so large. Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 × 10^5 Pa), Final volume = 100.5 litre.

Ans.

Given,

P = 100 atmosphere

= 100 x 1.013 x 10^5 Pa

Final volume, V_2 = 100.5 l = 100.5 ×10^{-3} m^3

Initial volume, V_1 = 100.0 l = 100.0 × 10^{-3} m^3

Increase in volume, ΔV = $V_2 - V_1$ = 0.5 × 10^{-3} m^3

Bulk modulus = Δp / (ΔV / V_1) = Δ p × V_1 / ΔV
= [100 × 1.013 × 10^5 × 100 × 10^{-3}] / (0.5 × 10^{-3})
= 2.026 × 10^9 Pa

We know ,Bulk modulus of air = 1 × 10^5 Pa
∴ Bulk modulus of water / Bulk modulus of air = 2.026 × 10^9 /(1 × 10^5) = 2.026 × 10^4
This ratio is very large because air has more intermolecular space thus it is more compressible than water.

Q13. At a depth where the pressure is 60 atm find the density of water if at the surface it is 1.03 x 10^3 kg m^{-3} ?

Ans.

let the depth be the alphabet 'd'.

Given,
Pressure at the given depth, p = 60.0 atm = 60 × 1.01 × 10^5 Pa
Density of water at the surface, ρ_1 = 1.03 × 10^3 kg m^{-3}

Let ρ_2 be the density of water at the depth d.
V_1 be the volume of water of mass m at the surface.
Then, let V_2 be the volume of water of mass m at the depth h
and ΔV is the change in volume.
ΔV = $V_1 - V_2$
= m [$(1/\rho_1) - (1/\rho_2)$]
∴ Volumetric strain = ΔV / V_1

$= m [(1/\rho_1) – (1/\rho_2)] \times (\rho_1 / m)$

$\Delta V / V_1 = 1 – (\rho_1/\rho_2)$ . . . . . . (1)

We know, Bulk modulus, $B = pV_1 / \Delta V$

=> $\Delta V / V_1 = p / B$

Compressibility of water = (1/B) = 45.8×10^{-11} Pa^{-1}

∴ $\Delta V / V_1 = 60 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 2.78 \times 10^{-3}$ . . . . . (2)

Using equation (1) and equation (2), we get:

$1 – (\rho_1/\rho_2) = 2.78 \times 10^{-3}$

$\rho_2 = 1.03 \times 10^3 / [1 – (2.78 \times 10^{-3})]$

$= 1.032 \times 10^3$ kg m^{-3}

Therefore, at the depth d water has a density of 1.034×10^3 kg m^{-3}.

Q14. Calculate the fractional change in the volume of a glass plate when it is subjected to a pressure of100 atm.

Ans.

Given,

Pressure acting on the glass plate, p = 100 atm = $100 \times 1.013 \times 10^5$ Pa

We know,

Bulk modulus of glass, $B = 37 \times 10^9$ Nm^{-2}

=> Bulk modulus, $B = p / (\Delta V/V)$

Where,

$\Delta V/V$ = Fractional change in volume

∴ $\Delta V/V = p / B$

$= [100 \times 1.013 \times 10^5] / (37 \times 10^9)$

$= 2.73 \times 10^{-4}$

Therefore, the fractional change in the volume of the glass plate is 2.73×10^{-4}.

Q15. A solid copper cube with edges of length 5 cm is subjected to a hydraulic pressure of 8 x 10^6 Pa. Calculate the volume contraction in it.

Ans.

A.Given,

Hydraulic pressure, $p = 8.0 \times 10^6$ Pa

Edge length of the cube, l = 5 cm = 0.05 m

Bulk modulus of copper, $B = 140 \times 10^9$ Pa

We know, bulk modulus, $B = p / (\Delta V/V)$

Where,

V = Original volume = $V = l^3$

ΔV = Change in volume

.$\Delta V/V$ = Volumetric strain.

$\Delta V = pV / B$

∴ $\Delta V = pl^3 / B$

$= [8 \times 10^6 \times (0.05)^3] / (140 \times 10^9)$

$= 7.142 \times 10^{-9}$ m^3 = 7.142×10^{-3} cm^{-3}

Hence, the volume contraction of the solid copper cube is 7.142×10^{-3} cm^{-3}.

Q16. Calculate the pressure on a litre of water if it is to be compressed by 0.15%.

Ans.

Given, volume of water,V= 1 L

And water needs to be compressed by 0.15%.

∴ Fractional change, $\Delta V / V = 0.15 / (100 \times 1) = 1.5 \times 10^{-3}$

We know,

Bulk modulus, $B = \rho / (\Delta V/V)$

=> $\rho = B \times (\Delta V/V)$

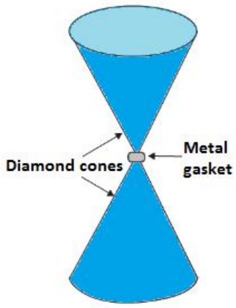
We know, bulk modulus of water, $B = 2.2 \times 10^9$ Nm^{-2}

=> $\rho = 2.2 \times 10^9 \times 1.5 \times 10^{-3} = 3.3 \times 10^6$ Nm^{-2}

Thus, a pressure of 3.3×10^6 Nm^{-2} should be applied on the water.

Q17. A diamond anvil cell is used to create extremely high-pressure environments. The narrow ends of the anvil have a diameter of 0.50 mm and the wide ends are subjected to a compressive force of 80,000 N. Calculate the pressure at the tip of the anvil.

Ans.

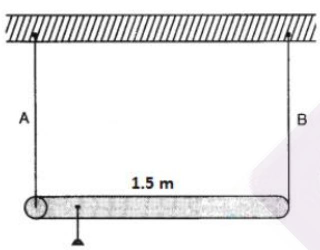


Given,

Diameter at the narrow ends, d = 0.50 mm = 0.5×10^{-3} m
Radius, r = d/2 = 0.25×10^{-3} m
Compressional force, F = 80000 N
Therefore the pressure at the tip of the anvil:
P = Force / Area = $80000 / \pi(0.25 \times 10^{-3})^2$
= 4.07×10^{11} Pa

Q18. A stick of length 1.5 m is supported by a steel wire (wire A) and an aluminum wire (wire B) from its two ends. The wires A and B have a cross-sectional area of 1 mm^2 and 2 mm^2 respectively. Calculate the location of a point along the stick from where we can hang a body of mass m that will produce (i) equal stresses, and (ii) equal strains in both the wires. Assume the stick to be weightless.

Ans.



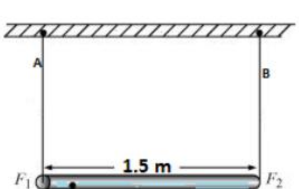
Given,

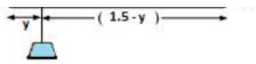
Cross-sectional area of wire A, a_1 = 1.0 mm^2 = 1.0×10^{-6} m^2
Cross-sectional area of wire B, a_2 = 2 mm^2 = 2×10^{-6} m^2

We know,Young's modulus for steel, $Y_1 = 2 \times 10^{11}$ Nm^{-2}
Young's modulus for aluminum, $Y_2 = 7.0 \times 10^{10}$ Nm^{-2}

(i) Let a mass m be hung on the stick at a distance y from the end where wire A is attached.
Stress in the wire = Force / Area = F / a
Now it is given that the two wires have equal stresses ;
$F_1 / a_1 = F_2 / a_2$

Where,
F_1 = Force acting on wire A
and F_2 = Force acting on wire B
$F_1 / F_2 = a_1 / a_2 = 1 / 2$ . . . . (1)
The above situation can be represented as :





Moment of forces about the point of suspension, we have:
$F_1 y = F_2 (1.5 – y)$
$F_1 / F_2 = (1.5 – y) / y$ . . . . (2)
Using equation (1) and equation (2), we can write:
$(1.5 – y) / y = 1 / 2$
$2 (1.5 – y) = y$
$y = 1$ m

Therefore, the mass needs to be hung at a distance of 1m from the end where wire A is attached in order to produce equal stress in the two wires.

(ii) We know,

Young's modulus = Stress / Strain
=> Strain = Stress / Young's modulus = (F/a)/ Y
It is given that the strain in the two wires is equal :
$(F_1/a_1) / Y_1 = (F_2/a_2) / Y_2$
$F_1 / F_2 = a_1Y_1 / a_2Y_2$
$a_1 / a_2 = 1 / 2$
$F_1 / F_2 = (1 / 2) (2 \times 10^{11} / 7 \times 10^{10}) = 10 / 7$ . . . . (3)

Let the mass m be hung on the stick at a distance y_1 from the end where the steel wire is attached in order to produce equal strain

Taking the moment of force about the point where mass m is suspended :

$F_1y_1 = F_2 (1.5 – y_1)$
$F_1 / F_2 = (1.5 – y_1) / y_1$ . . . . (4)

From equations (3) and (4), we get:
$(1.05 – y_1) / y_1 = 10 / 7$
$7(1.05 – y_1) = 10y_1$
$y_1 = 0.432$ m

Therefore, the mass needs to be hung at a distance of 0.432 m from the end where wire A is attached in order to produce equal strain in the two wires.

Q19. The deepest known point in our planet's oceans is the Marina Trench, it is 10,994 m deep and the water pressure at its bottom is 1000 atm. If a steel ball having an initial volume of 0.30 m ^{3} is dropped into the trench, find the change in the volume of this ball when it hits the bottom.

Ans.

Given,

Water pressure at the bottom, $p = 1000$ atm $= 1000 \times 1.013 \times 10^5$ Pa

$p = 1.01 \times 10^8$ Pa
Initial volume of the steel ball, $V = 0.30$ m^3

We know, bulk modulus of steel, $B = 1.6 \times 10^{11}$ Nm$^{–2}$

Let the change in the volume of the ball on reaching the bottom of the trench be ΔV.
Bulk modulus, $B = p / (\Delta V/V)$
$\Delta V = pV / B$
$= [1.01 \times 10^8 \times 0.30] / (1.6 \times 10^{11}) = 1.89 \times 10^{-4}$ m^3
Hence, volume of the ball changes by 1.89×10^{-4} m^3 on reaching the bottom of the trench.

Q20. Two metal bars are riveted together at their ends by four rivets, each having a diameter of 5 mm. Calculate the maximum tension the riveted bars can bear if the maximum shearing stress a rivet can take is 6.9 x 10^7 Pa. Consider that each rivet carries ¼ of the total load.

Ans.

Given,

Diameter of the metal bar, $d = 5.0$ mm $= 5.0 \times 10^{–3}$ m
Radius, $r = d/2 = 2.5 \times 10^{-3}$ m

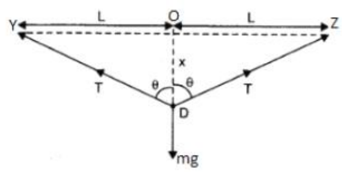
Maximum shearing stress $= 6.9 \times 10^7$ Pa

We know,

Maximum stress = Maximum force or tension / Area
=> Maximum force = Maximum stress × Area
$= 6.9 \times 10^7 \times \pi \times (2.5)^2$
$= 6.9 \times 10^7 \times \pi \times (2.5 \times 10^{-3})^2$
= 1354.125 N
Since each rivet carries ¼ of the load.
∴ Maximum tension on each rivet = 4 × 1354.125 = 5416.5 N.

Q22. A mild steel wire of cross-sectional area 0.60×10^{-2} cm^2 and length 2 m is stretched (not beyond its elastic limit) horizontally between two columns. If a 100g mass is hung at the midpoint of the wire, find the depression at the midpoint.

Ans.



Let YZ be the mild steel wire of length 2l = 2m and cross sectional area A = 0.60×10^{-2} cm^2 . Let the mass of m = 100 g = 0.1 kg be hung from the midpoint O, as shown in the figure. And let x be the depression at the midpoint i.e OD

From the figure;

ZO =YO = l = 1 m ;

M = 0.1 KG

ZD = YD = $(l^2 + x^2)^{1/2}$

Increase in length, Δl = YD + DZ – ZY

= 2YD – YZ (As DZ = YD)

$= 2(l^2 + x^2)^{1/2} - 2l$

$\Delta l = 2l(x^2/2l^2) = x^2 / l$

Therefore, longitudinal strain = $\Delta l / 2l = x^2/2l^2$ . . . . (i)

If T is the tension in the wires, then in equilibrium 2Tcosθ = 2mg

Or, T = mg / 2cos θ

$= [mg (l^2 + x^2)^{1/2}] / 2x = mgl / 2x$

Therefore, Stress = T / A = mgl / 2Ax . . . . (ii)

$Y = \frac{stress}{strain} = \frac{mgl}{2Ax} * \frac{2l^2}{x^2}$

$= \frac{mgl^3}{2Ax^3}$

x = $l[\frac{mg}{YA}]^{\frac{1}{3}}$ = $1[\frac{0.1*10}{20*10^{11}*0.6*10^{-6}}]^{\frac{1}{3}}$

= 9.41×10^{-3} m.