NCERT SOLUTIONS
CLASS-XI PHYSICS
CHAPTER-13
KINETIC THEORY

Q.1: Calculate the ratio of molecular volume to the actual volume of oxygen gas at standard temperature and pressure. (Diameter of oxygen molecule = 3Å)

Sol: Diameter of an oxygen molecule, **d = 3Å**

Radius, r =d/2 = 1.5 Å **= 1.5 × 10^{-8} cm**

We know:

Actual volume occupied by 1 mole of oxygen at **STP = 22400 cm^3**

Molecular volume of oxygen, **V = $N_A(4\pi r^3/3)$**

Where, **N** is **Avogadro's number** = **6.023 × 10^{23} molecules/mole**

Therefore, molecular volume of oxygen ,V = 6.023 × 10^{23} x 3.14 (1.5 × $10^{-8})^2$ x (4/3) = **8.51 cm^3**

Thus, ratio of the molecular volume to the actual volume of oxygen = 8.51/22400 = 3.8 x 10^{-4}

Q.2: The volume occupied by one mole of any (ideal) gas at STP is called molar volume (STP : 0 °C, 1 atmospheric pressure). Prove that molar volume is 22.4 litres.

Ans: We know that the ideal gas equation: PV = nRT

Where, R is the universal gas constant = 8.314 J mol^{-1} K^{-1}

n = Number of moles = 1

T = Standard temperature = 273 K

P = Standard pressure = 1 atm = 1.013 × 10^5 Nm^{-2}

Thus, V = (nRT)/p

= (1 x 8 314 x 273)/(1 013 × 10^5)

= 0.0224 m^3

= 22.4 liters

Thus, it is proved that molar volume of a gas at standard temperature and pressure is 22.4 liters.

Q.3. The figure below is a graph of PV/T versus P for 1 x 10^{-3} kg of oxygen at two different temperatures.

(i) What is the significance of the dotted plot?

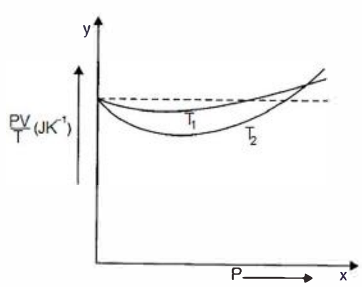
(ii) T1 < T2 or T1 > T2, which one is true?

(iii) Find the value of PV/T where the curves come together on the y axis.

(iv) In place of oxygen if we used 1 x 10^{-3} kg of hydrogen and we plotted similar graphs, would the value of PV/T be the same at the point where the curves come in contact with the y axis?

If the answer is no, find the mass of hydrogen that would give the same value (for high temperature low pressure region of the graph).

[Molecular mass of H_2 = 2.02 u, of O_2 = 32.0 u, R = 8.31 J mol^{-1} K^{-1}]



Sol:

(i) The dotted plot signifies the ideal gas behavior of oxygen as it is parallel to P –axis and it says that the ratio PV/T remains constant even when P is changed.

(ii) The dotted line in the plot stands for an ideal gas. At temperature T_1 the curve of the gas is closer to the dotted plot than at temperature T_2. A real gas behaves more like an ideal gas when its temperature rises. Thus dotted plot than at temperature T_2. A real gas behaves more like an ideal gas when its temperature rises. Thus, T1 > T2 is true for the given graph.

(iii) At the point where the curves meet PV/T = μR
Where μ = no. of moles = 1/32
R = 8.314 J mol^{-1} K^{-1}
Thus, PV/T = (1/32) x 8.314 **= 0.26 J K^{-1}**
(iv) Even if we obtain a similar curve for 1 x 10^{-3} kg of hydrogen, we will not get the same value for PV/T because the molar mass of H_2 is 2.02 u and not 32u.
We have:
PV / T = 0.26
Given:
Molecular mass of hydrogen , M = 2.02 u
PV/T = μ R
where, μ = m/M
i.e. PV/T = R (m/M)
i.e. m = MVP/ TR
= 0.26 x (2.02/8.31) **= 6.3 x 10^{-5} kg**

Hence, 6.3 x 10^{-5} kg of H_2 will give the value of PV/T = 0.26 J K^{-1}
Q.4: A 30 liters oxygen cylinder has an initial temperature and gauge pressure of 27 ^{0}C and 20 atm respectively. When a certain amount of oxygen escapes from the cylinder the temperature and gauge pressure drops to 17 ^{0}C and 22 atm, respectively. Find the mass of oxygen that escaped the cylinder. [R = 8.31 J mol^{-1} K $^{-1}$, molecular mass of O_2 = 32 u]

Sol:

Given:
Initial volume of oxygen, V_1 = 30 liters **= 30 × 10^{-3} m^3**
Gauge pressure, P_1 = 30, atm **= 30 × 1.013 × 10^5 Pa**
Temperature, T_1 = 27°C **= 300 K**
Universal gas constant, **R = 8.314 J $mole^{-1}$ K^{-1}**
Let the initial number of moles of oxygen in the cylinder be n_1 .
We know:

$P_1 V_1 = n_1 R T_1$
i.e. $n_1 = P_1 V_1 / RT_1$
= (30.39 × 10^5 × 30 × 10^{-3}) / (8.314 × 300)
= 36.552
But, $n_1 = m_1 / M$
Where,
m_1 = initial mass of oxygen
M = molecular mass of oxygen = 32 g

i.e. $m_1 = n_1$ x M = 36.552 x 32 = 1169.6 g
After some oxygen escapes:
Volume, V_2 = 30 x 10^{-3} m^3
Gauge pressure, P_2 = 22; atm = 22 × 1.013 × 10^5 Pa
Temperature, **T_2 =** 17°C **= 290 K**
Let the number of moles of oxygen left in the cylinder be n_2.
Now:
$P_2 V_2 = n_2 RT_2$
i.e. $n_2 = P_2 V_2 / RT_2$
= (22.286 × 10^5 × 30 × 10^{-3})/(8.314 × 290) = 27.72
But, **$n_2 = m_2 / M$**
Where, m_2 = remaining mass of oxygen
i.e. $m_2 = n_2$ x M = 27.72 x 32 **= 906.2g**
Therefore the mass of oxygen that escaped the cylinder = $m_1 - m_2$ = 1169.6 – 906.2 = 263.4 g

Q.5: An air bubble occupies a volume of 2 cm^3 at the bottom of 20m deep lake. Assuming the bottom temperature of the lake is 12 ^{0}C , find the volume of this air bubble when it rises up to the lake surface which is at 35 ^{0}C ?

Sol:

Given:
Volume of the air bubble, V_1 = 2.0 cm^3 = 2.0 x 10^{-6} m^3

Volume of the air bubble, $V = 2.0\ cm^3$ **= $2.0 \times 10^{-6}\ m^3$**
Bubble ascends a height of , **d = 20 m**
Temperature at a depth of 40 m, T = 12°C **= 285 K**
Temperature at the surface of the lake, T' = 35°C **= 308 K**

The pressure on the surface of the lake: $P' = 1\ atm = 1 \times 1.013 \times 10^5$ Pa

And, The pressure at the bottom: **P = 1atm + dρg**
Where, **ρ** is the density of water **= $10^3\ kg/m^3$**
g is the acceleration due to gravity **= $9.8\ m/s^2$**
i.e. $P = 1.013 \times 10^5 + 20 \times 10^3 \times 9.8$ **= 297300 Pa**
We know:

PV/T = P'V'/T'
Where, V' is the volume of the bubble at the surface.
V' = PVT'/P'T
$= (297300 \times 2 \times 10^{-6} \times 308) / (1.013 \times 10^5 \times 285)$ **= $6.34 \times 10^{-6}\ m^3$ or $6.34\ cm^3$**
Therefore, the volume of this bubble when it reaches the surface is $6.34\ cm^3$.

Q.6: In a $50\ m^3$ room, at a pressure of 1 atm and temperature 27^0C, what is the number of air molecules (oxygen, nitrogen, water vapour and other constituents) present?

Sol:

Given:
Volume of the room, V = $50.0\ m^3$

Temperature of the room, T = 27°C **= 300 K**

Pressure in the room, P = 1 atm **= $1 \times 1.013 \times 10^5$ Pa**

According to gas equation:

$PV = k_BNT$

Where, **k_B is Boltzmann constant = $1.38 \times 10^{-23}\ m^2\ kg\ s^{-2}\ K^{-1}$**
N is the number of air molecules in the room
Now, **$N = PV/k_BT$**

$= (1.013 \times 10^5 \times 50) / (1.38 \times 10^{-23} \times 300) = 1.22 \times 10^{27}$
Therefore there is 1.22×10^{27} molecules in the room.

Q.7: Calculate the average thermal energy of a helium atom at

(i) room temperature ($27\ ^0C$),

(ii) the core of the earth (6150 K),

(iii) at the core of the sun (10 million K)

Sol:

Given:
(i) At room temperature, T = 27°C = 300 K
Thus, average thermal energy = kT x (3/2)
Where k is Boltzmann constant = $1.38 \times 10^{-23}\ m^2\ kg\ s^{-2}\ K^{-1}$
Thus,
$kT \times (3/2) = 1.38 \times 10^{-23} \times 300 \times 1.5$ **= 6.21×10^{-21} J**

(ii) In the core of the earth, T = 6150 K
Thus, average thermal energy = kT x (3/2)
i.e. $kT \times (3/2) = 1.38 \times 10^{-19} \times 6150 \times 1.5$ **= 1.27×10^{-19} J**
(iii) At core of the sun, T = 10^7
Thus, average thermal energy = kT x (3/2)

i.e. $kT \times (3/2) = 1.38 \times 10^{-19} \times 10^7 \times 1.5$ **= 2.07×10^{-16} J**

Q.8: Three containers A, B and C , having the same capacity, contains neon (monatomic), chlorine (diatomic) and uranium hexafluoride (polyatomic) respectively at the same pressure and temperature .Do all the containers contain the same number of molecules? Also, do the molecules in the respective

containers have the same root mean square value of speed? If not, molecules of which gas has the highest value of V_{rms} ?

Sol:

According to **Avogadro's principle**, gases of the same volume at the same values of temperature and pressure will contain the same number of molecules. Thus, in the above case all the containers will contain **equal number of molecules.**
For a gas of mass (m) at temperature (T), its root mean square speed;
$V_{rms} = \sqrt{\frac{3kT}{m}}$
Where k is the **Boltzmann constant**.
As **k and T are constants**, we get:
$V_{rms} = \sqrt{\frac{1}{m}}$
Thus, V_{rms} is not the same for the molecules of the three gases.
As mass of neon is the least, it will have the highest V_{rms} .

Q.9: Calculate the temperature at which the root mean square speed of an argon atom is the same as the root mean square speed of a helium gas atom at – 20 ^{0}C.
[Atomic mass of Ar = 39.9 u, of He = 4.0 u]

Sol:

Given:
Temperature of the **helium atom, T' =** –20°C **= 253 K**
Atomic mass of **argon, M = 39.9 u**
Atomic mass of **helium, M' = 4.0 u**
Let, (V_{RMS}) Ar be the rms speed of argon and (V_{RMS}) He be the rms speed of helium.
Now, we know:

$(V_{RMS})Ar = \sqrt{\frac{3RT}{M}}$ . . . . **(1)**
Where, **R** is the **universal gas constant** and **T** is **temperature of argon** gas
Now, **(V_{RMS})He =** $\sqrt{\frac{3RT'}{M'}}$ . . . . **(2)**
According to the question :
(V_{RMS})Ar = (V_{RMS})He

i.e. $\sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT'}{M'}}$
i.e. T/M = T' / M'
T = M × (T'/M')
Therefore the temperature of argon, T = 39.9 × 253/4 = 2.52 × 10^3 K

Q.10: A cylinder contains nitrogen at 2 atm and 17 ^{0}C, find the collision frequency and the mean free path of a nitrogen molecule inside it. Considering the nitrogen molecule to have a radius of 1Å compare the time between two consecutive collisions and the collision time. [Molecular mass of N_2 = 28]

Sol:

Given:
Pressure inside the cylinder containing **nitrogen**, P = 1.0 atm = **1 × 1.013 × 10^5 Pa**

Temperature inside the cylinder, T = 17°C **= 290 K**
Radius of a nitrogen molecule, r = 1.0 Å **= 1 × 1010 m**
Diameter, d = 2 × 1 × 1010 **= 2 × 10^{-10} m**
Molecular mass of nitrogen, M = 28.0 g **= 28 × 10^{-3} kg**
We know, the **root mean square speed ,V_{RMS} =** $\sqrt{\frac{3RT}{M}}$
V_{RMS} = $\sqrt{\frac{3\times8.314\times290}{28\times10^{-3}}}$ **= 508.26 m/s**
For the mean free path (l) we have:
l = $\frac{kT}{\sqrt{2}\times\pi\times d^2\times P}$
Where, k is 1.38 x 10 $^{-23}$ kg m^2 s^{-2} K^{-1}
Therefore, l = $\frac{1.38\times10^{-23}\times290}{\sqrt{2}\times3.14\times(2\times10^{-10})^2\times1.013\times10^5}$ **= 2.22 x 10^{-7} m**
And, **Collision frequency** = V_{RMS} / l **= 2.29 x 10 ^{9} s^{-1}**
Collision time T = d / V_{RMS}

= 2 x 10^{-10} / 508.26 = 2.18 x 10^{-10} s **= 3.93 x 10^{-13} s**
Time between consecutive collisions:

T' = l / V_{RMS}

= 2.22 x 10^{-7} / 508.26 **= 4.36 x 10^{-10} s**

= 2.22 x 10^{-7} / 508.26 **= 4.36 x 10^{-10} s**

Thus, T'/ T = (4.36 x 10^{-10})/ (2.22 x 10^{-7}) **= 1109.41.**

Therefore the time between two consecutive collisions is 1109.41 times the collision time.

Q11: **A narrow bore a meter long held horizontally contains a mercury thread of 70 cm, which traps air column of 20 cm. What will happen if tube is vertically held with bottom end open?**

Sol:

Length of mercury thread, **l = 70 cm**

Length of the narrow bore, **L = 1 m = 100 cm**

The air column length in between the closed end & mercury, **l_a = 20 cm**

Since the bottom end is open and the bore is vertically held in air, the air space occupied by the mercury length is: **100 – (70 + 20) = 10 cm**

Hence, total air column length = 20 + 10 **= 30 cm**

Let, mercury out flow due to atmospheric pressure be **'h' cm**

Therefore,

The air column length in the bore **= (30 + h) cm**

And, mercury column length **= 80 – h cm**

Initial pressure, **P_1 = 80 cm of mercury**

Initial volume, **V_1 = 20 cm^3**

Final pressure, **P_2 = 80 – (80 – h) = h cm of mercury**

Final volume is **V_2 = (30 + h) cm^3**

Throughout the process the temperature is constant.

$$P_1V_1 = P_2V_2$$
$$70 \times 20 = h\,(30 + h)$$
$$h^2 + 30h – 1400 = 0$$

Therefore, $h = \frac{-30 \pm \sqrt{(30)^2 + 4 \times 1 \times 1400}}{2 \times 1}$

$= -55.3\ cm\ or\ 25.3\ cm$

Height is always positive. **Hence, mercury that flow out from bore is 25.3 cm and mercury that remains in it is 54.7 cm. The air column length is 30 + 25.3 = 55.3 cm**

Q12. Hydrogen gas's diffusion rate from one certain apparatus has average value 30 cm^2/ s. Under same condition the average diffusion rate of another gas is 8 cm^2/s. What gas is it?

[Hint: Graham's law of diffusion states that: $(M_2/ M_1)^{1/2} = R_1/ R_2$, where diffusion rates of gas 1 and gas 2 are given by R_1, R_2 and M_1 and M_2 are their molecular masses]

Sol:

Diffusion rate of hydrogen, R_1 = 30 cm^3/ s

Diffusion rate of the other gas, R_2 = 8 cm^3/ s

According to Graham's Law of diffusion, we have:

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

Where, Molecular mass of hydrogen **M_1 = 2.020 g**

Molecular mass of the unknown gas is M_2

Therefore, $M_2 = M_1\left(\frac{R_1}{R_2}\right)^2$

$= 2.02\left(\frac{30}{8}\right)^2$

$= 28.40\ g$

Nitrogen has the molecular mass 28. Hence the other gas is nitrogen.

Q.13: Throughout the volume of a gas in equilibrium the density and pressure is uniform. It is true only if no external influences are used. Gas column because of gravity doesn't have uniform density or pressure. Density of the gas decreases with height. The dependence precise is given by law of atmosphere $n_2 = n_1 \exp [-mg(h_2 - h_1)/k_BT]$ Where n_1, n_2 are referred to density at h_1 and h_2 respectively. The sedimentation equilibrium equation of liquid column can be derived by using this relation: $n_2 = n_1 exp[-mgN_A(\rho - \rho')(h_2 - h_1)/(\rho RT)]$ **Where ρ is the density of the particle suspended, and ρ' is surrounding medium's density. [NA = Avogadro's number & R the universal gas constant.] [To find the suspended particle's apparent weight use Archimedes principle]**

Sol:

From law of atmosphere, we have:

$n_2 = n_1 \exp[-mg(h_2 - h_1)/k_BT]$.... **(i)**

Where, at **height h_1, number density is n_1**, and at **height h_2, number density is n_2**

Weight of suspended particle in gas column is mg

Medium density = ρ'

Suspended particle density = ρ

Suspended particle mass **= m'**

Displaced medium's mass **= m**

Suspended particle's volume **= V**

Archimedes' principle states that the weight of the suspended particle in the liquid column is given by

Now, Displaced medium weight – suspended particle weight

$$= mg - m'g$$
$$= mg - V\rho' g = mg - \left(\frac{m}{\rho}\right)\rho' g$$
$$= mg\left(1 - \frac{\rho'}{\rho}\right) \ldots . (ii)$$
$$Gas\ constant,\ R = k_B N$$
$$K_B = \frac{R}{N} \ldots . (iii)$$
$$Substitution\ equation\ (ii)\ in\ place\ of\ mg\ in\ equation (i)\ and\ then\ using\ equation (iii), we; get:$$
$$n_2 = n_1\ exp[-mg(h_2 - h_1)/k_B T]$$
$$= n_1\ exp[-mg\left(1 - \frac{\rho'}{\rho}(h_2 - h_1)\frac{N}{RT\rho}\right)]$$

Q14: Density of some of the solids and liquids are given below. Provide rough estimates of their atom sizes:

Substance	**Atomic mass**	**Density (10^3 kg m^{-3})**
Carbon (diamond)	**12.01**	**2.22**
Gold	**197.00**	**19.32**
Nitrogen	**14.01**	**1.00**
Lithium	**6.94**	**0.53**
Fluorine (liquid)	**19.00**	**1.14**

[Assume in solid and liquid phase the atoms are tightly packed, and use Avogadro's number. Do not take actual numbers obtain for different atomic sizes? Because of tight packing approximation of the crudeness, the range of atomic size in between few Å]

Sol:

Substance	Radius (Å)
Carbon (diamond)	1.29
Gold	1.59

Nitrogen (liquid)	1.77
Lithium	1.73
Fluorine (liquid)	1.88

Substance's atomic mass = M

Substance's density = ρ

$$Avogadro's\ number = N = 6.023 \times 10^{23}$$
$$Each\ atom's\ volume = \frac{4}{3}\pi r^3$$
$$N\ number\ of\ molecules'\ volume = \frac{4}{3}\pi r^3 N \ldots.(i)$$
$$One\ mole's\ volume = \frac{M}{\rho} \ldots.(iii)$$
$$\frac{4}{3}\pi r^3 N = \frac{M}{\rho}$$

Therefore, r = $\sqrt[3]{\frac{3M}{4\pi\rho N}}$

For carbon:

$$M = 12.01 \times 10^{-3}\ kg,$$
$$\rho = 2.22 \times 10^3 kgm^{-3}$$

Therefore, r = $\left(\frac{3\times12.01\times10^{-3}}{4\pi\times2.22\times10^3\times6.023\times10^{23}}\right)^{\frac{1}{3}} = 1.29$

Hence, radius of carbon atom = 1.29 Å

For gold:

$$M = 197.01 \times 10^{-3}\ kg,$$
$$\rho = 19.32 \times 10^3 kgm^{-3}$$

Therefore, r = $\left(\frac{3\times197\times10^{-3}}{4\pi\times19.32\times10^3\times6.023\times10^{23}}\right)^{\frac{1}{3}} = 1.59$

Hence, radius of gold atom = 1.59 Å

For nitrogen (liquid):

$$M = 14.01 \times 10^{-3}\ kg,$$
$$\rho = 1.00 \times 10^3 kgm^{-3}$$

Therefore, r = $\left(\frac{3\times14.01\times10^{-3}}{4\pi\times1.00\times10^3\times6.23\times10^{23}}\right)^{\frac{1}{3}} = 1.77$

Hence, radius of nitrogen (liquid) atom = 1.77 Å

For lithium:

$$M = 6.94 \times 10^{-3}\ kg,$$
$$\rho = 0.53 \times 10^3 kgm^{-3}$$

Therefore, r = $\left(\frac{3\times6.94\times10^{-3}}{4\pi\times0.53\times10^3\times6.23\times10^{23}}\right)^{\frac{1}{3}} = 1.73$

Hence, radius of lithium atom = 1.73 Å

For fluorine (liquid):

$$M = 19.00 \times 10^{-3}\ kg,$$
$$\rho = 1.14 \times 10^3 kgm^{-3}$$

Therefore, r = $\left(\frac{3\times19\times10^{-3}}{4\pi\times1.14\times10^3\times6.023\times10^{23}}\right)^{\frac{1}{3}} = 1.88$

Hence, radius of fluorine (liquid) atom = 1.88 Å