Chapter 15

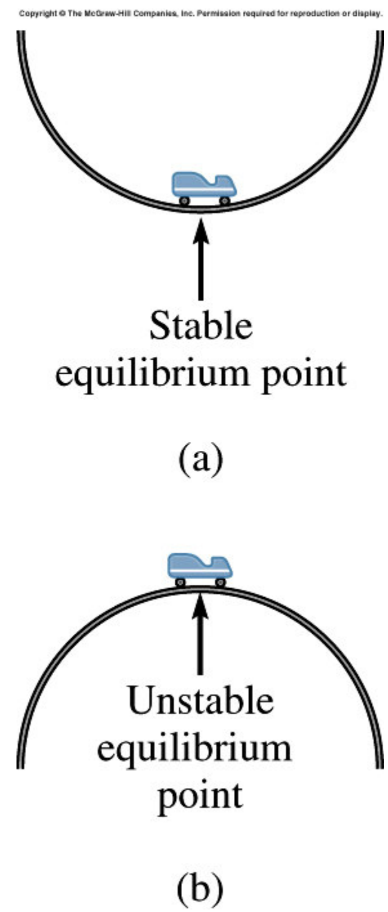
Oscillations and Waves

Oscillations and Waves

- Simple Harmonic Motion
- Energy in SHM
- Some Oscillating Systems
- Damped Oscillations
- Driven Oscillations
- Resonance

Simple Harmonic Motion

Simple harmonic motion (SHM) occurs when the **restoring force** (the force directed toward a stable **equilibrium point**) is proportional to the displacement from equilibrium.

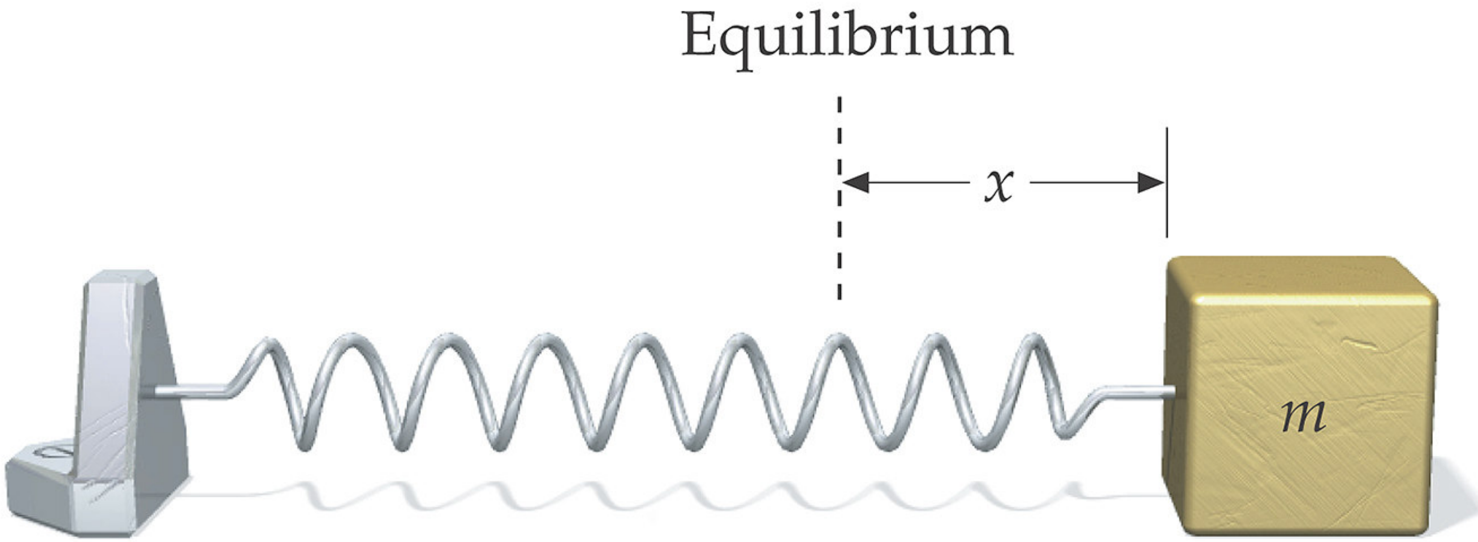


Characteristics of SHM

- Repetitive motion through a central equilibrium point.
- Symmetry of maximum displacement.
- Period of each cycle is constant.
- Force causing the motion is directed toward the equilibrium point (minus sign).
- F directly proportional to the displacement from equilibrium.

$$\text{Acceleration} = -\,\omega^2 \times \text{Displacement}$$

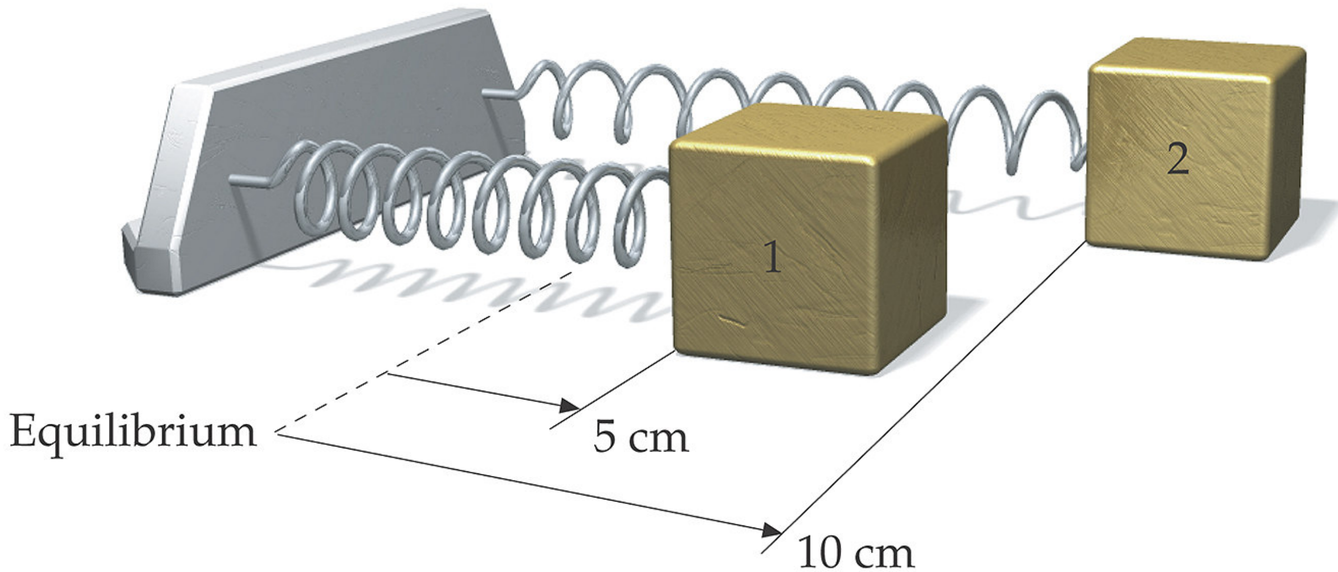
A Simple Harmonic Oscillator (SHO)



Frictionless surface

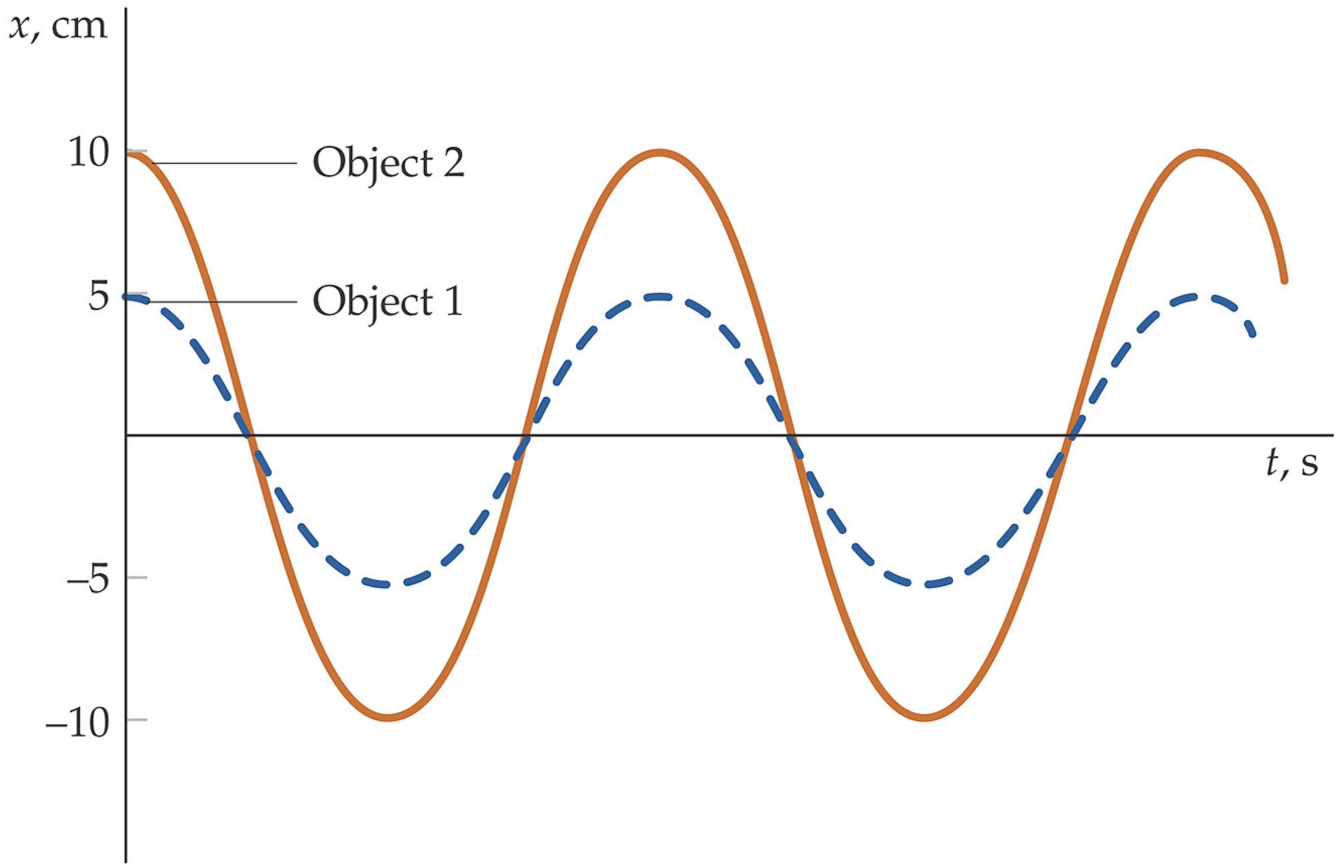
The restoring force is $F = -kx$.

Two Springs with Different Amplitudes



Frictionless surface

SHO Period is Independent of the Amplitude



The Period and the Angular Frequency

The period of oscillation is $T = \dfrac{2\pi}{\omega}.$

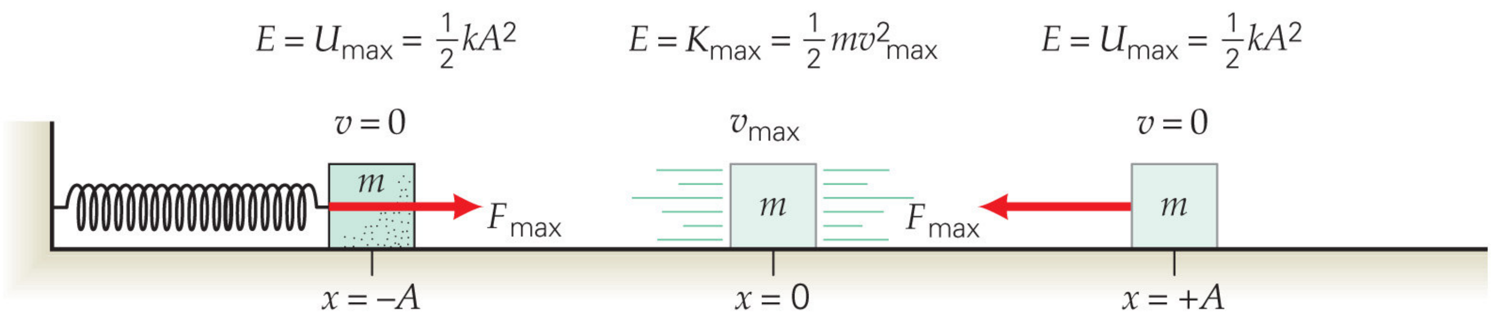
where ω is the angular frequency of the oscillations, k is the spring constant and m is the mass of the block.

$$\omega = \sqrt{\frac{k}{m}}$$

Simple Harmonic Motion

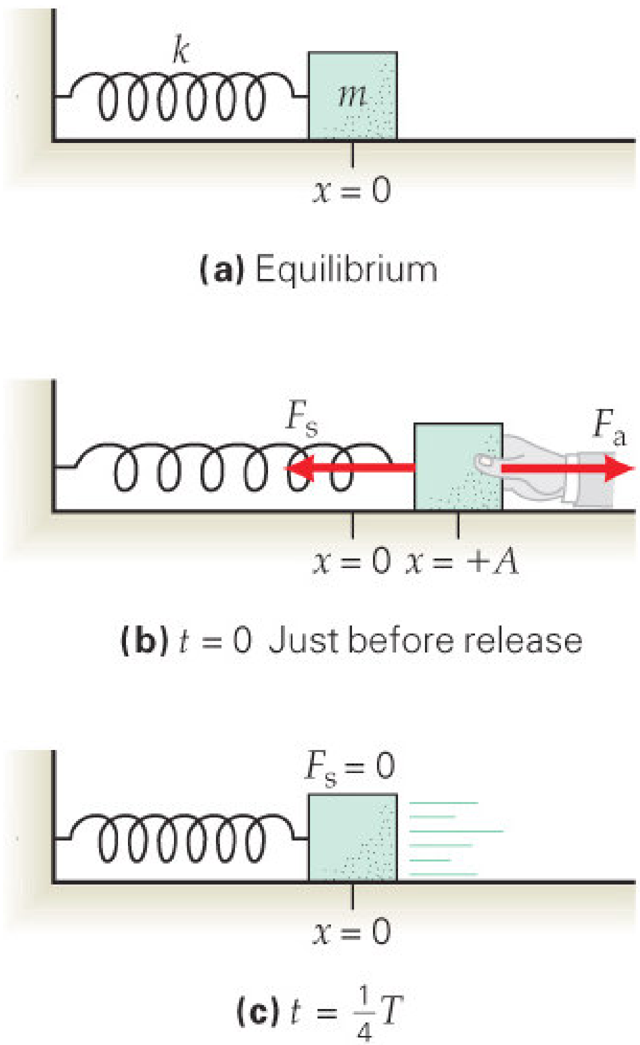
At the equilibrium point $x = 0$ so, $a = 0$ also.

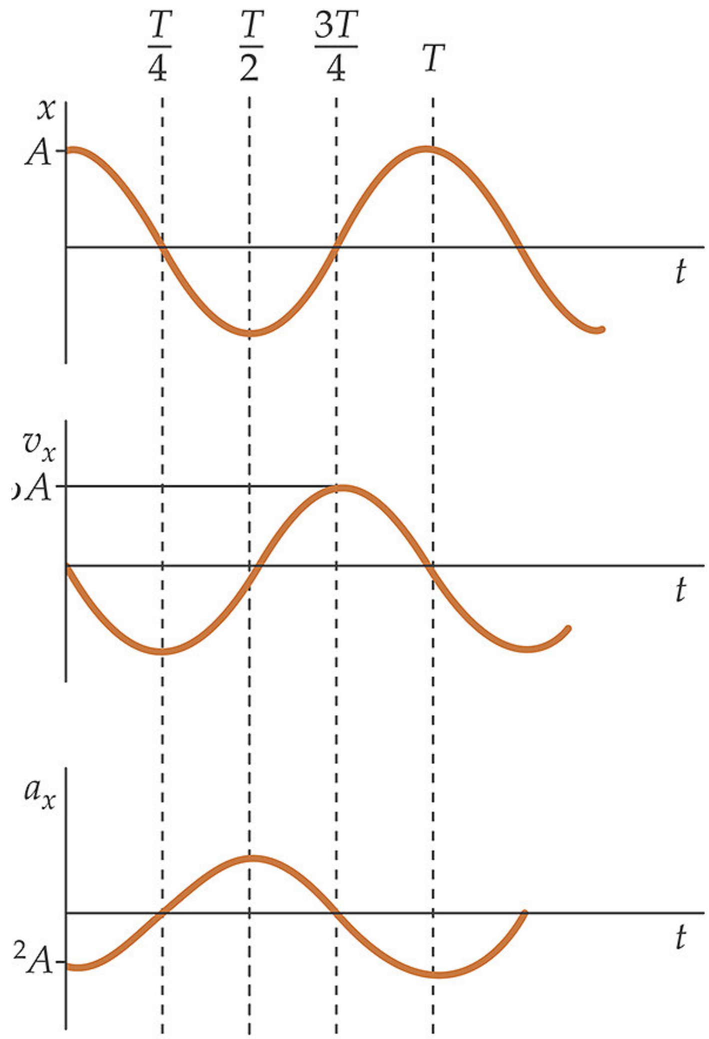
When the stretch is a maximum, a will be a maximum too.



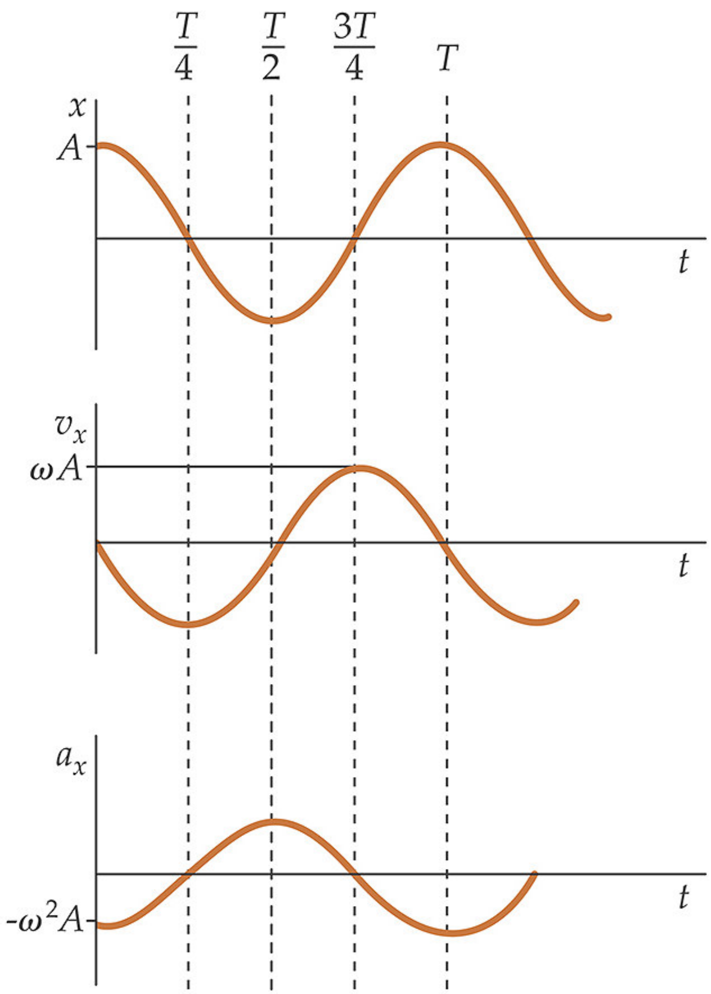
The velocity at the end points will be zero, and it is a maximum at the equilibrium point.

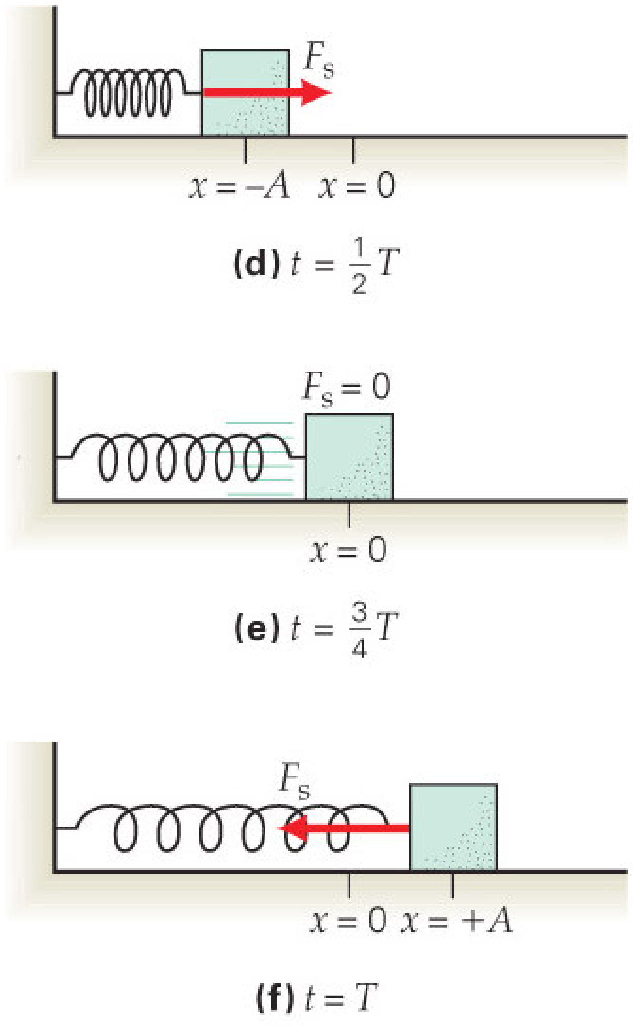
Representing Simple Harmonic Motion





Representing Simple Harmonic Motion



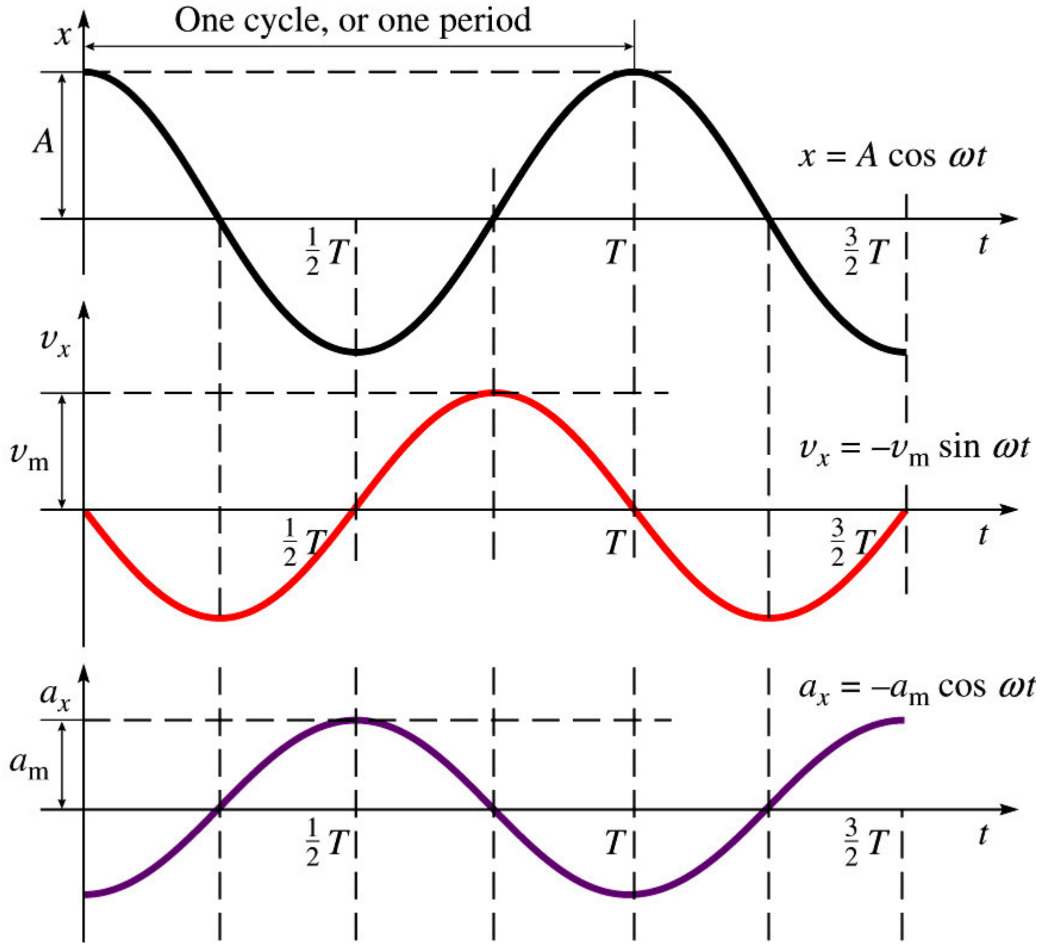


Representing Simple Harmonic Motion

Position - $x_{max} = A$

Velocity - $v_{max} = \omega A$

Acceleration - $a_{max} = \omega^2 A$



A simple harmonic oscillator can be described mathematically by:

$$x(t) = Acos\omega t$$

$$v(t) = \frac{dx}{dt} = -A\omega sin\omega t$$

$$a(t) = \frac{dv}{dt} = -A\omega^2 cos\omega t$$

Or by:

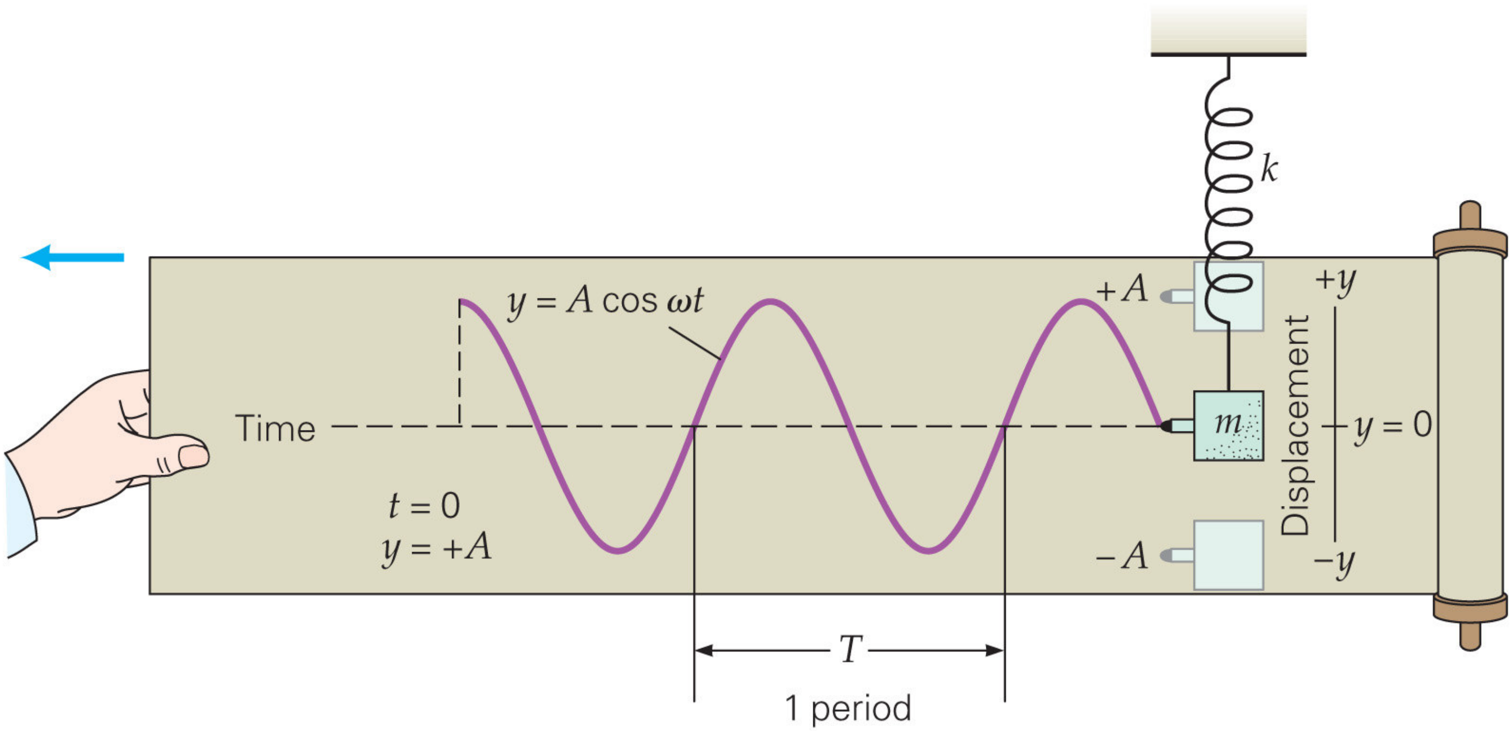
$$x(t) = Asin\omega t$$

$$v(t) = \frac{dx}{dt} = A\omega cos\omega t$$

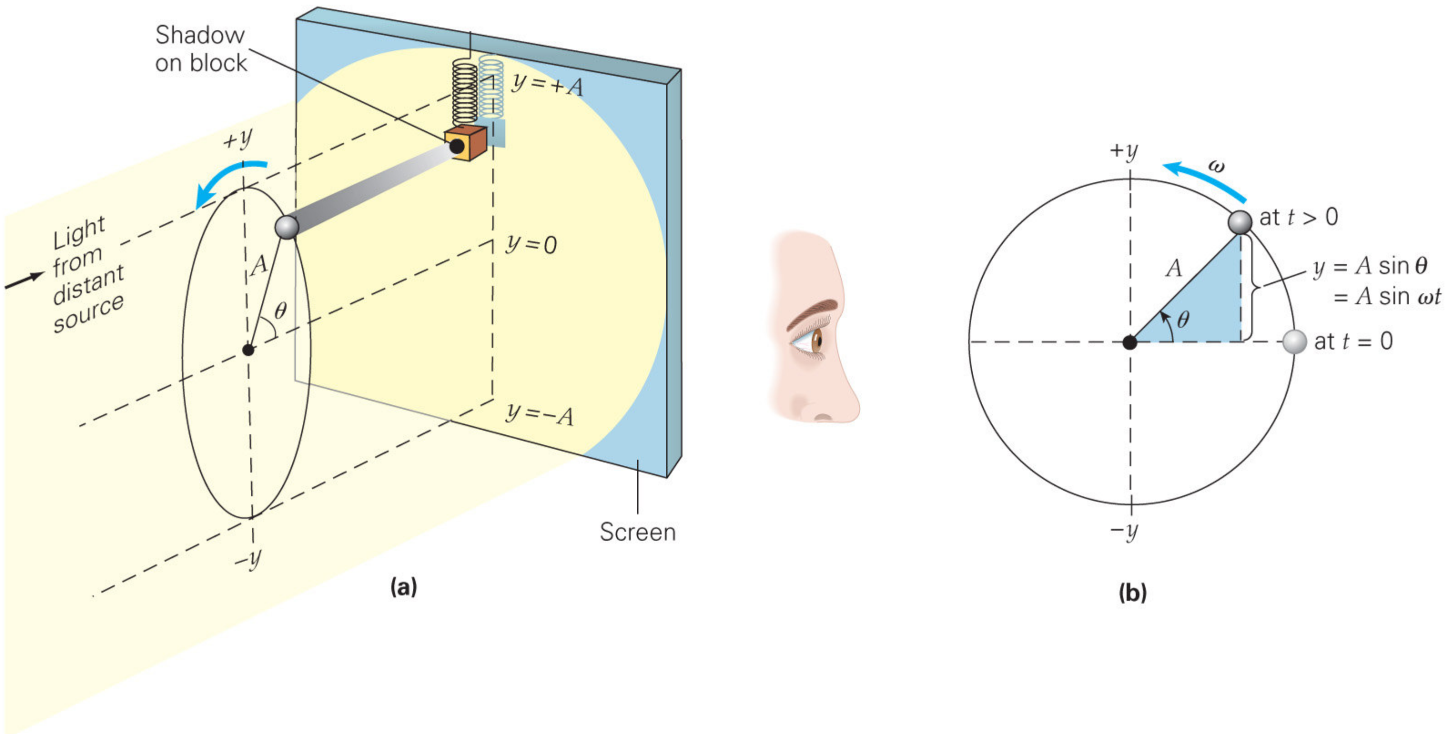
$$a(t) = \frac{dv}{dt} = -A\omega^2 sin\omega t$$

where A is the amplitude of the motion, the maximum displacement from equilibrium, $A\omega = v_{max}$, and $A\omega^2 = a_{max.}$

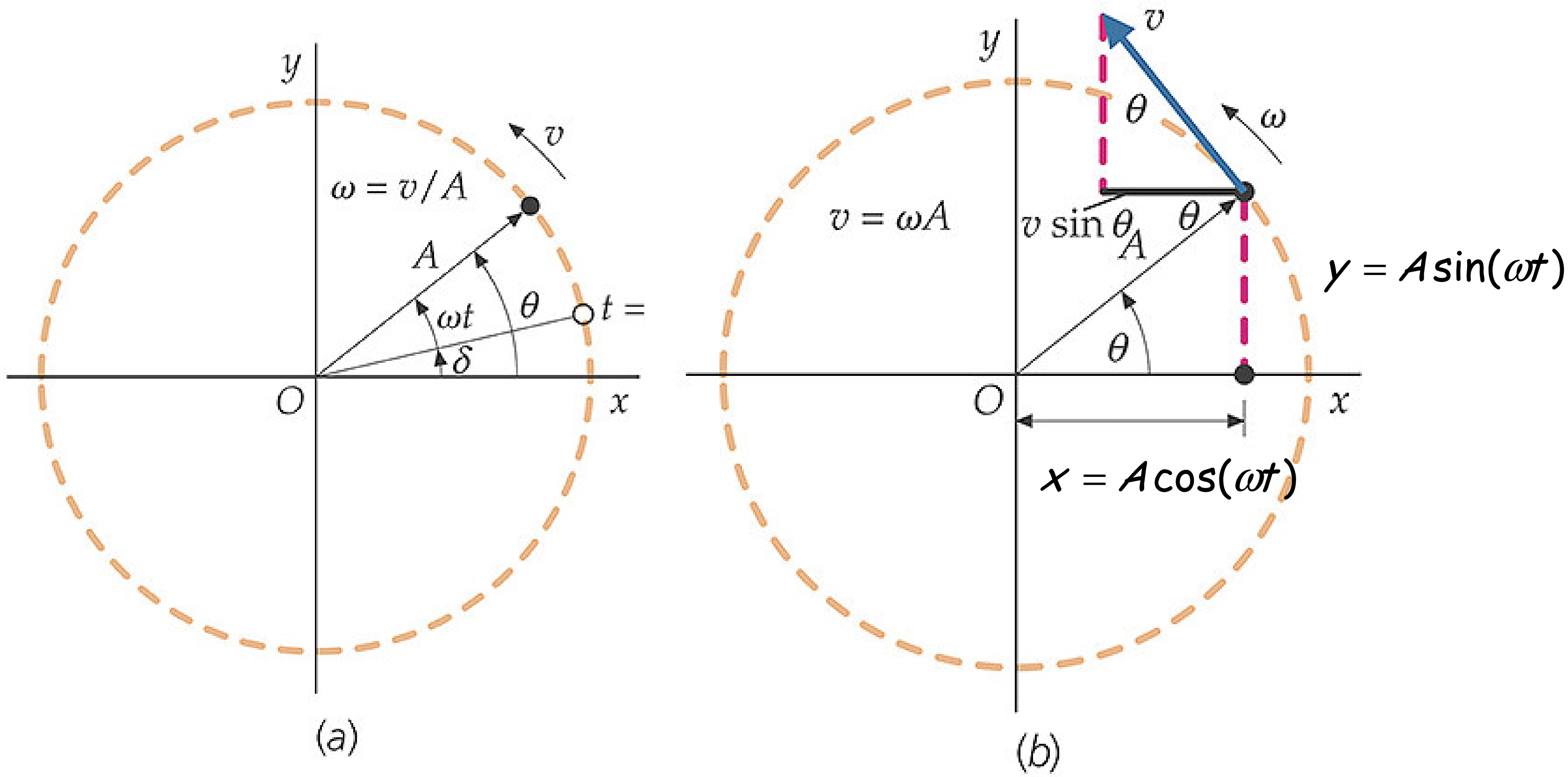
Linear Motion - Circular Functions



Projection of Circular Motion



Circular Motion is the superposition of two linear SHO that are 90^0 out of phase with each other



Shifting Trig Functions

$$x = A\left\{\frac{sin}{cos}\right\}\left[\omega t - \varphi\right]$$

$$x = A\left\{\frac{sin}{cos}\right\}\left[2\pi\frac{t}{T} - \varphi\right]$$

The minus sign means that the phase is shifted to the right.

A plus sign indicated the phase is shifted to the left

$$x = A sin\left[\omega t - \tfrac{\pi}{2}\right]$$

$$x = A\left(sin\omega t\, cos\tfrac{\pi}{2} - sin\tfrac{\pi}{2}\, cos\omega t\right)$$

$$x = A\left(sin\omega t\,(0) - (1) cos\omega t\right)$$

$$x = -A cos\omega t$$

Shifting Trig Functions

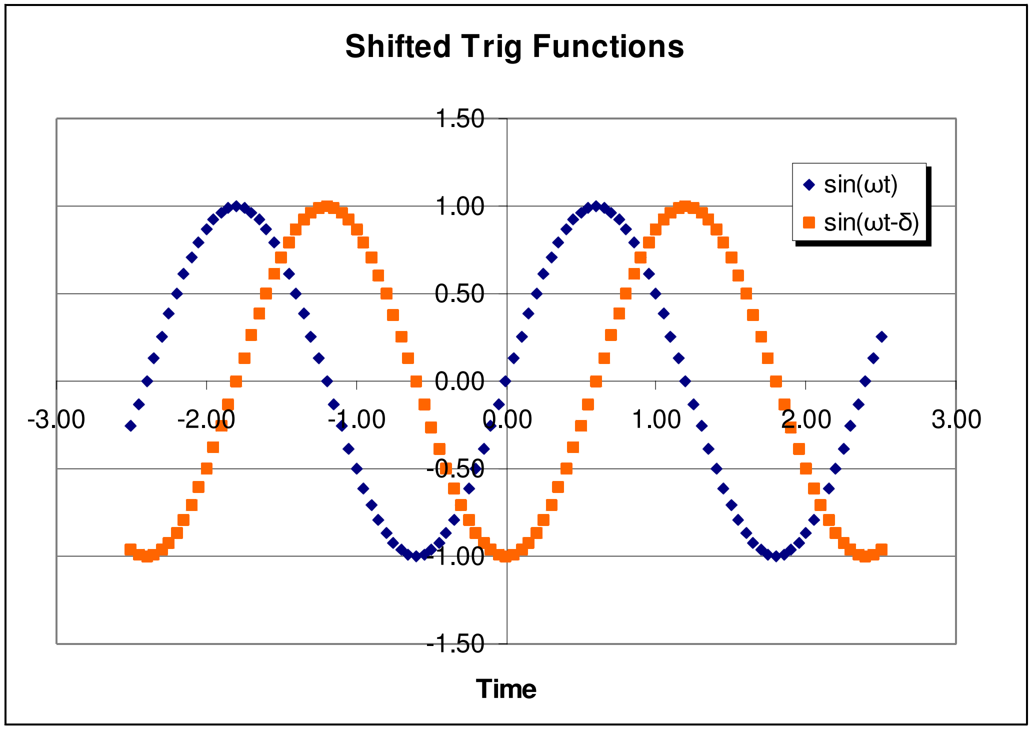
$$\sin\left(\omega t - \frac{\pi}{2}\right) = 0$$

$$\omega t - \frac{\pi}{2} = 0$$

$$\omega t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2}\frac{1}{\omega}; \qquad \frac{1}{\omega} = \frac{T}{2\pi}$$

$$t = \frac{\pi}{2}\frac{T}{2\pi} = \frac{T}{4}$$



Energy

Equation of Motion & Energy

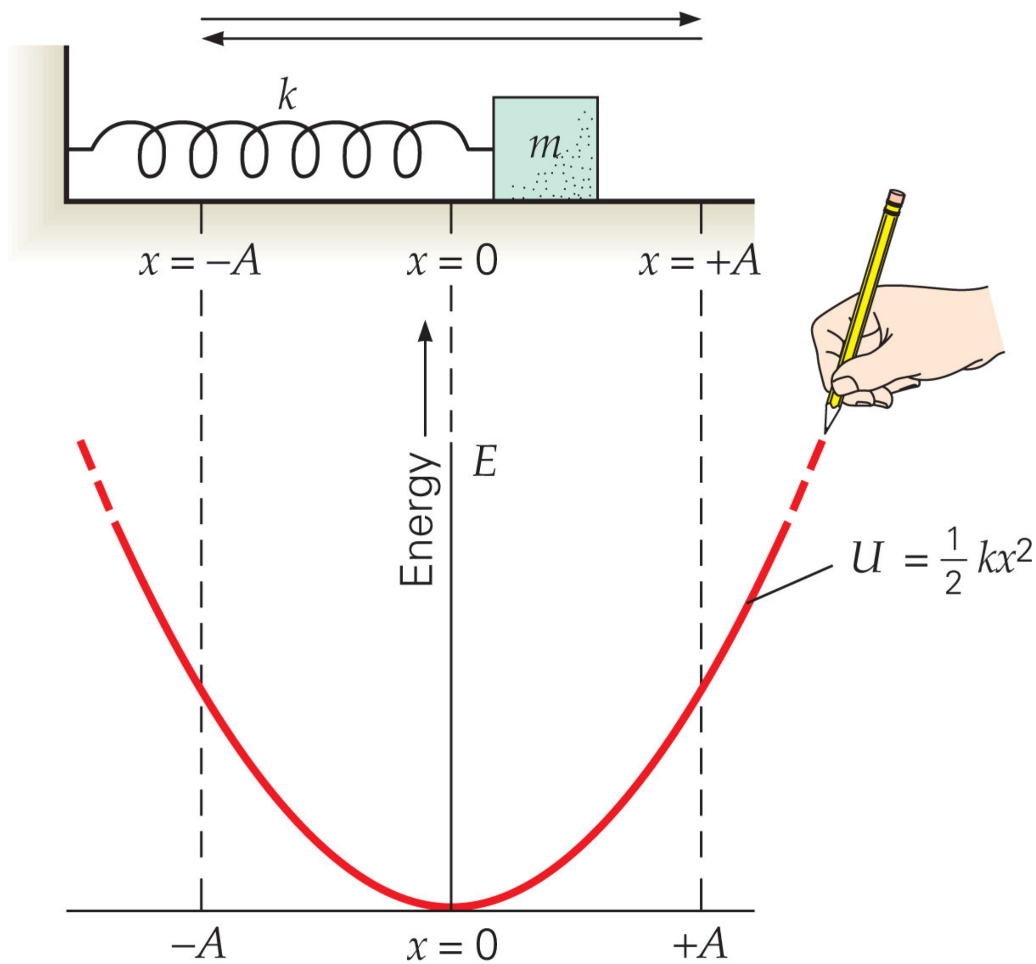
Assuming the table is frictionless:

$$\sum F_x = -kx = ma_x$$

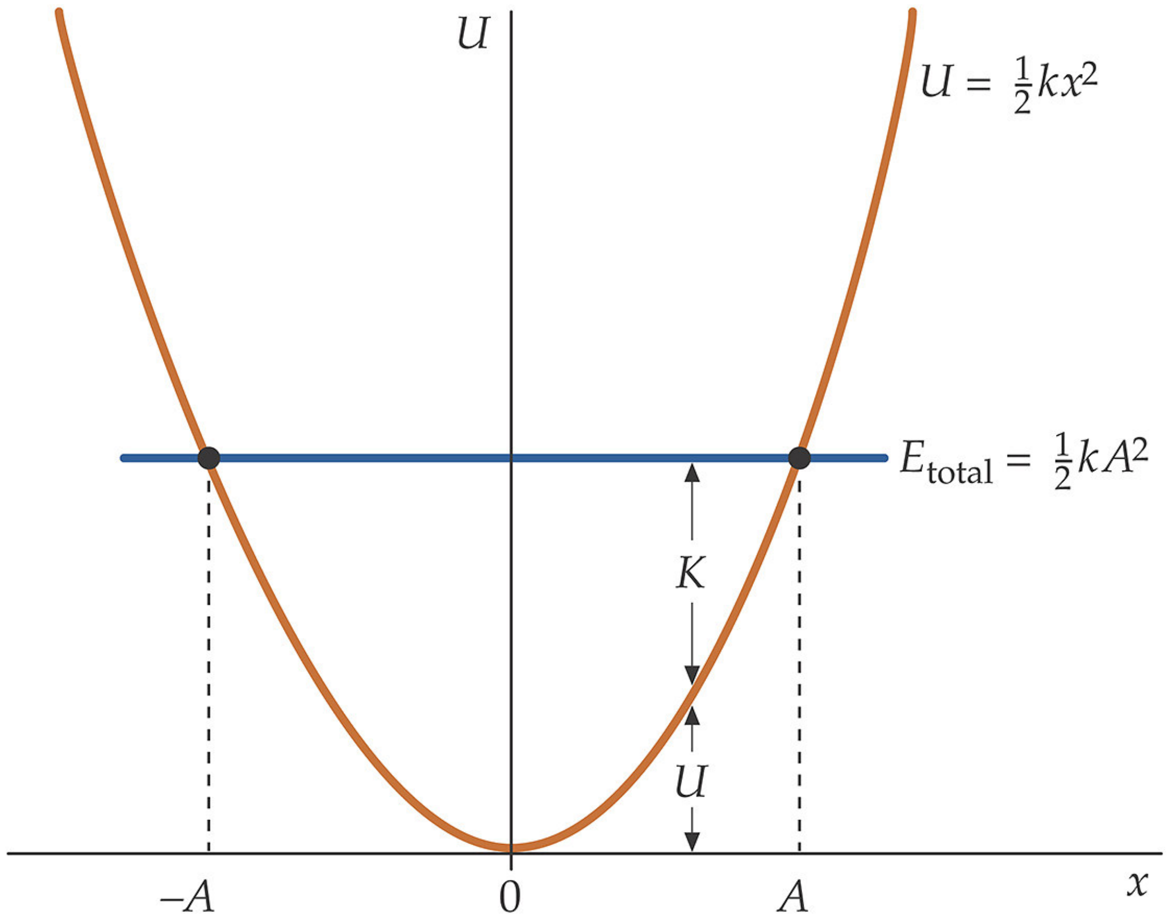
Classic form for SHM $\quad a_x(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$

Also, $\quad E = K(t) + U(t) = \frac{1}{2}mv^2(t) + \frac{1}{2}kx^2(t)$

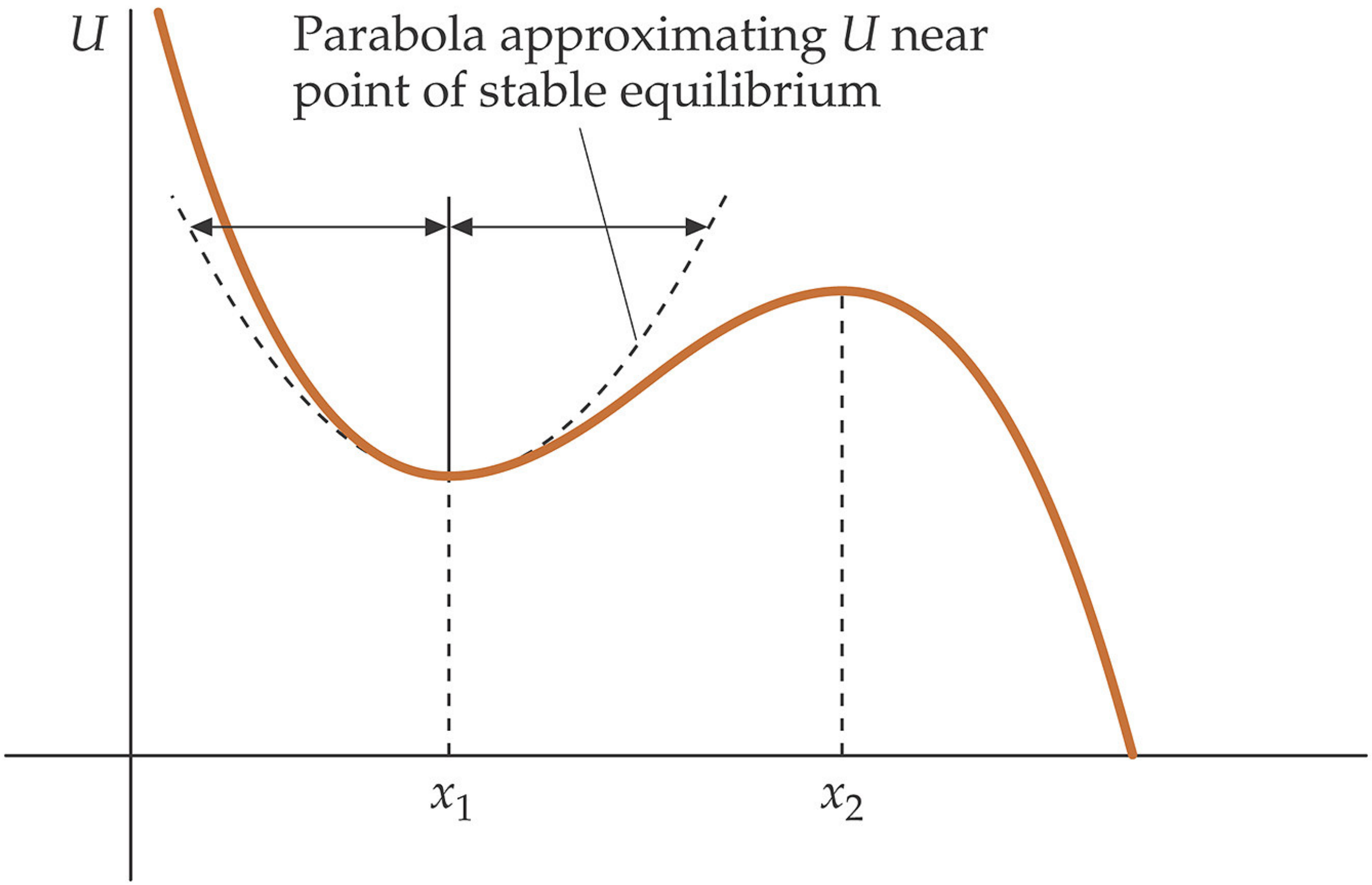
Spring Potential Energy



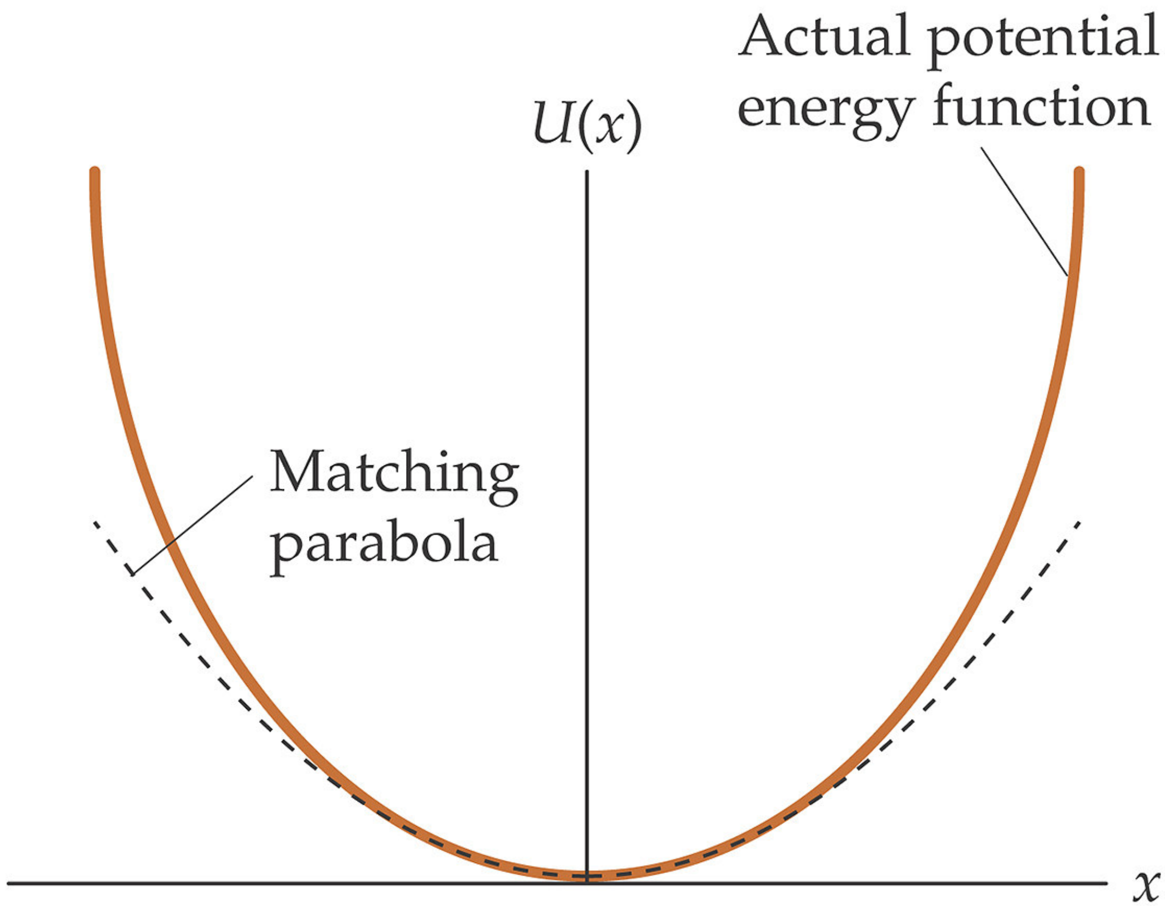
Spring Total Energy



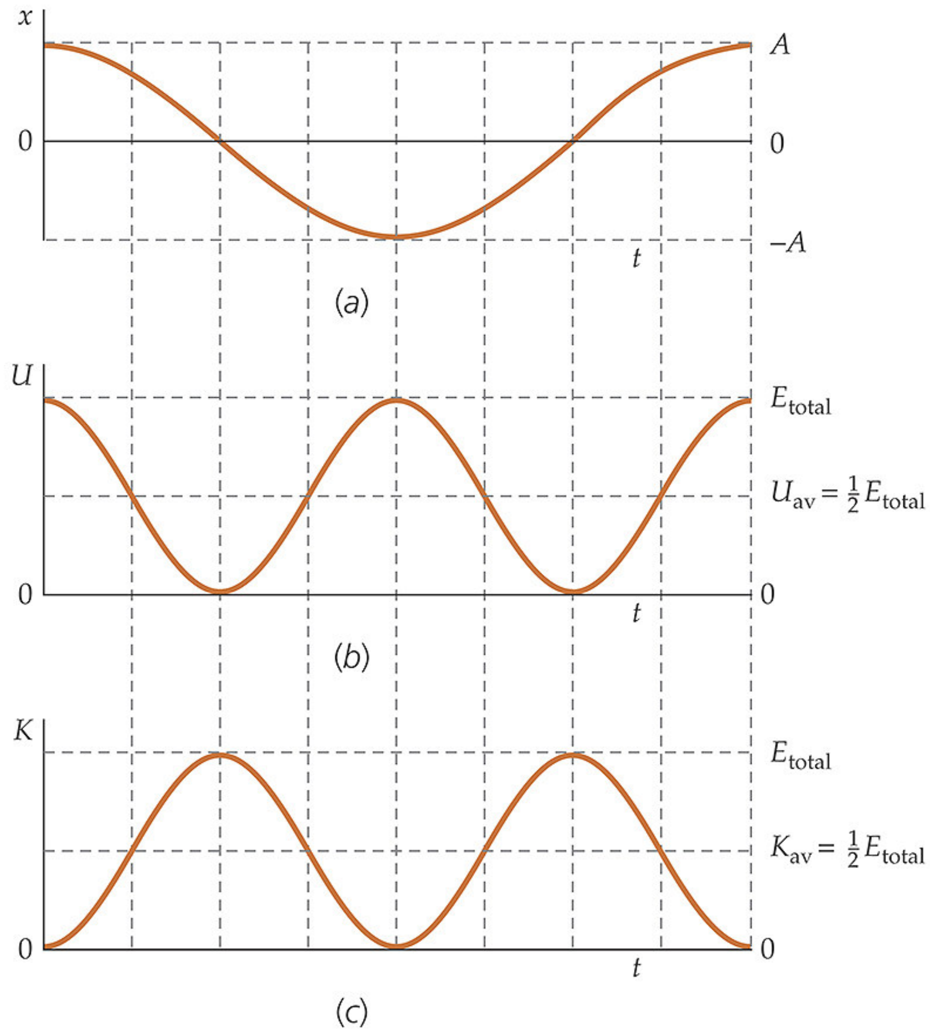
Approximating Simple Harmonic Motion



Approximating Simple Harmonic Motion



Potential and Kinetic Energy



The period of oscillation of an object in an ideal mass-spring system is 0.50 sec and the amplitude is 5.0 cm.

What is the speed at the equilibrium point?

At equilibrium x = 0:

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

Since E = constant, at equilibrium (x = 0) the KE must be a maximum. Here $v = v_{max} = A\omega$.

Example continued:

The amplitude A is given, but ω is not.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.50\ \text{s}} = 12.6\ \text{rads/sec}$$

and $v = A\omega = (5.0\ \text{cm})(12.6\ \text{rads/sec}) = 62.8\ \text{cm/sec}$

The diaphragm of a speaker has a mass of 50.0 g and responds to a signal of 2.0 kHz by moving back and forth with an amplitude of 1.8×10^{-4} m at that frequency.

(a) What is the maximum force acting on the diaphragm?

$$\sum F = F_{\text{max}} = ma_{\text{max}} = m\left(A\omega^2\right) = mA\left(2\pi f\right)^2 = 4\pi^2 mAf^2$$

The value is F_{max}=1400 N.

Example continued:

(b) What is the mechanical energy of the diaphragm?

Since mechanical energy is conserved, $E = K_{max} = U_{max.}$

$$U_{\max} = \frac{1}{2}kA^2$$

$$K_{\max} = \frac{1}{2}mv_{\max}^2$$

The value of k is unknown so use K_{max}.

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}m(A\omega)^2 = \frac{1}{2}mA^2(2\pi f)^2$$

The value is K_{max}= 0.13 J.

Example: The displacement of an object in SHM is given by:

$$y(t) = (8.00\,\text{cm})\sin\left[(1.57\text{ rads/sec})t\right]$$

What is the frequency of the oscillations?

Comparing to y(t) = A sinωt gives A = 8.00 cm and ω = 1.57 rads/sec. The frequency is:

$$f = \frac{\omega}{2\pi} = \frac{1.57\text{ rads/sec}}{2\pi} = 0.250\text{ Hz}$$

Example continued:

Other quantities can also be determined:

The period of the motion is $T=\frac{2\pi}{\omega}=\frac{2\pi}{1.57\ \text{rads/sec}}=4.00\ \text{sec}$

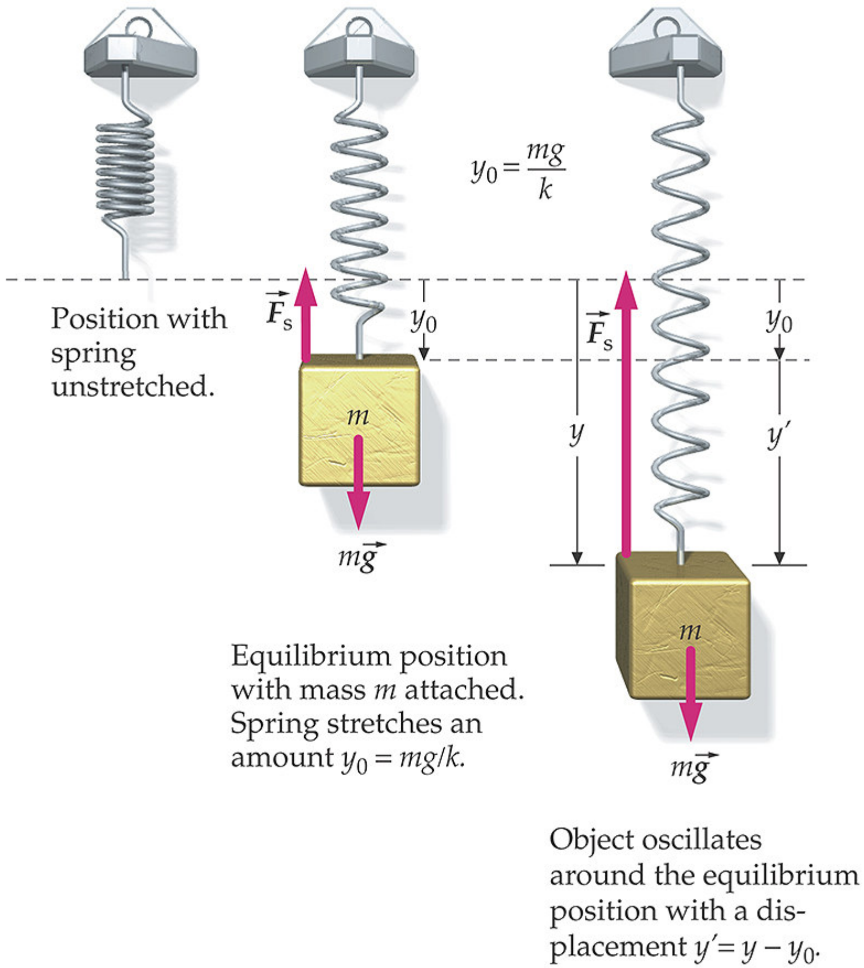
$$x_{\max}=A=8.00\ \text{cm}$$
$$v_{\max}=A\omega=(8.00\ \text{cm})(1.57\ \text{rads/sec})=12.6\ \text{cm/sec}$$
$$a_{\max}=A\omega^2=(8.00\ \text{cm})(1.57\ \text{rads/sec})^2=19.7\ \text{cm/sec}^2$$

What About Gravity?

When a mass-spring system is oriented vertically, it will exhibit SHM with the same period and frequency as a horizontally placed system.

The effect of gravity is canceled out.

Why We Ignore Gravity with Vertical Springs



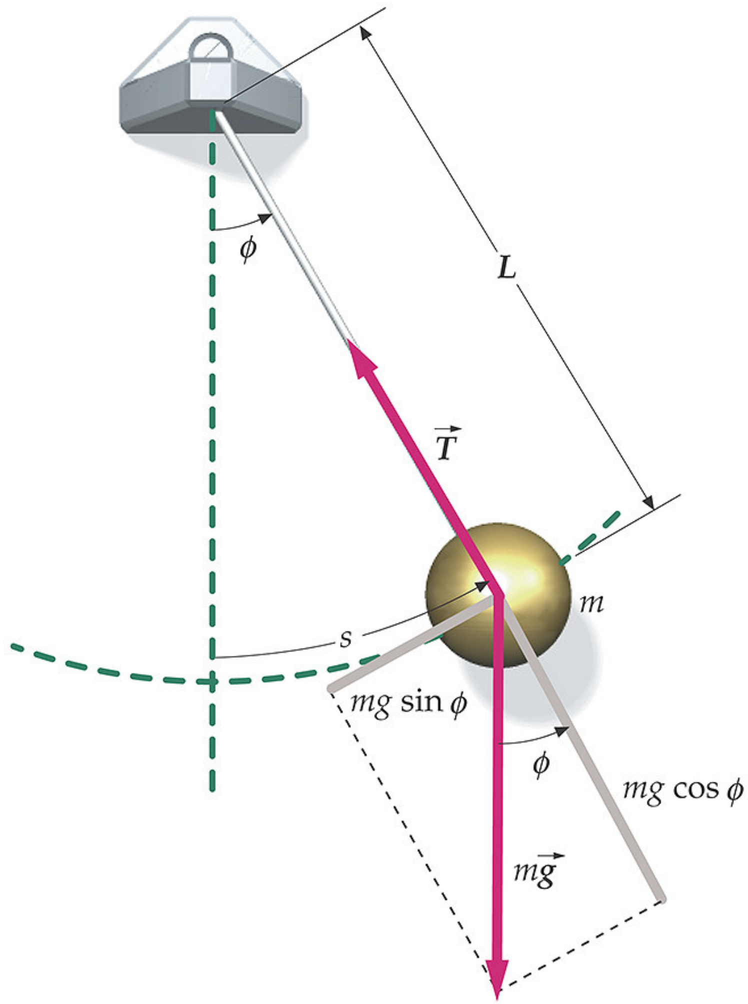
The Simple Pendulum

A **simple pendulum** is constructed by attaching a mass to a thin rod or a light string. We will also assume that the amplitude of the oscillations is small.

The Simple Pendulum

The pendulum is best described using polar coordinates.

The origin is at the pivot point. The coordinates are (r, φ). The r-coordinate points from the origin along the rod. The φ-coordinate is perpendicular to the rod and is positive in the counter clockwise direction.



Apply Newton's 2nd Law to the pendulum bob.

$$\sum F_\phi = -mg\sin\phi = ma_\phi$$

$$\sum F_r = T - mg\cos\phi = m\frac{v^2}{r}$$

If we assume that φ <<1 rad, then sin φ ≈ φ and cos φ ≈1, the angular frequency of oscillations is then:

$$\sum F_\phi = -mg\sin\phi = ma_\phi = mL\alpha$$

$$-mg\sin\phi = mL\alpha$$

$$\alpha = -(g/L)\sin\phi$$

$$\alpha = -(g/L)\phi$$

$$\omega = \sqrt{\frac{g}{L}}$$

The period of oscillations is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

Example: A clock has a pendulum that performs one full swing every 1.0 sec. The object at the end of the string weighs 10.0 N.

What is the length of the pendulum?

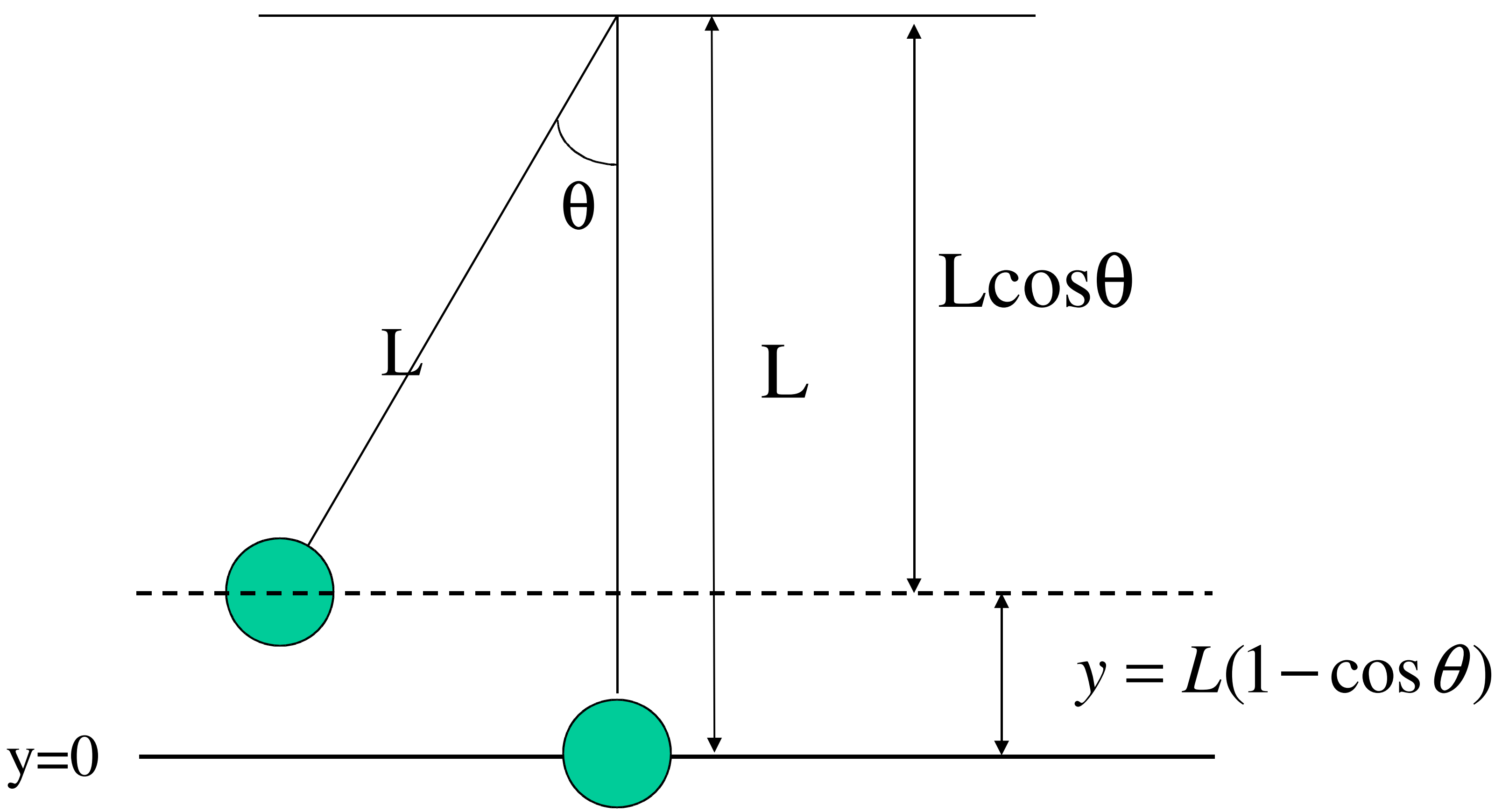
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Solving for L: $$\mathrm{L} = \frac{gT^2}{4\pi^2} = \frac{\left(9.8\ \mathrm{m/s^2}\right)\left(1.0\ \mathrm{s}\right)^2}{4\pi^2} = 0.25\ \mathrm{m}$$

The gravitational potential energy of a pendulum is

$$U = mgy.$$

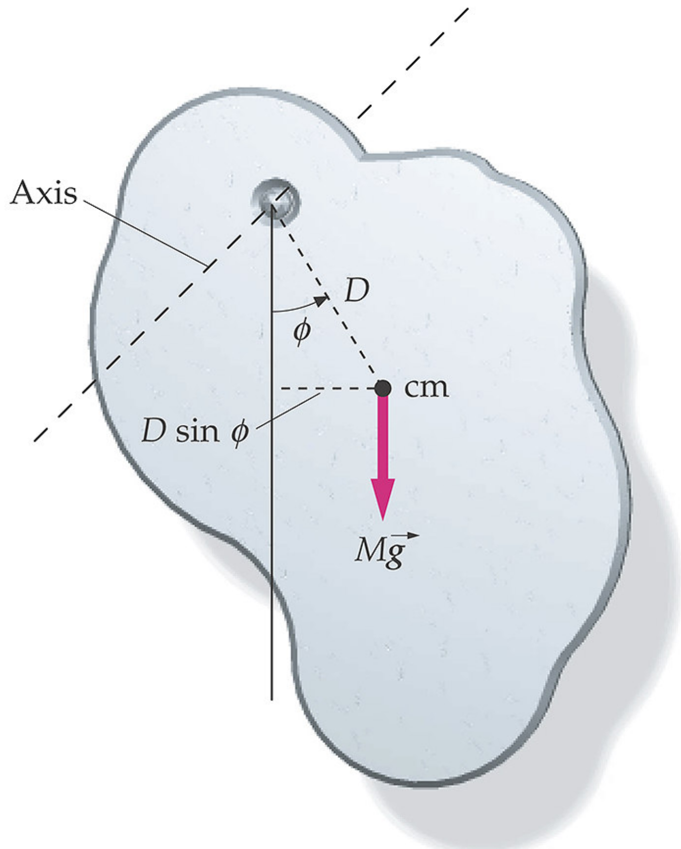
Taking $y = 0$ at the lowest point of the swing, show that $y = L(1-\cos\theta)$.



The Physical Pendulum

A **physical pendulum** is any rigid object that is free to oscillate about some fixed axis. The period of oscillation of a physical pendulum is not necessarily the same as that of a simple pendulum.

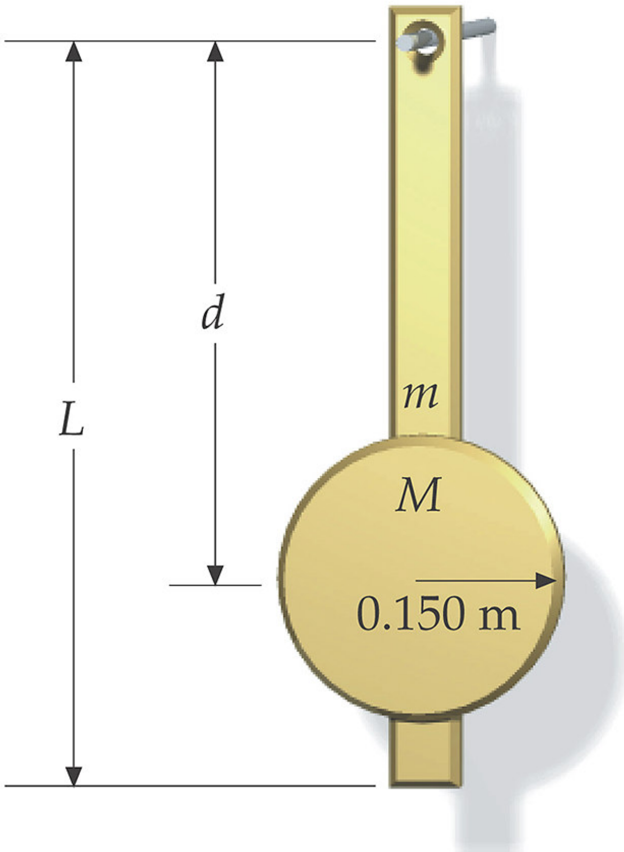
The Physical Pendulum



$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

I is the moment of inertia about the given axis. The I_{cm} from the table will need to be modified using the parallel axis theorem.

Compound Pendulum



$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

$I = I_{rod} + I_{disk}$

$M = m_{rod} + M_{disk}$

D = distance from the axis to the center of mass of the rod and disk.

Damped Oscillations

When dissipative forces such as friction are not negligible, the amplitude of oscillations will decrease with time. The oscillations are damped.

Damped Oscillations Equations

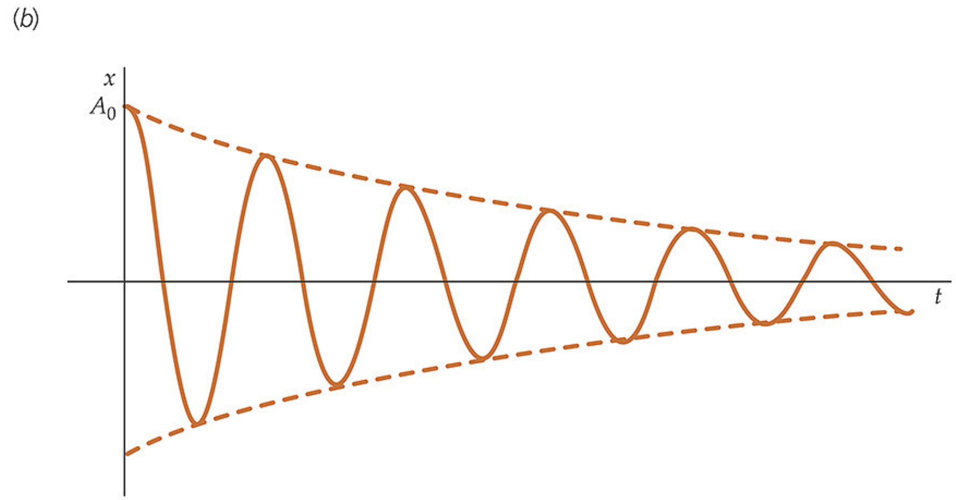
$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + m\omega_0 x = 0$$

$$x(t) = A_0 exp\left[\frac{-t}{2\tau}\right]cos(\,\omega' t + \delta)$$

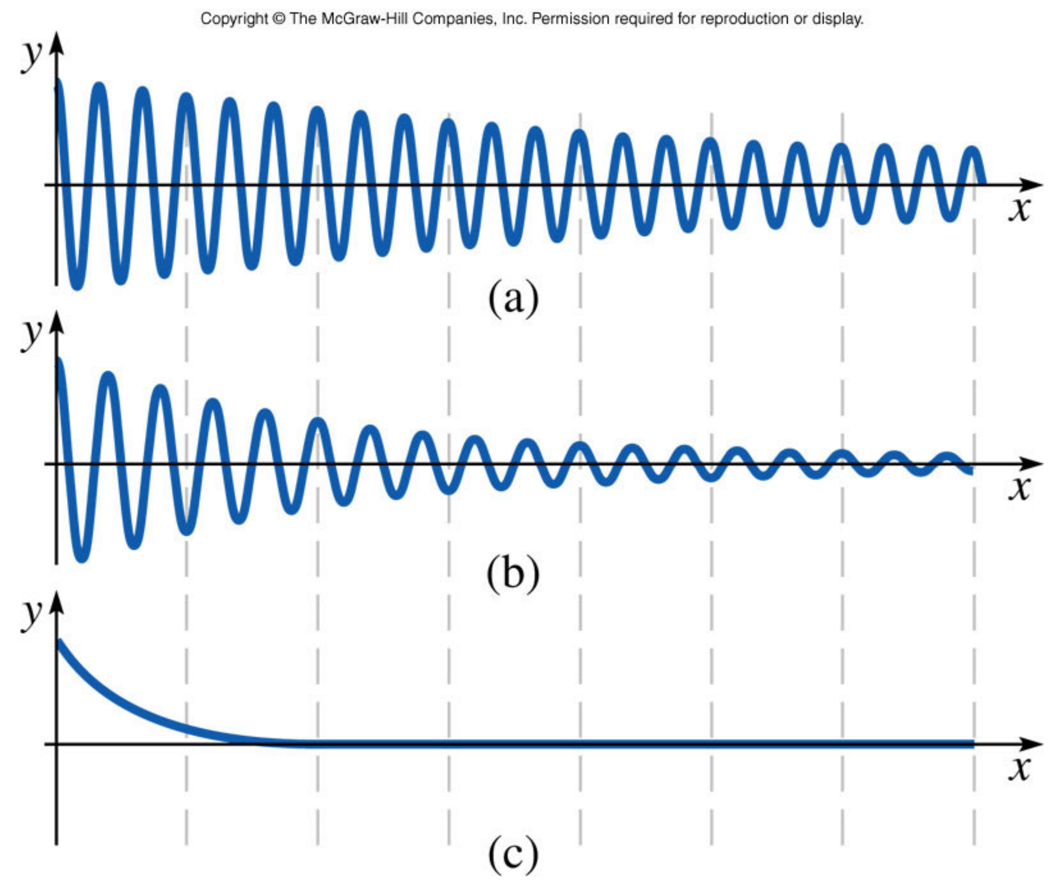
$$\omega' = \omega_0\sqrt{1 - \left(\frac{b}{2m\omega_0}\right)^2} \qquad \omega_0 = \sqrt{\frac{k}{m}} \qquad \tau = \frac{m}{b};\; b_c = 2m\omega_0$$

For b > b_c the system is overdamped. For b = b_c the system is critically damped. The object doesn't oscillate and returns to its equilibrium posion very rapidly.

Damped Oscillations



Graphical representations of damped oscillations:



Damped Oscillations

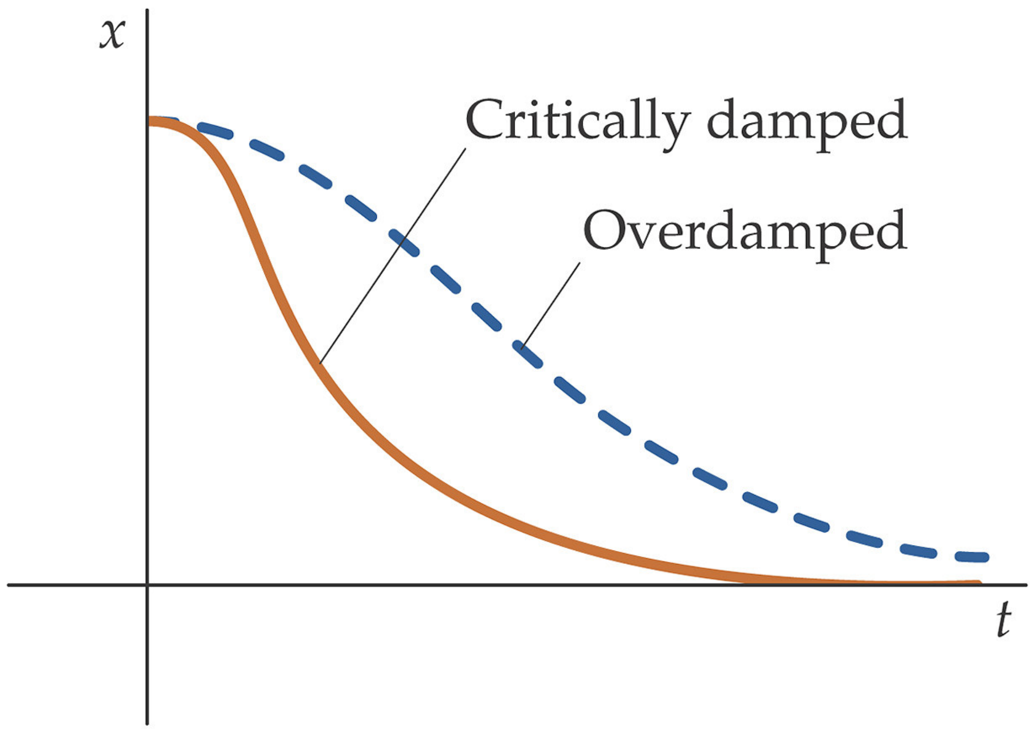
- *Overdamped*: The system returns to equilibrium without oscillating. Larger values of the damping the return to equilibrium slower.

- *Critically damped* : The system returns to equilibrium as quickly as possible without oscillating. This is often desired for the damping of systems such as doors.

- *Underdamped* : The system oscillates (with a slightly different frequency than the undamped case) with the amplitude gradually decreasing to zero.

Source: Damping @ Wikipedia

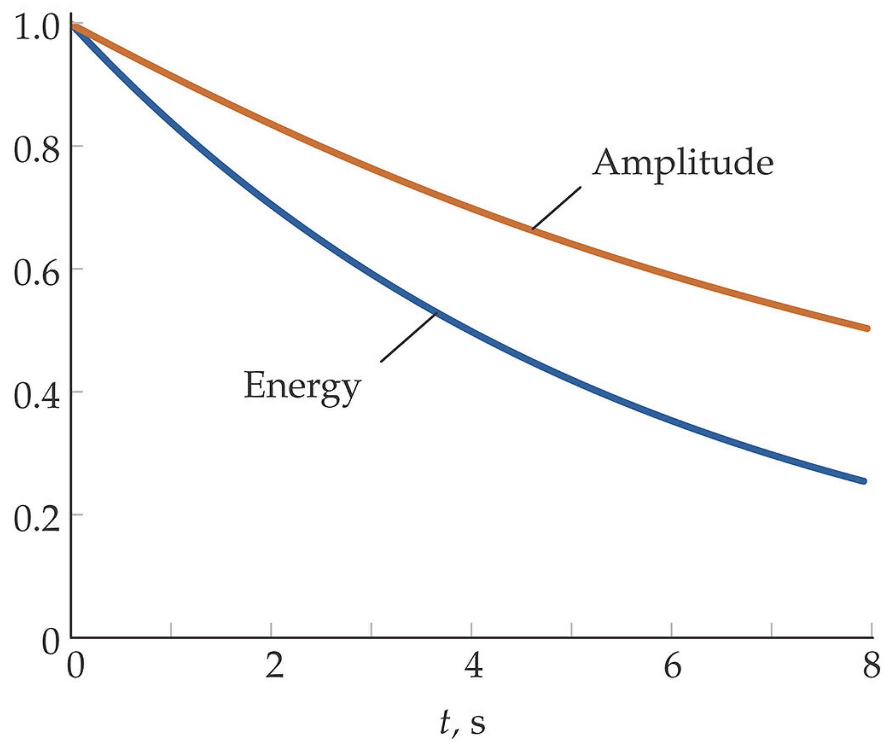
Damped Oscillations



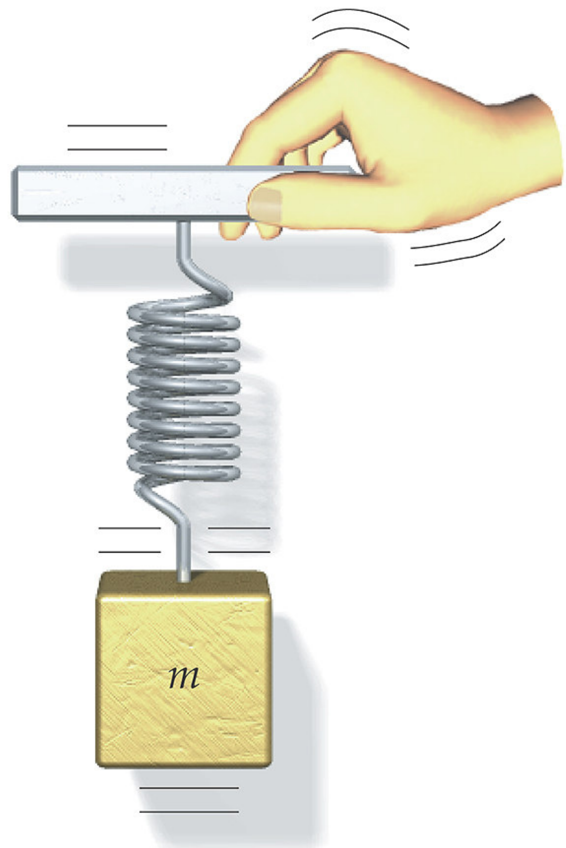
The larger the damping the more difficult it is to assign a frequency to the oscillation.

Damped Oscillations

$$E \, \alpha \, A^2$$



Forced Oscillations



Forced Oscillations and Resonance

A force can be applied periodically to a damped oscillator (a forced oscillation).

When the force is applied at the natural frequency of the system, the amplitude of the oscillations will be a maximum. This condition is called **resonance**.

Forced Oscillations Equations

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + m\omega_0 x = F_0 cos\omega t$$

ma friction spring applied force

$$x = Acos(\omega t - \delta)$$

$$A = \frac{F_0}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2\omega^2}} \qquad tan\delta = \frac{b\omega}{m(\omega_0^2 - \omega^2)}$$

Energy and Resonance

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+m\omega_0 x = F_0 cos\omega t$$

At resonance v and F_o are in phase

$$v_x = \frac{dx}{dt} = -\omega Asin(\omega t - \delta)$$

$$v_x = -\omega Asin(\omega t - \frac{\pi}{2}) = +\omega Acos\omega t$$

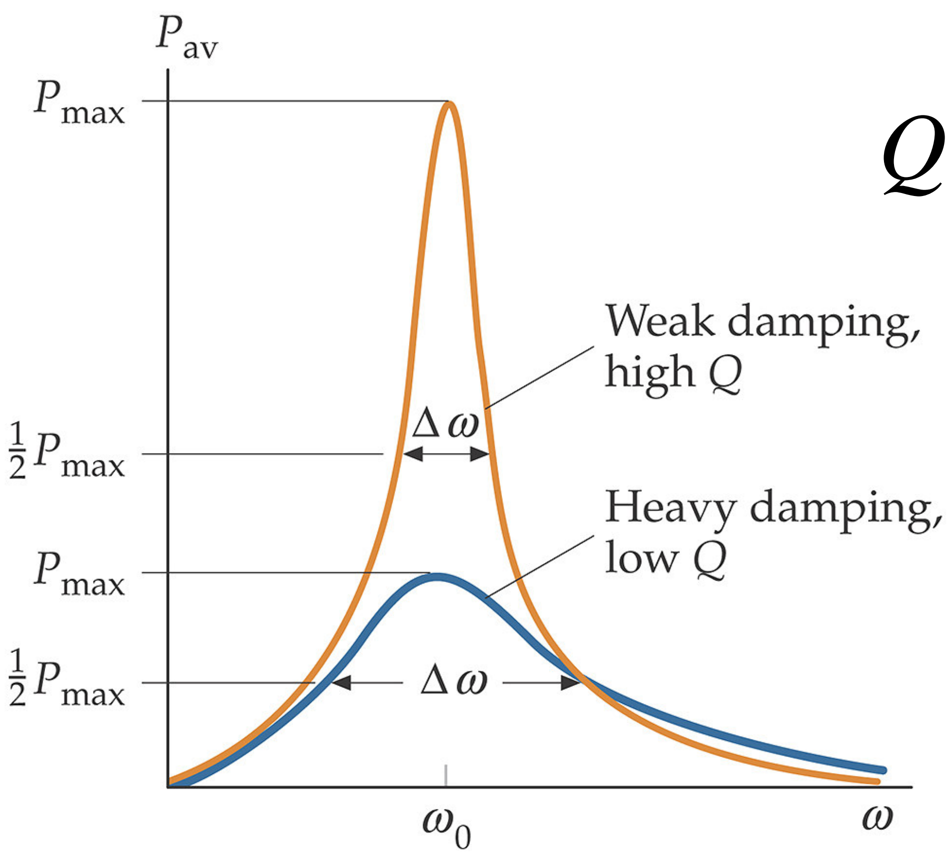
$$Energy \;\; \alpha \;\; A^2 = A_0^2 exp\left[\frac{-t}{\tau}\right]$$

$$Q = \omega_0 \tau = \frac{\omega_0 m}{b}$$

$$E = \tfrac{1}{2} m\omega^2 A^2 = E_0 exp\left[\frac{-t}{\tau}\right]$$

$$E_0 = \tfrac{1}{2} m\omega^2 A_0^2; \quad \tau = m/b$$

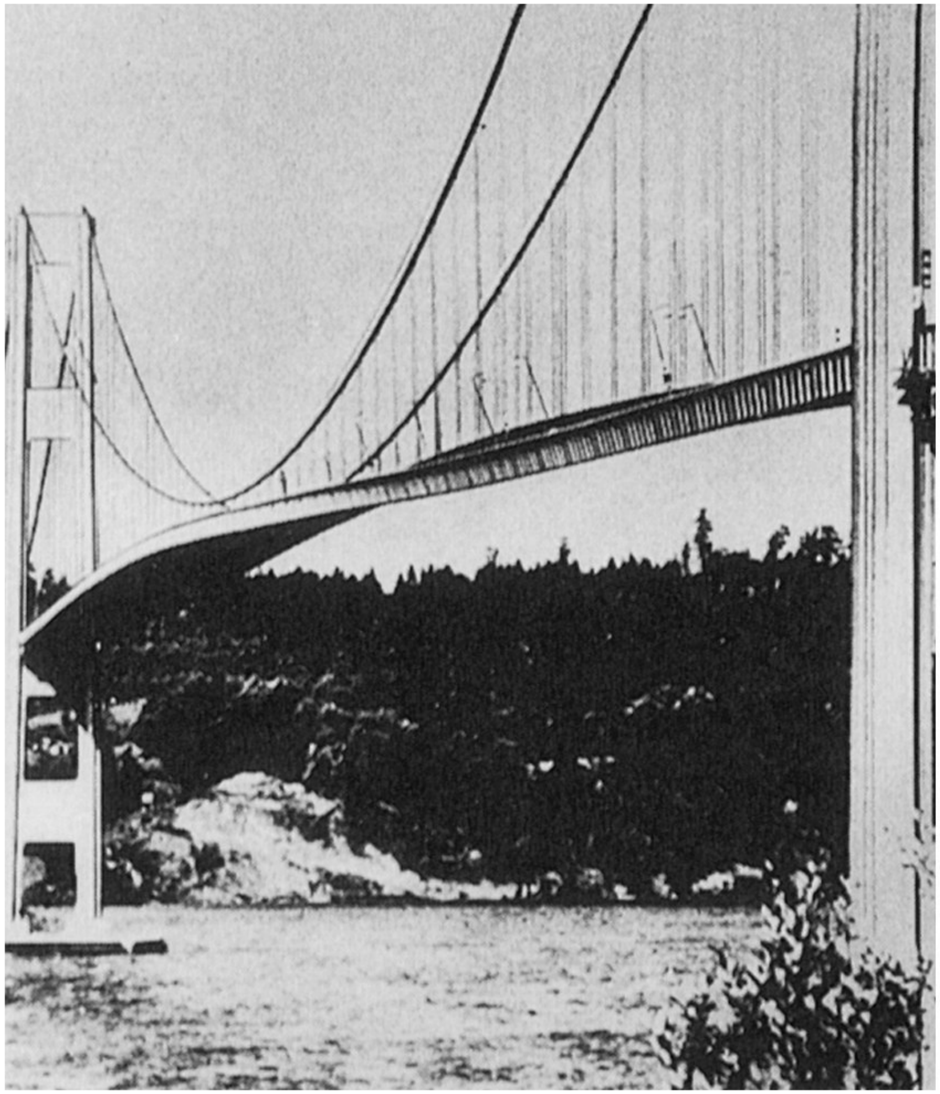
Power Transfer



$$Q = \frac{\omega_0}{\Delta\omega}$$

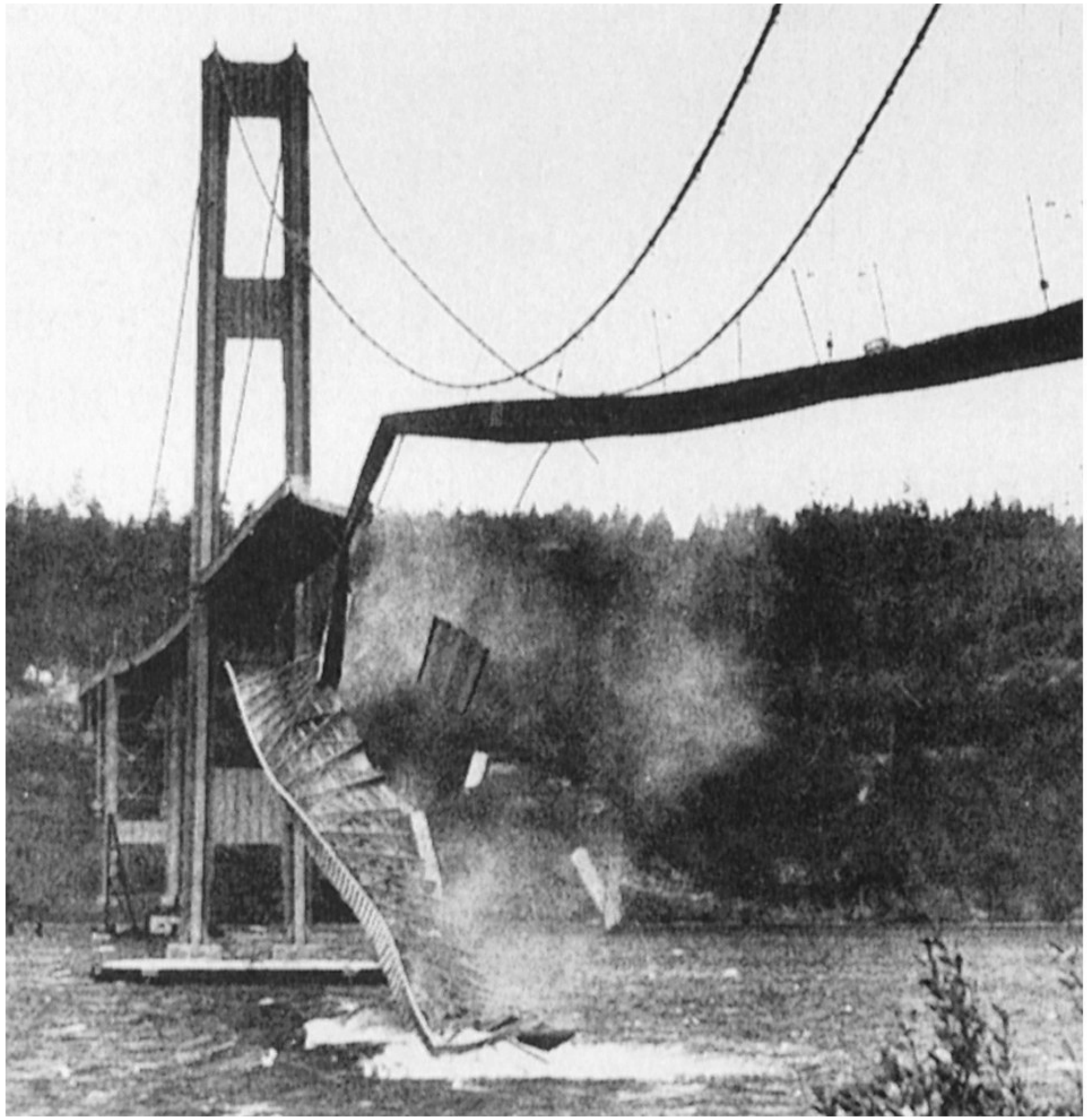
The dissipation in the system, represented by "b" keeps the amplitude from going to infinity.

Tacoma Narrows Bridge



Nov. 7, 1940

Tacoma Narrows Bridge



Nov. 7, 1940

Tacoma Narrows Bridge

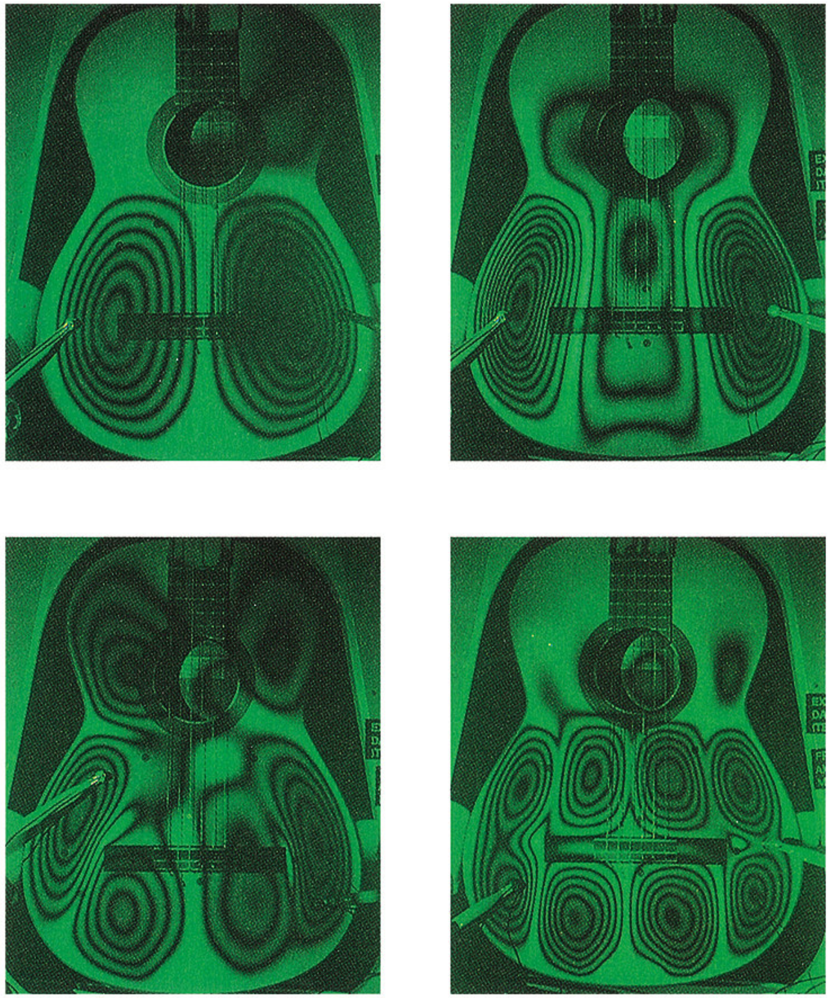
The first Tacoma Narrows Bridge opened to traffic on July 1, 1940. It collapsed four months later on November 7, 1940, at 11:00 AM (Pacific time) due to a physical phenomenon known as aeroelastic flutter caused by a 67 kilometres per hour (42 mph) wind.

The bridge collapse had lasting effects on science and engineering. In many undergraduate physics texts the event is presented as an example of elementary forced resonance with the wind providing an external periodic frequency that matched the natural structural frequency (even though the real cause of the bridge's failure was aeroelastic flutter[1]).

Its failure also boosted research in the field of bridge aerodynamics/ aeroelastics which have themselves influenced the designs of all the world's great long-span bridges built since 1940. - Wikipedia

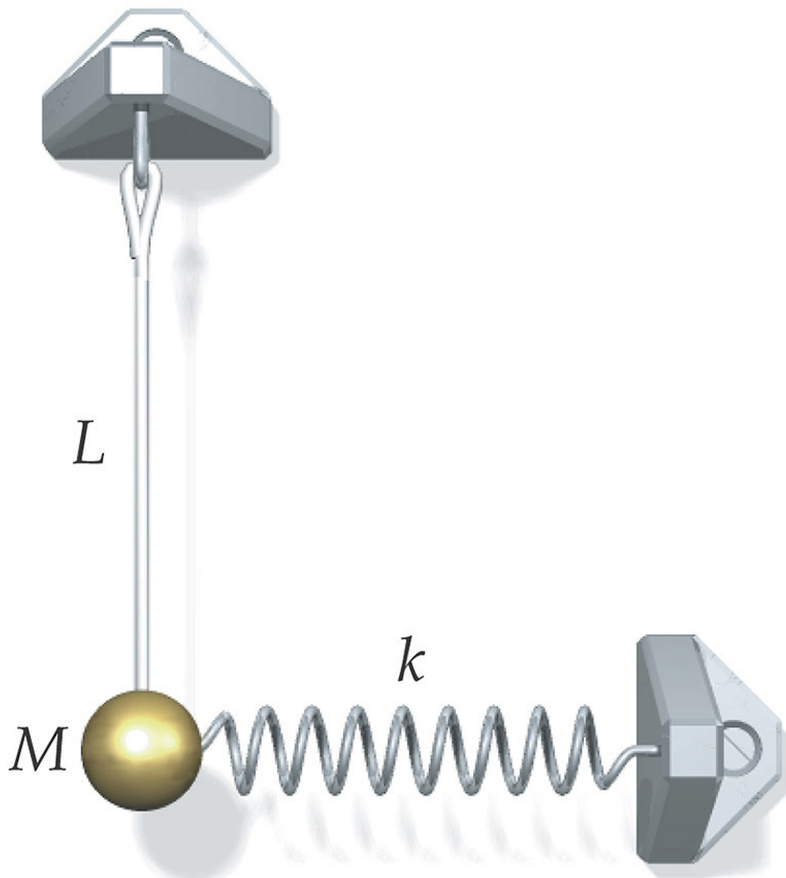
http://www.youtube.com/watch?v=3mclp9QmCGs

Normal Mode Vibrations



End of Chapter Problems

Chap 14 - #92



Extra Slides

The Full Wave Equation

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$$

$$y(t) = Asin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}; \quad \omega = 2\pi f = \frac{2\pi}{T}$$

$$y(t) = Asin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

Traveling with the wave the phase is constant

$$\frac{x}{\lambda} - \frac{t}{T} = Constant$$

$$\frac{dx}{\lambda} - \frac{dt}{T} = 0$$

$$\frac{dx}{dt} = \frac{\lambda}{T} = \lambda f = v$$ Wave velocity