

## periodic motion

- → often a physical system will repeat the same motion over and over
  - → we call this periodic motion, or an oscillation

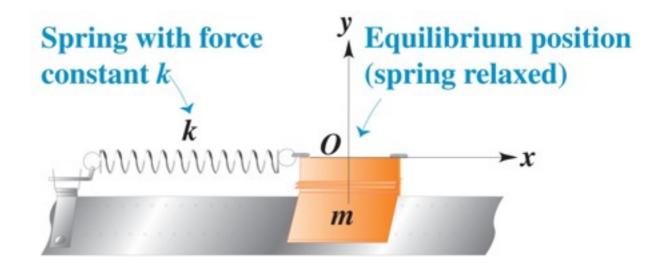
the time it takes for the motion to complete one cycle is called the period, **T** the number of times the motion repeats in unit time is called the frequency, **f** 

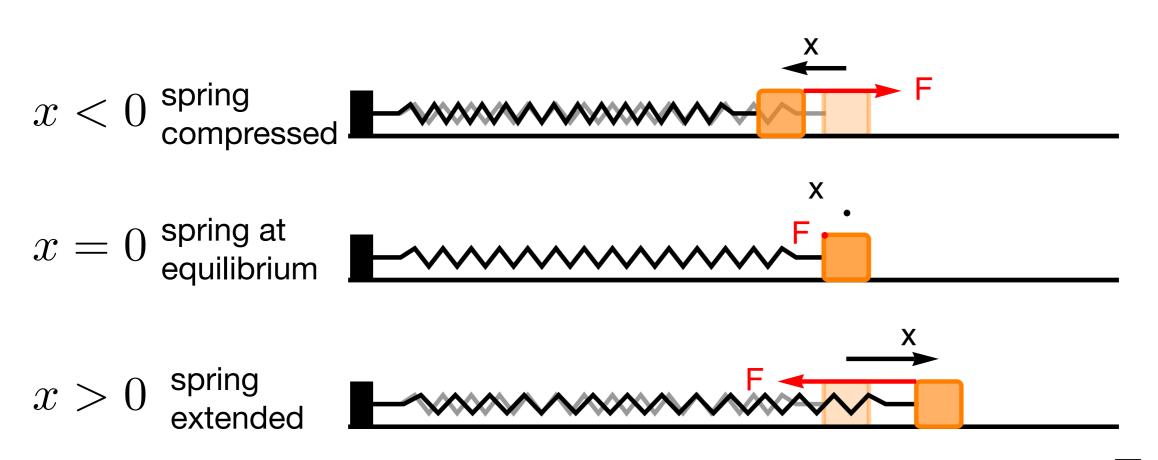
$$f = \frac{1}{T}$$

so e.g. if the period is 0.1 s, the frequency is  $10 \, \text{s}^{-1}$ 

→ a very special form of periodic motion where the restoring force is proportional to the displacement

→ once way to construct such an oscillator is to use a spring obeying Hooke's law

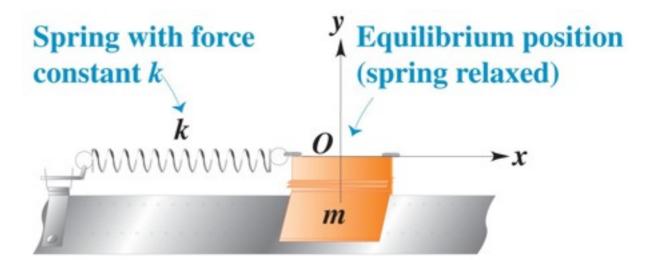




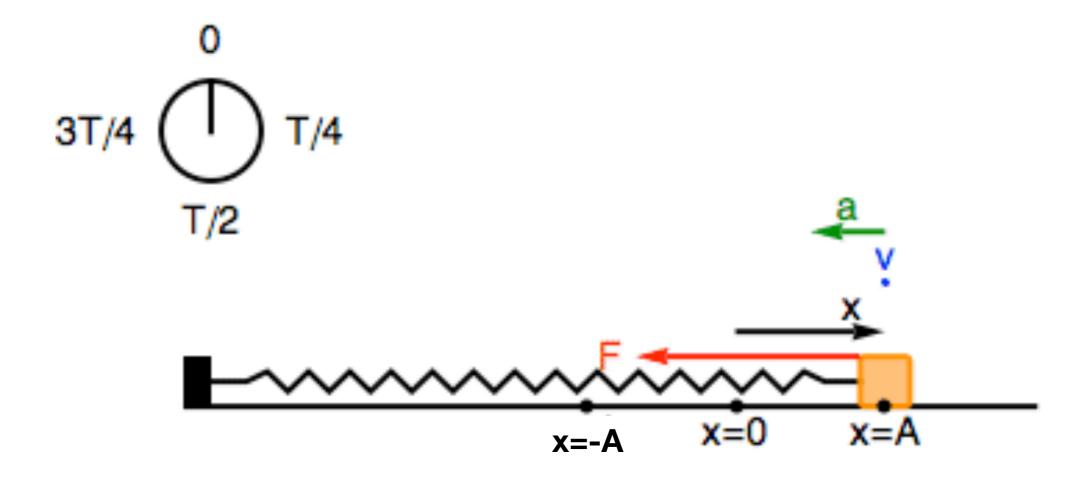
F = -kx

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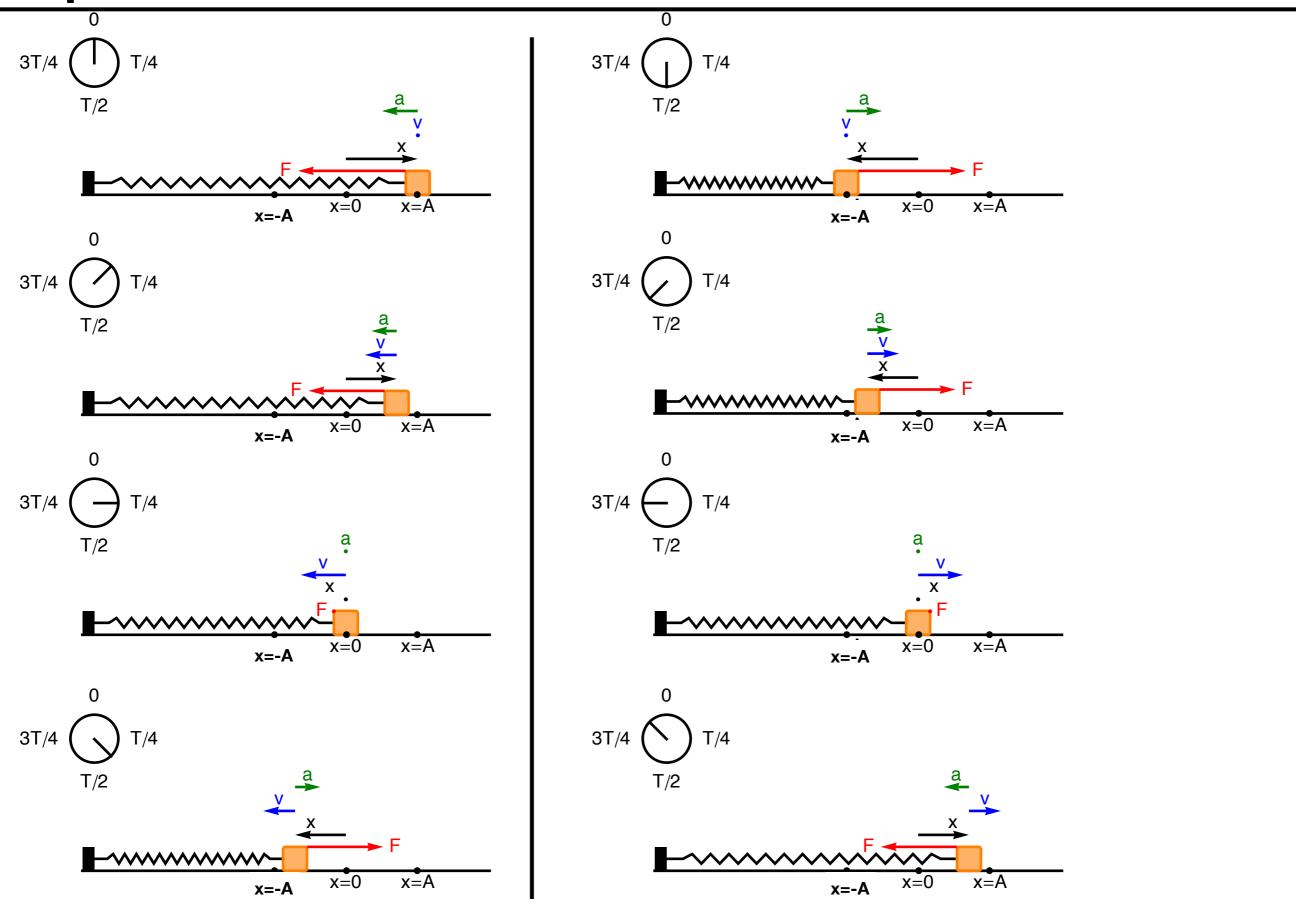
→ once way to construct such an oscillator is to use a spring obeying Hooke's law





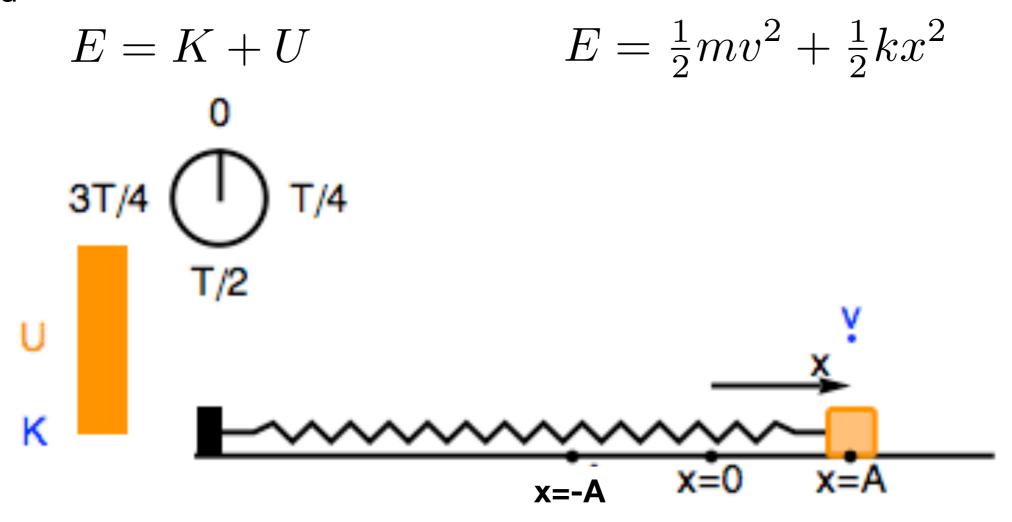


the amplitude of motion, A, is the maximum displacement from equilibrium



### energy in simple harmonic motion

→ the elastic restoring force is conservative & there is no friction, so total energy is conserved



when x=A, the block is at rest 
$$K=0,~U=\frac{1}{2}kA^2 \implies E=\frac{1}{2}kA^2$$

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

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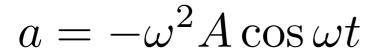
$$v = \pm \sqrt{\frac{k}{m}} \sqrt{A^2 - x^2}$$

$$v_{\text{max}} = \pm \sqrt{\frac{k}{m}} A \qquad @ \qquad x = 0$$

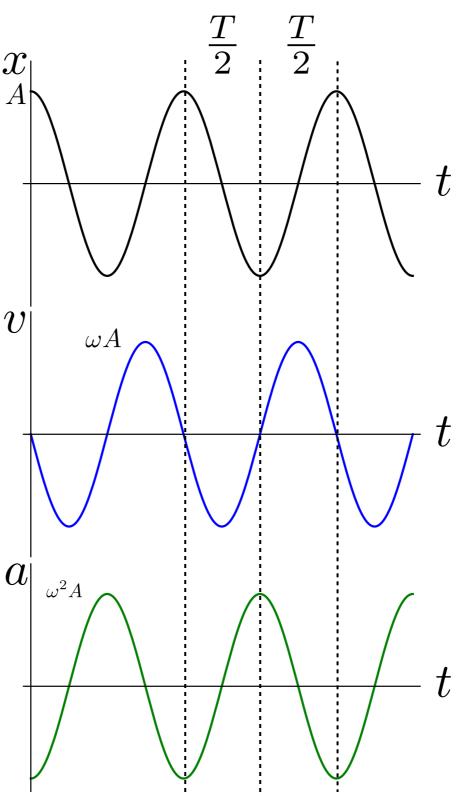
the position, velocity and acceleration in simple harmonic motion can be expressed as functions of time (assuming here that the block is released from rest at the maximum displacement)

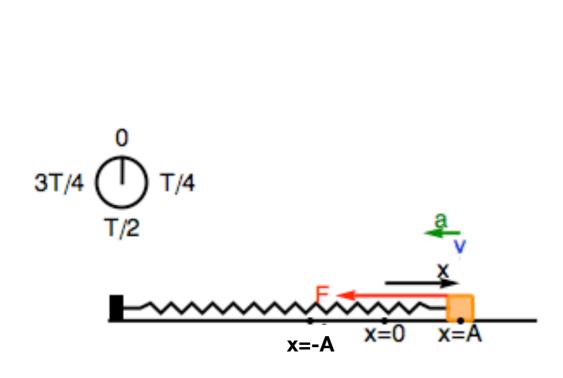


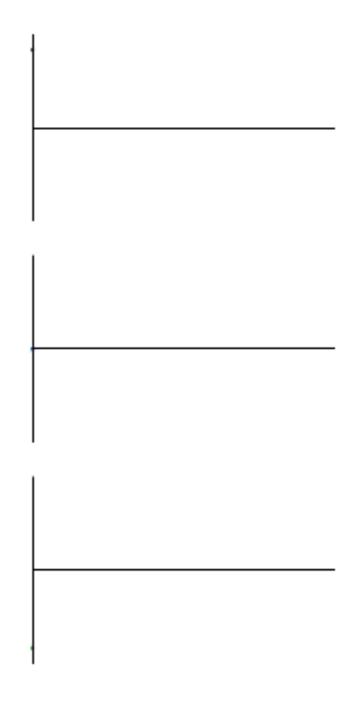
$$v = -\omega A \sin \omega t$$



these formulae can be derived using calculus or a graphical construction (read the textbook)







we can find the frequency & time period from these formulae

$$x = A\cos\omega t \qquad a = -\omega^2 A\cos\omega t$$

$$F = -kx = ma$$

$$-kA\cos\omega t = -m\omega^2 A\cos\omega t$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

cos goes through one whole cycle when  $\omega t = 2\pi$ 

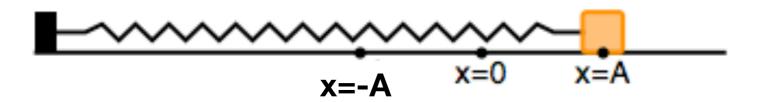
$$T = \frac{2\pi}{\omega}$$

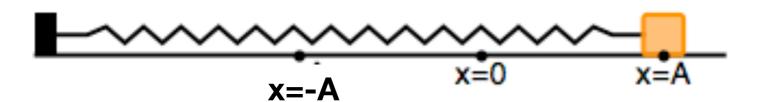
$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

notice that the time period is independent of the amplitude, *A* this is a special property of simple harmonic motion

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

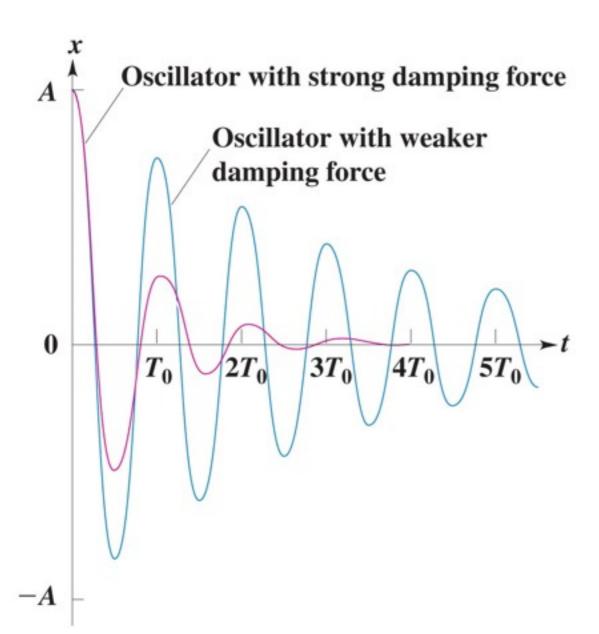




## damped & forced oscillations

- → imagine there was some friction between the block and the surface
  - → then energy would be 'lost' to the non-conservative force and the amplitude of oscillation would have to decrease

we call this effect "damping" of the oscillation

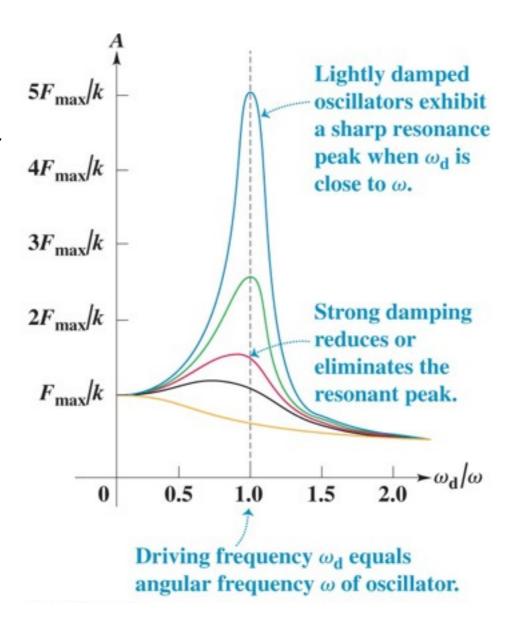


damping can be introduced deliberately to reduce oscillations, e.g. the shock absorbers attached to your car wheels

## damped & forced oscillations

- → in some mechanical systems we want the oscillation to continue despite damping
- → we need to continuously supply energy to make up for the 'loss' to damping
- → apply an extra periodic force that does net positive work on the system

→ we don't have to drive the system at or near its 'natural' frequency, but if we do we can get a 'resonance'



# Driven Mechanical Oscillator

MIT Physics Lecture Demonstration Group

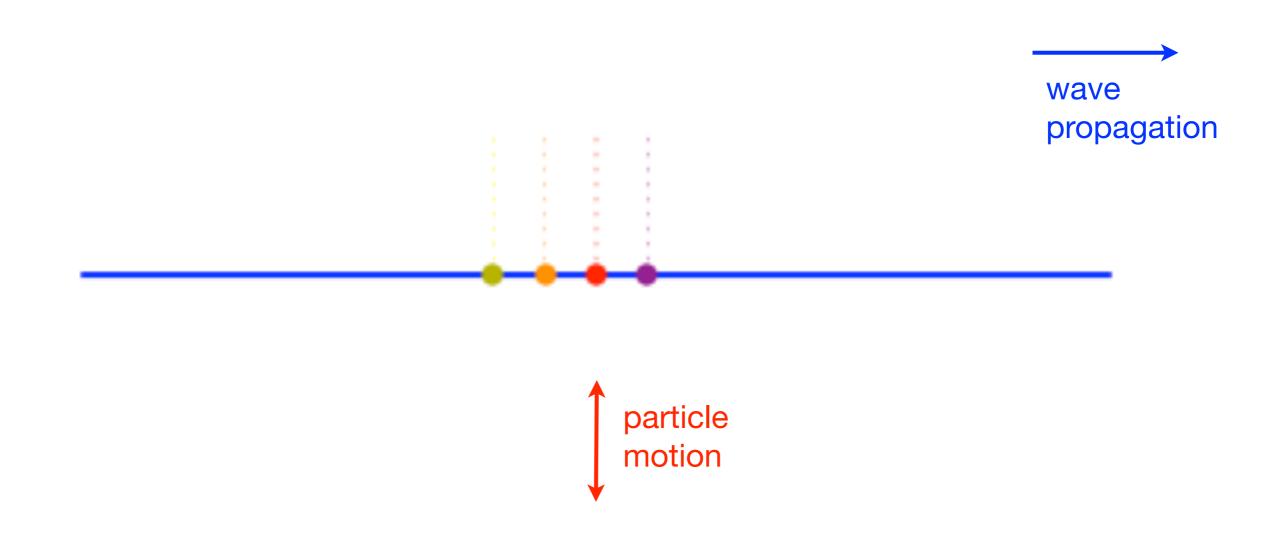
## mechanical waves





#### transverse waves

transverse waves - the particles move transverse to the direction of propagation of the wave

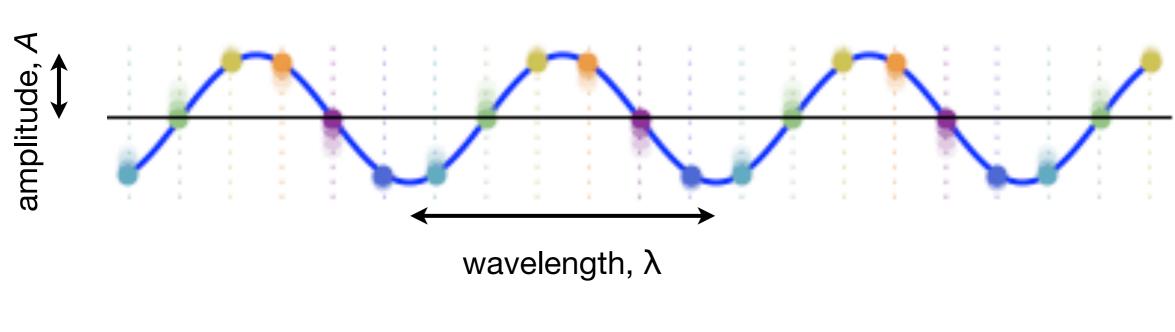


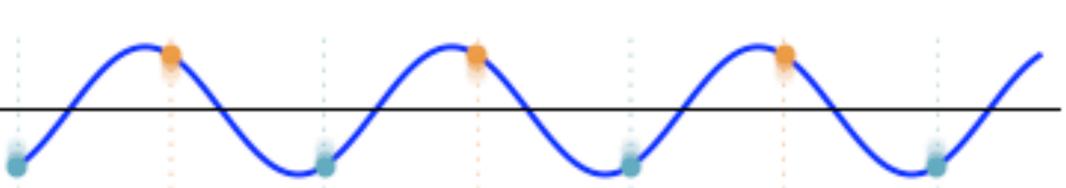
#### sinusoidal transverse waves

a special case is where the particles move with Simple Harmonic Motion - such waves are called **sinusoidal** 

wave

propagation





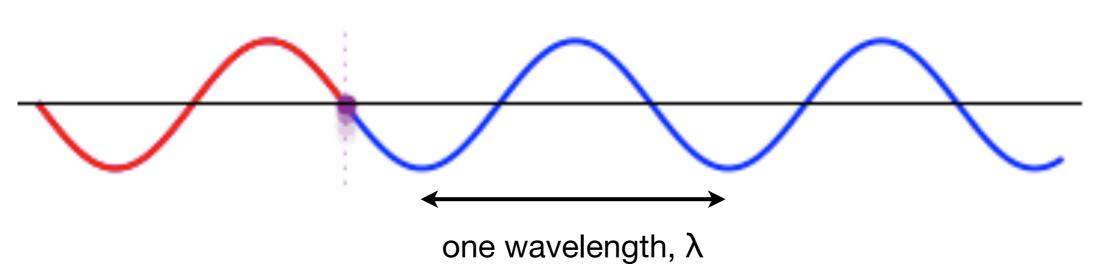
particles separated by half a wavelength oscillate out of phase

#### sinusoidal transverse waves

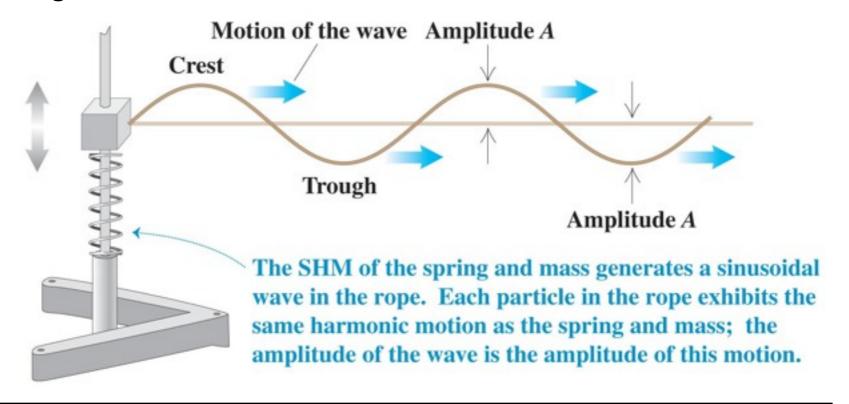
a special case is where the particles move with Simple Harmonic Motion

- such waves are called sinusoidal





in the time period for a particle to undergo a single cycle of oscillation, the wave moves along one wavelength



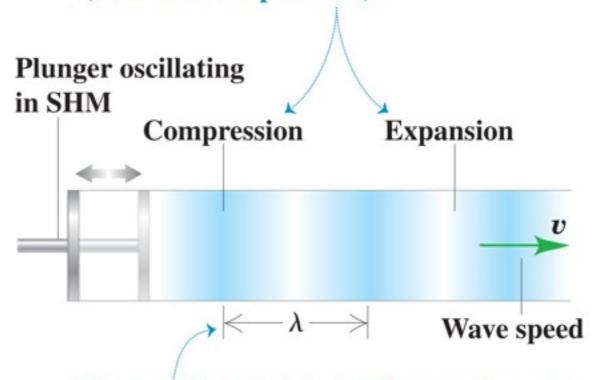
## **longitudinal waves**

if the particles move **parallel** to the direction of wave propagation, the wave is called **longitudinal** 

wave propagation

e.g. pressure waves in a fluid - i.e. sound

Forward motion of the plunger creates a compression (a zone of high pressure); backward motion creates an expansion (a zone of low pressure).



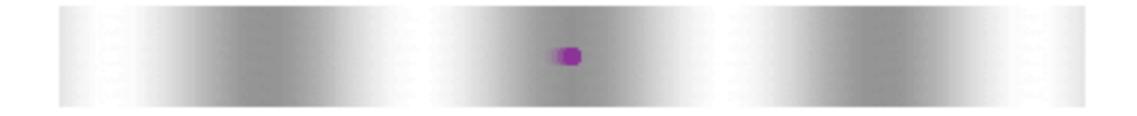
The wavelength  $\lambda$  is the distance between corresponding points on successive cycles.

## **longitudinal waves**

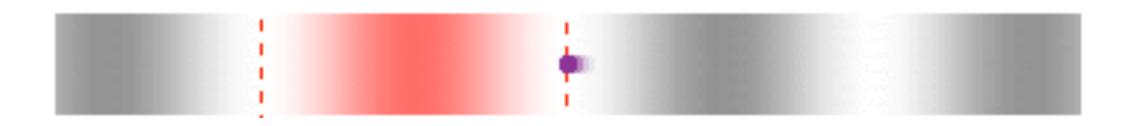
you can see the compression and expansion if I show you the motion of some of the particles



this is a sinusoidal wave so each particle is undergoing Simple Harmonic Motion



in one time period the wave moves along one wavelength



## wave propagation

notice that for both transverse & longitudinal waves the disturbance is transported without individual particles being transported

waves transport energy & momentum without transporting matter

## wave speed

notice that for periodic waves, the wave moves one wavelength in one time period

$$v = \frac{\lambda}{T} \qquad \qquad \boxed{v = \lambda f}$$

the wave speed is determined by the properties of the medium (e.g. tension & mass density of a rope)

so if we drive a system at a particular frequency, the wavelength will be fixed

## wave speed on a tensioned string

the speed of waves traveling on a string under tension is given by

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension force on the string and  $\mu$  is the mass per unit length of the string

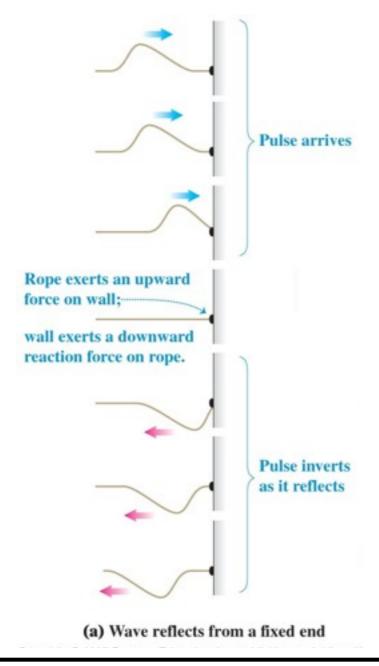
so 'heavier' strings have slower waves and 'tighter' springs have faster waves

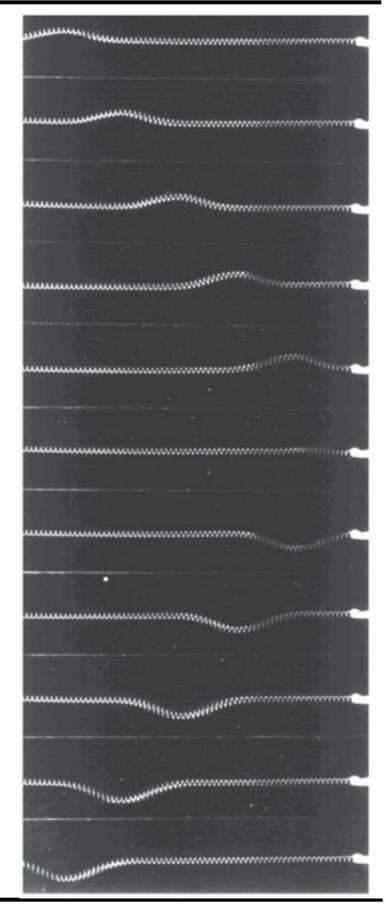
## reflections at a boundary

what happens when a wave arrives at the end of the medium supporting it?

consider these strobe images of a wave on a stretched spring with a **fixed** end at the right

notice that the pulse is reflected but with inversion



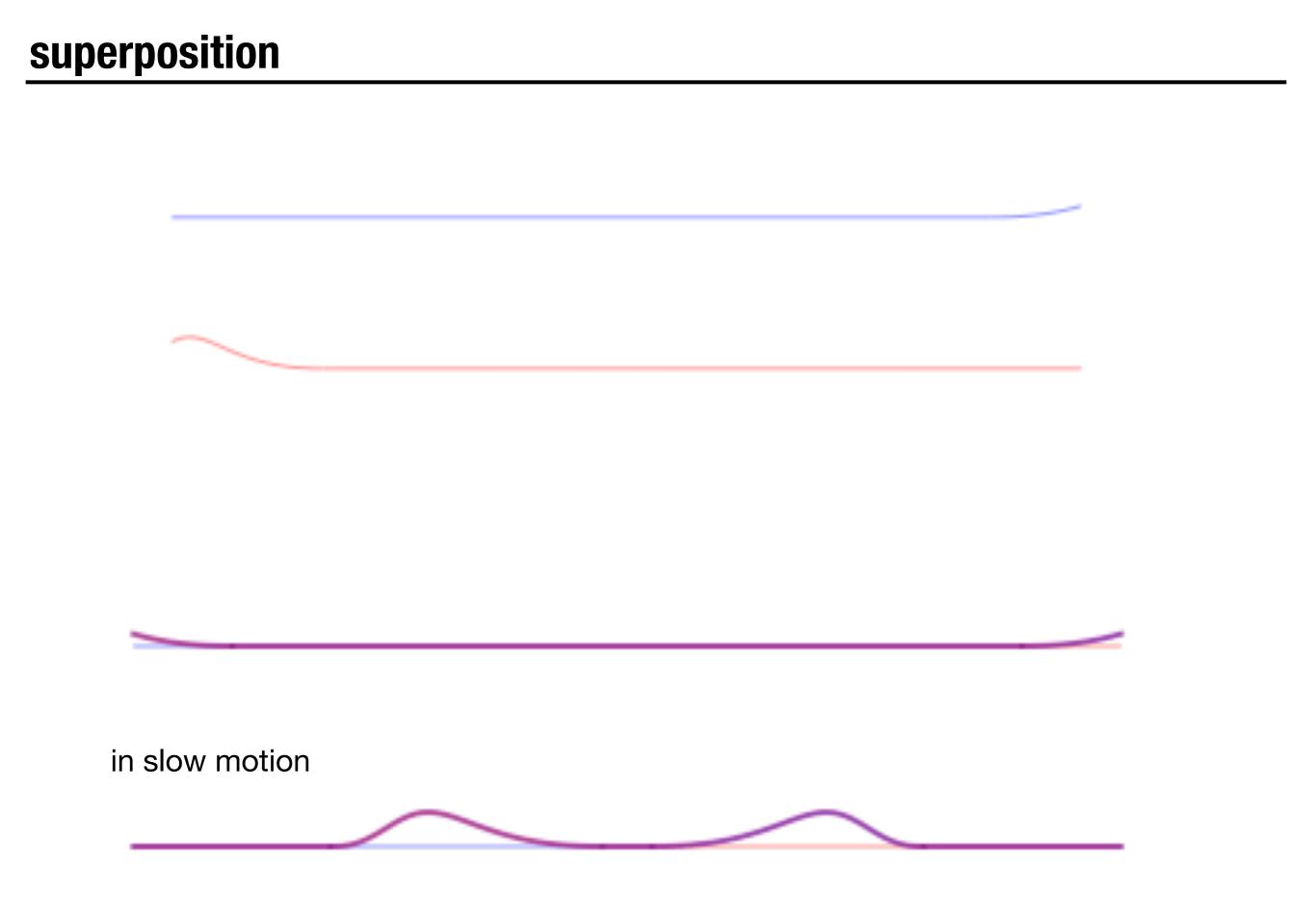


## superposition

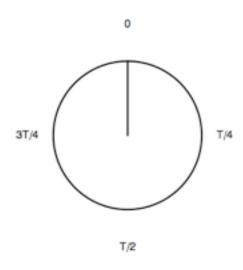
what happens if two wave pulses meet?

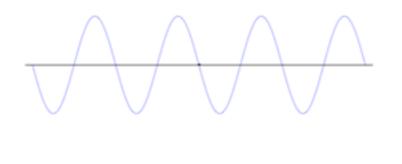
they **superpose** - or "add together"

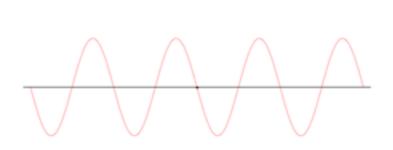
in slow motion



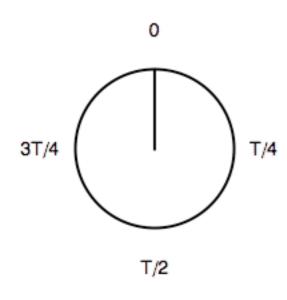
## standing waves

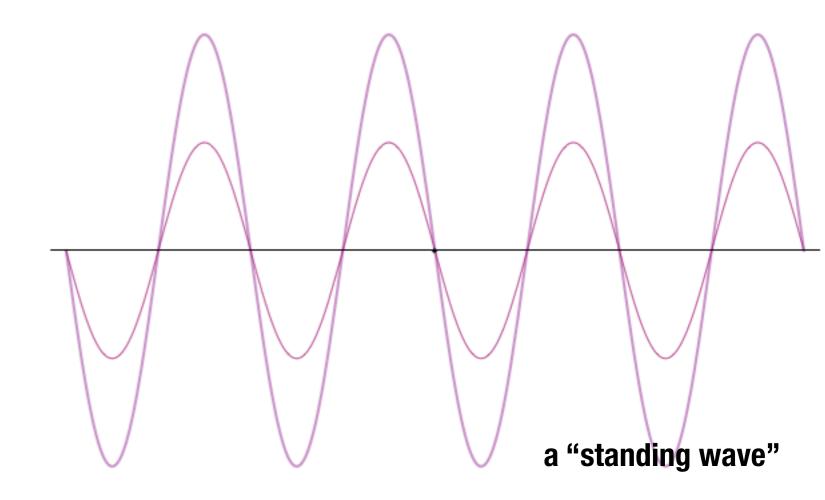




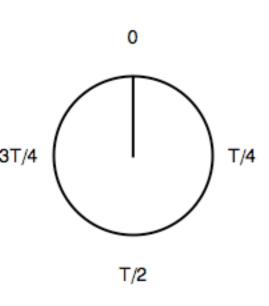


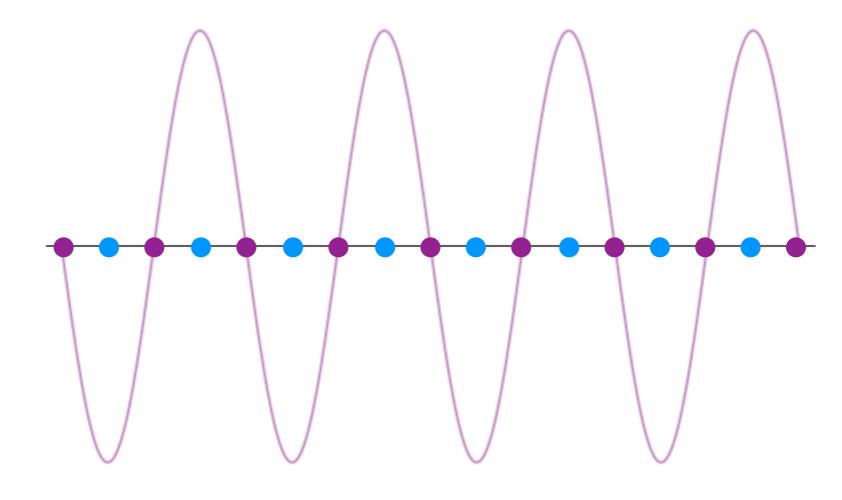
what happens if two sinusoidal waves of the same wavelength traveling in opposite directions superpose?





# standing waves





- nodes
- anti-nodes

## standing wave

for a string of a given length whose ends are fixed, only certain standing waves are allowed - those which fit a whole number of half wavelengths on the string

 $L = n\frac{\lambda}{2}$ 

$$n = 1$$

"fundamental"

$$\lambda_n = \frac{2L}{n}$$

$$n=2$$

"second harmonic"

wave speed is fixed by the properties of the string &

$$v = \lambda f$$

n = 3

$$f_n = n \frac{v}{2L} = n f_1$$

$$n=4$$

"fourth harmonic"

"third harmonic"

