#419349

Topic: Acceleration in SHM

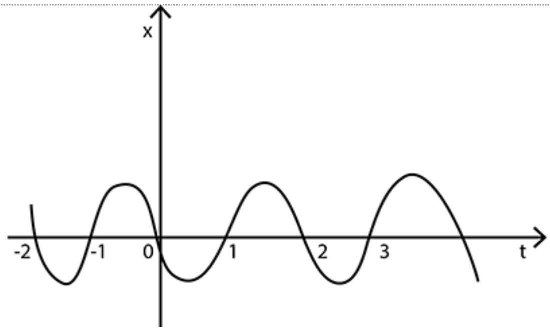


Figure gives the x-t plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at

$t = 0.3s, 1.2s, -1.2s.$

Solution

For simple harmonic motion (SHM) of a particle, acceleration (a) is given by the relation:

$a = -\omega^2 x. \ldots\ldots\ldots\ldots. (i)$

ω is angular frequency

$t = 0.3s$

x is negative. The slope of the x-t plot is also negative. Therefore, both position and velocity are negative. However, using equation (i), acceleration of the particle will be positive.

$t = 1.2s$

x is positive. The slope of the x-t plot will also be positive. Therefore, both position and velocity are positive. However, using equation (i), acceleration of the particle comes to be negative.

$t = -1.2s$

In this time interval, x is negative. The slope of the x-t plot is positive. Hence, the velocity comes to be positive. From equation (i), it can be inferred that the acceleration of the particle will be positive.

#420310

Topic: Fundamentals of Oscillations and Periodic motion

Among the following which are the examples of periodic motion?

(a) A hydrogen molecule rotating about its centre of mass.

(b) A freely suspended bar magnet displaced form its N-S direction and released.

(c) A swimmer completing one (return) trip from one bank of a river to the other and back.

(d) An arrow release from a bow.

Solution

(a) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such a motion is periodic.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic because the magnet oscillates about its position with a definite period of time.

(c) The swimmer's motion is not periodic. Though the motion of a swimmer is to and fro but will not have a definite period.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

#420321

Topic: Fundamentals of Oscillations and Periodic motion

Which of the following examples represent (nearly) simple harmonic motion and which represents periodic but not simple harmonic motion ?

(a) the rotation of earth about its axis.

(b) motion of an oscillating mercury column in a U-tube.

(c) motion of a ball bearing inside a smooth curved bowl when released from a point slightly above the lower most point.

(d) general vibrations of a poly-atomic molecule about its equilibrium position.

Solution

(a) It is periodic but not Simple Harmonic Motion because it is not to and fro about a fixed point.

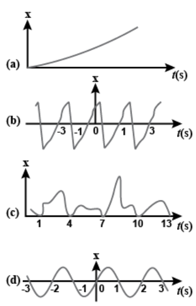
(b) It is a simple harmonic motion because the mercury moves to and fro on the same path, about the fixed position, with a certain period of time.

(c) It is simple harmonic motion because the ball moves to and fro about the lowermost point of the bowl when released. Also, the ball comes back to its initial position in the same period of time, again and again.

(d) A poly-atomic molecule has many natural frequencies of oscillation. Its vibration is the superposition of individual simple harmonic motions of a number of different molecules. Hence, it is not simple harmonic, but periodic.

#420344

Topic: Fundamentals of Oscillations and Periodic motion



Among the four plots of linear motion of a particle which of the following plots represent periodic motion and what is the period of the motion of that motion?

Solution

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

#420378

Topic: Displacement in SHM

Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(a) $\sin \omega t - \cos \omega t$

(b) $\sin^3 \omega t$

(c) $3\cos(\pi/4 - 2\omega t)$

(d) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$

(e) exp $\left(-\omega^2 t^2\right)$

(f) 1+$\omega t + \omega^2 t^2$

Solution

(a) SHM

The given function is:

$sin\,\omega t - cos\,\omega t$

$= \sqrt{2}\left[\frac{1}{\sqrt{2}} sin\,\omega t - \frac{1}{\sqrt{2}} cos\,\omega t\right]$

$= \sqrt{2}\left[sin\,\omega t \times cos\frac{\pi}{4} - cos\,\omega t \times sin\frac{\pi}{4}\right]$

$= \sqrt{2}\,sin(\omega t - \frac{\pi}{4})$

This function represents SHM as it can be written in the form: a $sin(\omega t + \phi)$ Its period is: $2\pi/\omega$.

(b) Periodic but not SHM

The given function is:

$sin^3\omega t = 1/4[3\,sin\,\omega t - sin\,3\omega t]$

The terms sin ωt and sin 3ωt individually represent simple harmonic motion (SHM). However, the superposition of two SHM is periodic and not simple harmonic.

Its period is: $2\pi/\omega$ (LCM of time periods).

(c) SHM

The given function is:

$3\,cos\,\left[\frac{\pi}{4} - 2\,\omega t\right]$

$= -3\,cos\,\left[2\,\omega t - \frac{\pi}{4}\right]$

This function represents simple harmonic motion because it can be written in the form: a $cos(\omega t + \phi)$. Its period is: $2\pi/2\omega = \pi/\omega$

(d) Periodic, but not SHM

The given function is $cos\,\omega t + cos\,3\,\omega t + cos\,5\omega t$ Each individual cosine function represents SHM. However, the superposition of three simple harmonic motions is periodic, but not simple harmonic.

Its period is LCM of the period of the three sinusoids = $2\pi/\omega$

(e) Non-periodic motion

The given function exp$(-\omega^2 t^2)$ is an exponential function. Exponential functions do not repeat themselves. Therefore, it is a non-periodic motion.

(f) The given function $1 + \omega t + \omega^2 t^2$is non-periodic.

#420393

Topic: Fundamentals of Oscillations and Periodic motion

A particle is in linear S.H.M. between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration, and force on the particle when it is:

(a) at the end A,

(b) at the end B,

(c) at the mid point of AB going towards A,

(d) at 2 cm away from B going towards A,

(e) at 3 cm away from A going towards B, and

(f) at 4 cm away from A going towards A.

Solution

Answer is:

a)Zero, Positive, Positive

b) Zero, Negative, Negative

c) Negative, Zero, Zero

d) Negative, Negative, Negative

e) Zero, Positive, Positive

f)Negative, Negative, Negative

Explanation:

a,b) The given situation is shown in the following figure. Points A and B are the two end points, with AB = 10 cm. O is the midpoint of the path.

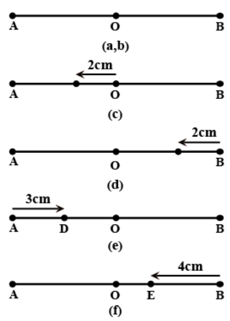
A particle is in linear simple harmonic motion between the end points At the extreme point A, the particle is at rest momentarily. Hence, its velocity is zero at this point. Its acceleration is positive as it is directed along AO. Force is also positive in this case as the particle is directed rightward. At the extreme point B, the particle is at rest momentarily.Hence, its velocity is zero at this point.

c) The particle is executing a simple harmonic motion. O is the mean position of the particle. Its velocity at the mean position O is the maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

d) The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A and B.Hence the particle's velocity and acceleration, and the force on it are all negative.

e) The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction.Hence, the values for velocity, acceleration, and force are all positive

f)-This case is similar to the one given in (d)



#420396

Topic: Acceleration in SHM

Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion ?

(a) a = 0.7 x

(b) a = -200x^2

(c) a = - 10x

(d) a = 100x^3

Solution

$F = -kx$

$ma = -kx$

$a = \frac{-kx}{m}$

For SHM, acceleration should be directly proportional to displacement and sign should be negative. Hence, (c) represents SHM.

#420408

Topic: Displacement in SHM

The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\cos(\omega t + \phi)$

If the initial $(t = 0)$ position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle ? The angular frequency of the particle is πs^{-1}. If instead of the cosine function we choose the sine function to describe the $SHM : x = B\sin(\omega t + \alpha)$ what are the amplitude and initial phase of the particle with the above initial conditions.

Solution

Intially, at t = 0;

Displacement, $x = 1cm$

Intial velocity, $v = \omega\, cm/sec.$

Angular frequency, $\omega = \pi\, rad/s^{-1}$

It is given that,

$x(t) = A\, cos(\omega t + \phi)$

$1 = A\, cos(\omega \times 0 + \phi) = A\, cos\,\phi$

$A\, cos\,\phi = 1$ ...(i)

Velocity, $v = dx/dt$

$\omega = -A\,\omega\, sin(\omega t + \phi)$

$1 = -A\, sin(\omega \times 0 + \phi) = -A\, sin\,\phi$

$A\, sin\,\phi = -1$ ...(ii)

Squaring and adding equations (i) and (ii), we get:

$A^2(sin^2\phi + cos^2\phi) = 1 + 1$

$A^2 = 2$

$\therefore A = \sqrt{2}\, cm$

Dividing equation (ii) by equation (i), we get:

$tan\,\phi = -1$

$\therefore \phi = 3\pi/4, 7\pi/4$

SHM is given as:

$x = B\, sin(\omega t + \alpha)$

Putting the given values in this equation, we get:

$1 = B\, sin(\omega \times 0 + \alpha] = 1 + 1$

$B\, sin\,\alpha = 1$ ...(iii)

Velocity, $v = \omega\, cos\,(\omega t + \alpha)$

Substituting the given values, we get:

$\pi = \pi\, B\, cos\,\alpha$

$B\, cos\,\alpha = 1$ ...(iv)

Squaring and adding equations(iii) and (iv), we get:

$B^2[sin^2\alpha + cos^2\alpha] = 1 + 1$

$B^2 = 2$

$\therefore B = \sqrt{2}$

Dividing equation (iii) by equation (iv), we get:

$B\, sin\,\alpha / B\, cos\,\alpha = 1/1$

$tan\,\alpha = 1 = tan\,\pi/4$

$\therefore \alpha = \pi/4, 5\pi/4,$.....

#420411

Topic: Springs

A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with period of 0.6 s. What is the weight of the body ?

Solution

Maximum mass that the scale can read, $M = 50kg$

Maximum displacement of the spring = Length of the scale, $l = 20cm = 0.2m$

Time period, $T = 0.6s$

Maximum force exerted on the spring, $F = Mg$

where,

$g =$ acceleration due to gravity $= 9.8\,m/s^2$

$F = 50 \times 9.8 = 490$

$\therefore$ Spring constant, $k = F/l = 490/0.2 = 2450\,Nm^{-1}$

Mass m, is suspended from the balance.

Times period, $t = 2\pi\sqrt{\dfrac{m}{k}}$

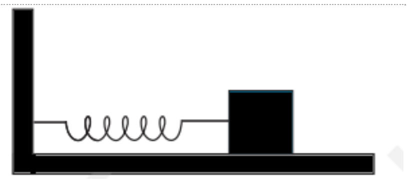
$\therefore m = \left(\dfrac{T}{2\pi}\right)^2 \times k = \left(\dfrac{0.6}{2 \times 3.14}\right)^2 \times 2450 = 22.36kg$

$\therefore$ Weight of the body $= mg = 22.36 \times 9.8 = 219.16\,N$

Hence, the weight of the body is about 219 N.

#420414

Topic: Springs



A spring having with a spring constant 1200 N m^{-1} is mounted on a horizontal table as shown in the Figure. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.

Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.

Solution

Spring constant, $k = 1200\,m^{-1}$

Mass, $m = 3kg$

Displacement, $A = 2.0\,cm = 0.02m$

(i) Frequency of oscillation v, is given by the relation:

$v = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

where, T is time period

$\therefore v = \dfrac{1}{2 \times 3.14}\sqrt{\dfrac{1200}{3}} = 3.18Hz$

Hence, the frequency of oscillations is 3.18 cycles per second.

(ii) Maximum acceleration (a) is given by the relation:

$a = \omega^2 A$

where,

$\omega =$ Angular frequency $= \sqrt{\frac{k}{m}}$

A = maximum displacement

$\therefore a = \dfrac{k}{m}A = \dfrac{1200 \times 0.02}{3} = 8\,ms^{-2}$

Hence, the maximum acceleration of the mass is $8.0\,m/s^2$

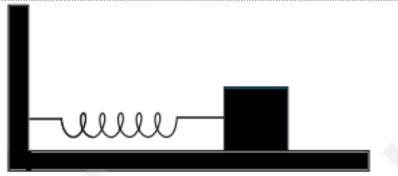
(iii) Maximum velocity, $v_{max} = A\omega$

$= A\sqrt{\dfrac{k}{m}} = 0.02 \times \sqrt{\dfrac{1200}{3}} = 0.4\,m/s$

Hence, the maximum velocity of the mass is 0.4 m/s.

#420425

Topic: Springs



Exercise:

[A spring with a spring constant of 1200 N m^{-1} is mounted on a horizontal table as shown in the Figure. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.]

In Exercise, let us take the position of mass when the spring is unstretched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position.

In what way do these function for SHM differ from each other in frequency, in amplitude or the initial phase?

Solution

Distance travelled by the mass sideways, $a = 2.0cm$

Angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}}$$
$$= \sqrt{\frac{1200}{3}} = \sqrt{400} = 20\, rad\, s^{-1}$$

(a) As time is noted from the mean position, hence using

$x = a\, sin\, \omega t$ we hav $x = 2\, sin\, 20t$

(b) At maximum stretched position, the body is at the extreme right position, with an intial phase of $\pi/2$ rad. Then,

$$x = a\, sin\left(\omega t + \frac{\pi}{2}\right) = a\, cos\, \omega t = 2\, cos\, 20t$$

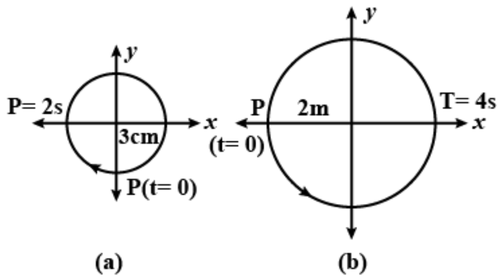
(c) At maximum compressed position, the body is at left position, with an intial phase of $3\,\pi/2\, rad$. Then,

$$x = a\, sin\left(\omega t + \frac{3\pi}{2}\right) = -a\, cos\, t = -2\, cos\, 20t$$

The functions neither differ in amplitude nor in frequency. They differ in intial phase.

#420431

Topic: Displacement in SHM



Figures given correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure. Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Solution

(a) Time period, $t = 2s$

Amplitude, $A = 3cm$

At time, $t = 0$, the radius vector OP makes an angle $\pi/2$ with the positive x-axis, i.e., phase angle $\phi = +\pi/2$

Therefore, the equation of simple harmonic motion for the x-projection of OP, at the time t, is given by the displacement equation:

$$x = A\,cos\left[\frac{2\pi t}{T} + \phi\right]$$
$$= 3\,cos\left(\frac{2\pi t}{2} + \frac{\pi}{2}\right) = -3\,sin\left(\frac{2\pi t}{2}\right)$$
$$\therefore x = -3\,sin\,(\pi t)\,cm$$

(b) Time Period, $t = 4s$

Amplitude, $a = 2m$

At time $t = 0$, OP makes an angle π with the x-axis, in the anticlockwise direction, Hence, phase angle $\phi = +\pi$

Therefore, the equation of simple harmonic motion for the x-projection of OP, at the time t, is given as:

$$x = a\,cos\left[\frac{2\pi t}{T} + \phi\right]$$
$$= 2\,cos\left(\frac{2\pi t}{4} + \pi\right)$$
$$\therefore x = -2\,cos\left(\frac{\pi}{2}t\right)\,m$$

#420439

Topic: Displacement in SHM

Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(a) $x = -2\sin(3t + \pi/3)$

(b) $x = \cos(\pi/6 - t)$

(c) $x = 3\sin(2\pi t + \pi/4)$

(d) $x = 2\cos\pi t$

Solution

(a) $x = -2\,sin\left(3t + \frac{\pi}{3}\right) = +2\,cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$
$$= 2\,cos\left(3t + \frac{5\pi}{6}\right)$$

If this equation is compared with the standard SHM equation

$x = A\,cos\left(\frac{2\pi}{T}t + \phi\right)$, then we got:

Amplitude, $A = 2\,cm$

Phase angle, $\phi = 5\pi/6 = 150^o$.

Angular velocity $\omega = 2\pi/T = 3\,rad/sec$

The motion of the particle can be plotted as shown in fig.(a).

(b) $x = cos\left(\frac{\pi}{6} - t\right)$

$= cos\left(t - \frac{\pi}{6}\right)$

If this equation is compared with the standard SHM equation

$x = A\,cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, $A = 1$

Phase angle, $\phi = -\pi/6 = 30^o$.

Angular velocity, $\omega = 2\pi/T = 1\,rad/s$

The motion of the particle can be plotted as shown in fig. (b).

(c) $x = 3\,sin\left(2\pi t + \frac{\pi}{4}\right)$

$= -3\,cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right] = -3\,cos\left(2\pi t + \frac{3\pi}{4}\right)$

If this equation is compared with the standard SHM equation

$x = A\,cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, $A = 3\,cm$

Phase angle, $\phi = 3\pi/4 = 135^o$.

Angular velocity, $\omega = 2\pi/T = 2\pi\,rad/s$

The motion of the particle can be plotted as shown in fig. (c).

(d) $x = 2\,cos\,\pi t$

If this equation is compared with the standard SHM equation

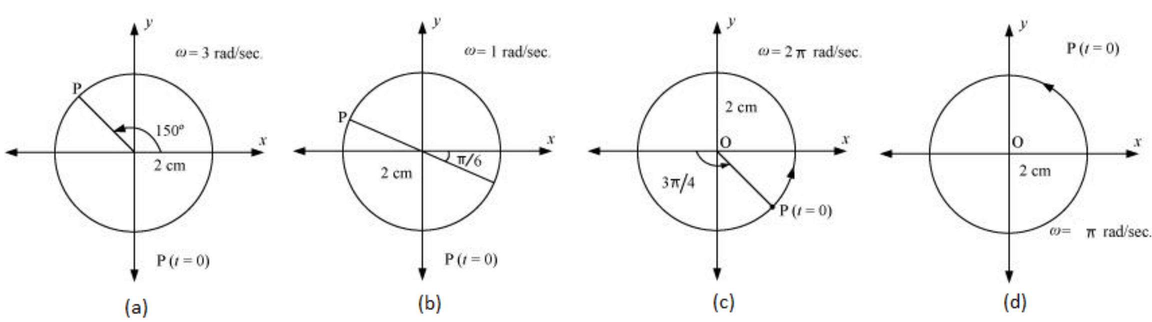
$x = A\,cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, $A = 2\,cm$

Phase angle, $\phi = 0$

Angular velocity, $\omega = \pi\,rad/s$.

The motion of the particle can be plotted as shown in fig. (d).



#420443
Topic: Springs

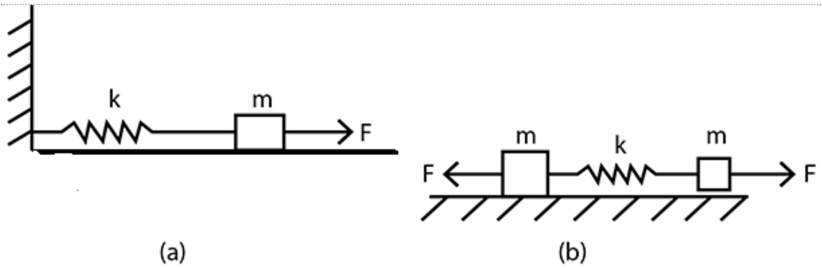


Figure (a) attached shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. (b) is stretched by the same force F.

(a) What is the maximum extension of the spring in the two cases ?

(b) If the mass in Fig (a) and the two masses in Fig (b) are released, what is the period of oscillation in each case ?

Solution

(a) The maximum extension of the spring in both cases will $= F/k$, where k is the spring constant of the springs used.

(b) In Figure (a) if x is the extension in the spring, when mass m is returning to its mean position after being released free, then restoring force on the mass is $F = -kx$ i.e., $F \propto x$

As this F is directed towards mean position of the mass, hence the mass attached to the spring will execute SHM.

Spring factor $=$ spring constant $= k$

inertia factor $=$ mass of the given mass $= m$

As time period,

$T = 2\pi\sqrt{\dfrac{inertia\ factor}{spring\ factor}}$

$\therefore T = 2\pi\sqrt{\frac{m}{k}}$

In Figure (b), we have a two body system of spring constant k and reduced mass, $\mu = m \times m/m + m = m/2$

Inertia factor $= m/2$

Spring factor $= k$

$\therefore$ time peroid, $T = 2\pi\sqrt{\dfrac{m/2}{k}} = 2\pi\sqrt{\dfrac{m}{2k}}$

#420444
Topic: Velocity in SHM

The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what it its maximum speed ?

Solution

The amplitude is 0.5m. The maximum velocity is given by

$v_{max} = \omega A = 200 \times 0.5 = 100\ m/min$

#420450
Topic: Pendulums

Answer the following questions :

(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle:

$T = 2\pi\sqrt{\dfrac{m}{k}}$. A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angle of oscillation a more involved analysis shows that T is greater than $2\pi\sqrt{\dfrac{l}{g}}$. Think of a qualitative argument to appreciate this result.

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall ?

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity ?

Solution

(a) For a simple pendulum, force constant or spring factor k is proportional to mass m, therefore, m cancels out in denominator as well as in numerator. That is why the time period of simple pendulum is independent of the mass of the bob.

(b) In the case of a simple pendulum, the restoring force acting on the bob of the pendulum is given as:

$F = -mg\,sin\,\theta$

where,

$F =$ Restoring force

$m =$ Mass of the bob

$g =$ Acceleration due to gravity

$\theta =$ Angle of displacement

For small θ , $sin\,\theta \simeq\ \theta$

For large θ, $sin\,\theta$ is greater than θ.

This decreases the effective value of g.

Hence, the time period increases as:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(c) Yes, because the working of the wrist watch depends on spring action and it has nothing to do with gravity.

(d) Gravity disappears for a object under free fall, so frequency is zero.

#420453

Topic: Pendulums

A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period ?

Solution

The bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.

Acceleration due to gravity $= g$

Centripetal acceleration $= v^2/R$

where,

v is the uniform speed of the car

R is the radius of the track

Effective acceleration (g') is given as:

$$g' = \sqrt{g^2 + \frac{v^4}{R^2}}$$

$\therefore$ Time peroid, $t = 2\pi\sqrt{\frac{l}{g'}}$

$$= 2\pi\frac{l}{g^2 + \frac{v^4}{R^2}}$$

#420460

Topic: Time Period and Frequency

A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_1. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period

$$T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$$

where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

Solution

Base area of the cork $= A$

Height of the cork $= h$

Density of the liquid $= \rho_1$

Density of the cork$= \rho$

In equilibrium:

Weight of the cork $=$ Weight of the liquid displaced by the floating cork

Let the cork be depressed slightly by x. As a result, some extra water of a certain volume is displaced. Hence, an extra up-thrust acts upward and provides the restoring force to the cork.

Up-thrust $=$ Restoring force, $F =$ Weight of the extra water displaced

$F = -(Volume \times Density \times g)$

Volume $=$ Area $\times$ Distance through which the cork is depressed

Volume $= Ax$

$\therefore F = -Ax \times \rho_1 g$...(i)

Accroding to the force law:

$F = kx$

$k = F/x$

where, k is constant

$k = F/x = -A\rho_1 g$ ...(ii)

The time period of the oscillations of the cork:

$T = 2\pi\sqrt{\dfrac{m}{k}}$ ....(iii)

where,

$m =$ Mass of the cork

$=$ Volume of the cork $\times$ Density

$=$ Base area of the cork $\times$ Height of the cork $\times$ Density of the cork

$= Ah\rho$

Hence, the expression for the time period becomes:

$$T = 2\pi\sqrt{\frac{Ah\rho}{A\rho_1 g}} = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$$

#420462

Topic: Time Period and Frequency

The end which contain mercury of a U-tube is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

Solution

Area of cross-section of the U-tube $= A$

Density of the mercury column $= \rho$

Acceleration due to gravity $= g$

Restoring force, $F =$ Weight of the mercury column of a certain height

$F = -(Volume \times Density \times g)$

$F = -(A \times 2h \times \rho \times g) = -2A\rho gh = -k \times$ Displacement in one of the arms (h)

Where,

$2h$ is the height of the mercury column in the two arms

k is a constant, given by $k = -F/h = 2A\rho g$

Time Period, $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{m}{2A\rho g}}$

where,

m is the mass of the mercury column

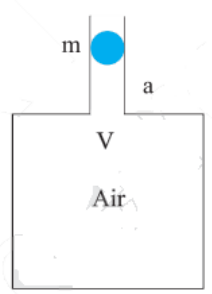
Let l be the length of the total mercury in the U-tube.

Mass of mercury, $m =$ Volume of mercury $\times$ Density of mercury $= Al\rho$

$\therefore T = 2\pi\sqrt{\dfrac{Al\rho}{2A\rho g}} = 2\pi\sqrt{\dfrac{l}{2g}}$

#420464

Topic: Time Period and Frequency



An air chamber having a volume V and a cross-sectional area of the neck is a into which a ball of mass m can move up and down without friction. Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal.

Solution

Volume of the air chamber $= V$

Area of cross-section of the neck $= a$

Mass of the ball $= m$

The pressure inside the chamber is equal to the atmospheric pressure.

Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber, $\triangle V = ax$

Volumetric strain $=$ Change in volume/original volume

$\Rightarrow \triangle V/V = ax/V$

Bulk modulus of air, $B =$ Stress/Strain $= -p/ax/V$

In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with decrease in volume.

$p = -Bax/V$

The restoring force acting on the ball, $F = p \times a$

$= -Bax/V \times a$

$= -Ba^2x/V$ ...(i)

In simple harmonic motion, the equation for restoring force is:

$F = -kx$ ...(ii)

where, k is the spring constant

Comparing equations (i) and (ii), we get:

$k = Ba^2/V$

Time Period,

$$\therefore T = 2\pi\sqrt{\frac{m}{k}}$$
$$= 2\pi\sqrt{\frac{Vm}{Ba^2}}$$

#421116

Topic: Free, Forced and Damped Oscillations

You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is places on it. Also the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Solution

The equation for the system,

$$mg - 4kx = 0 \Rightarrow k = \frac{3000 \times 10}{4 \times 15 \times 10^{-2}} = 5 \times 10^4 N/m$$

$T = 2\pi\sqrt{M/k} = 0.77$

Mass on each wheel support is $M/4 = 750kg$

For damping factor $x = x_o e^{-bt/2M}$

Now according to question, $x = x_o/2$

$\Rightarrow b = \frac{2M \ln 2}{T}$ where T is time period

$T = 2\pi\sqrt{\frac{m}{4K}} = 0.7691\ s$

Now putting the value of T we have

$\Rightarrow b = 1351.58\ kg/s$

#421117

Topic: Energy in SHM

Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Solution

Let $x = Asin\omega t$

Then, $v = dx/dt = A\omega\cos(\omega t)$

$K.E. = \frac{1}{2}mv^2$

Average kinetic energy is $\frac{1}{T}\int_0^T \frac{1}{2}mv^2dt = \frac{1}{4}mA^2\omega^2$

Average potential energy is $\frac{1}{T}\int_0^T \frac{1}{2}kx^2dt = \frac{1}{4}mA^2\omega^2$

#421119

Topic: Acceleration in SHM

A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find acceleration and velocity of the body when the displacement is (a) 5 cm (b) 3 cm (c) 0 cm.

Solution

r = 5 cm = 0.05 m

T = 0.2 s

$\omega = 2\pi/T = 2\pi/0.2 = 10\pi\, rad/s$

When displacement is y, then acceleration, $A = -\omega^2 y$

Velocity, $V = \omega\sqrt{r^2 - y^2}$

Case (a) When $y = 5cm = 0.05m$

$A = -(10\pi)^2 \times 0.05 = -5\pi^2\ m/s^2$

$V = 10\pi \times \sqrt{(0.05)^2 - (0.05)^2} = 0$

Case (b) When $y = 3cm = 0.03m$

$A = -(10\pi)^2 \times 0.03 = -3\pi^2\ m/s^2$

$V = 10\pi \times \sqrt{(0.05)^2 - (0.03)^2} = 10\pi \times 0.04 = 0.4\pi\ m/s$

Case (c) When $y = 0$

$A = -(10\pi)^2 \times 0 = 0$

$V = 10\pi \times \sqrt{(0.05)^2 - 0^2} = 10\pi \times 0.05 = 0.5\pi\ m/s$

#421121

Topic: Velocity in SHM

A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time t = 0. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0. [Hint : Start with the equation $x = a\cos(\omega t + \theta)$and note that the initial velocity in negative.]

Solution

The displacement equation for an oscillating mass is given by:

$x = A\,cos(\omega t + \theta)$

Velcoity, $v = dx/dt = -A\,\omega\,sin(\omega t + \theta)$

At $t = 0,\ x = x_0$

$x_0 = A\,cos\,\theta = x_0$ ...(i)

and, $dx/dt = -v_0 = A\,\omega\,sin\,\theta$

$A\,sin\,\theta = v_0/\omega$ ...(ii)

Squaring and adding equations (i) and (ii), we get:

$A^2(cos^2\theta + sin^2\theta) = x_0^2 + \left(\frac{v_0^2}{\omega^2}\right)$

$\therefore A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$

Therefore, the amplitude of the resulting oscillation is $(x_0^2 + \left(\frac{v_0}{\omega}\right)^2)^{\frac{1}{2}}$.