Chapter 15

Wave Motion

Units of Chapter 15

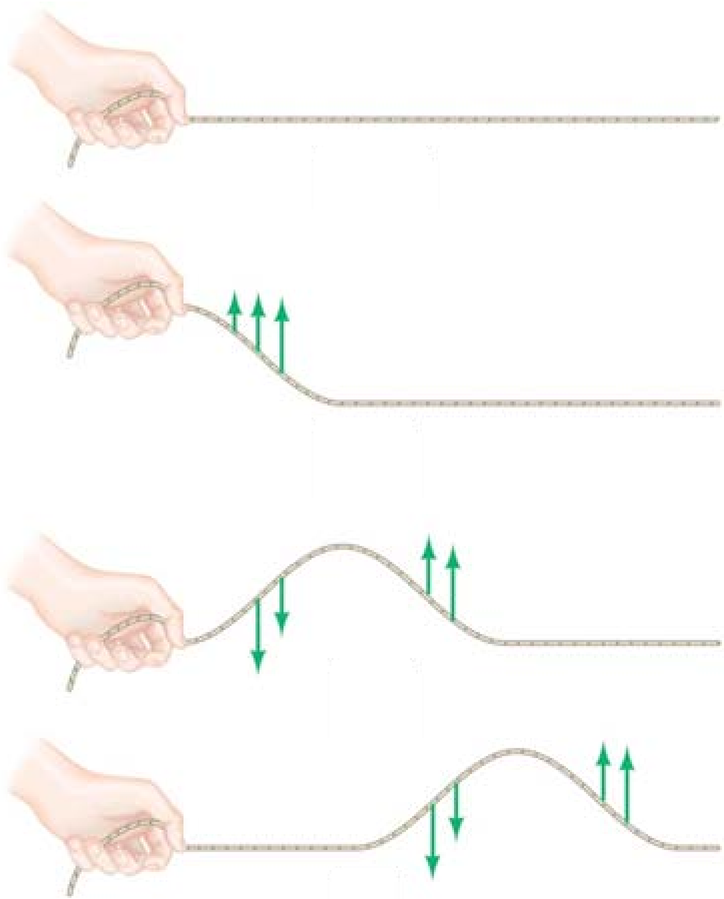
- Characteristics of Wave Motion
- Types of Waves: Transverse and Longitudinal
- Energy Transported by Waves
- Mathematical Representation of a Traveling Wave
- The Wave Equation
- The Principle of Superposition
- Reflection and Transmission

Units of Chapter 15

- Interference
- Standing Waves; Resonance
- Refraction
- Diffraction

15-1 Characteristics of Wave Motion

All types of traveling waves transport **energy**.



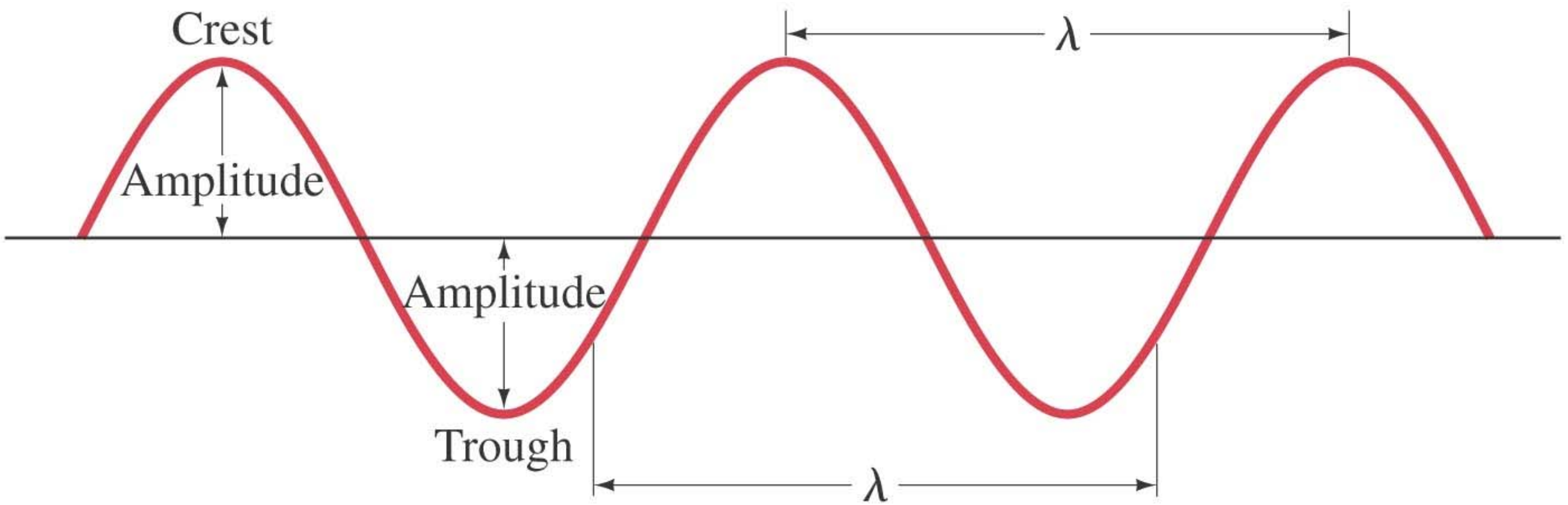
Study of a single wave **pulse** shows that it is begun with a **vibration** and is transmitted through internal forces in the medium.

Continuous waves start with vibrations, too. If the vibration is **SHM**, then the wave will be **sinusoidal**.

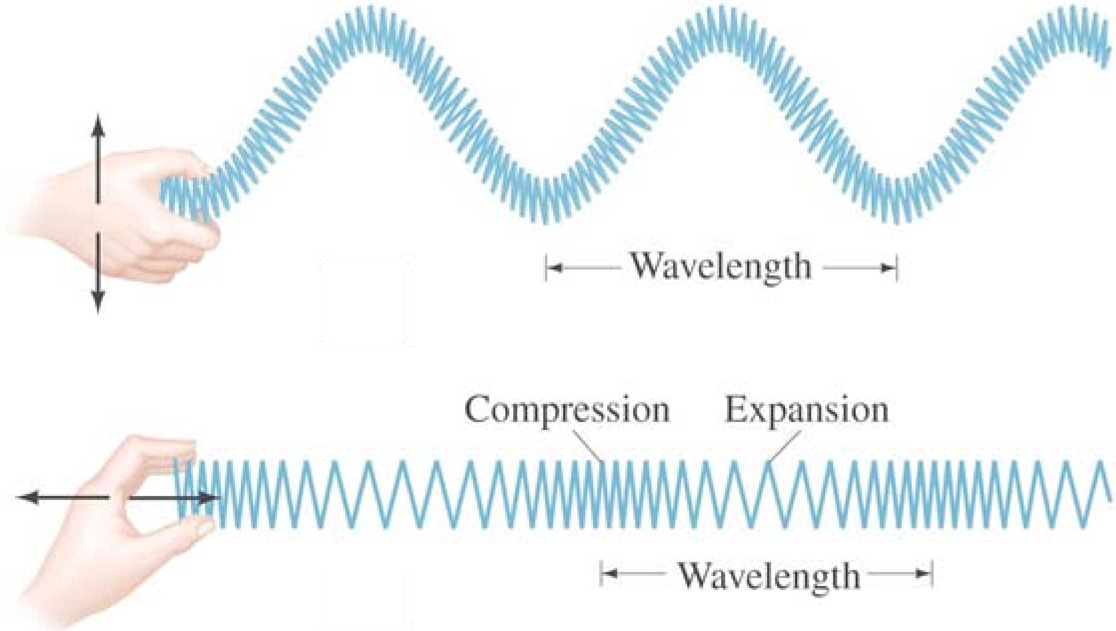
15-1 Characteristics of Wave Motion

Wave characteristics:

- **Amplitude**, A
- **Wavelength**, λ
- **Frequency**, f and **period**, T
- Wave **velocity**, $v = \lambda f$



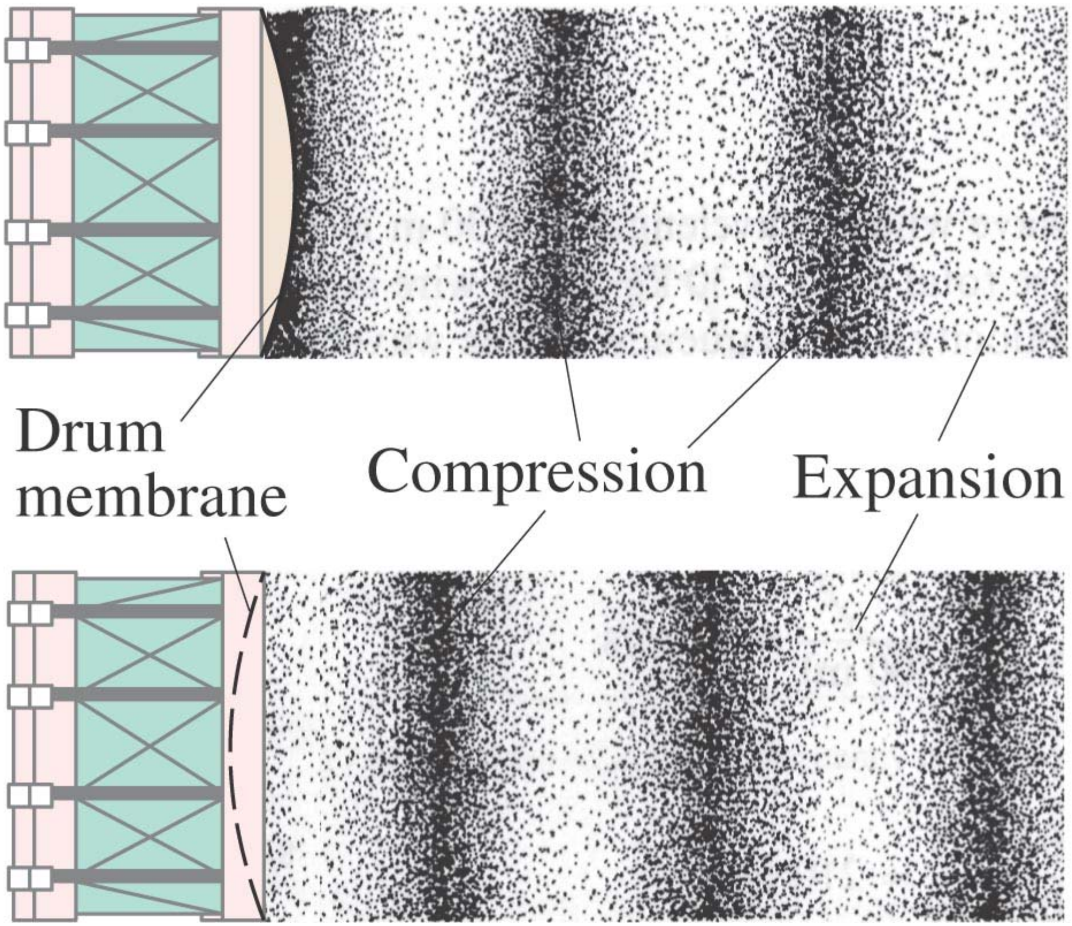
15-2 Types of Waves: Transverse and Longitudinal



The motion of particles in a wave can be either perpendicular to the wave direction (transverse) or parallel to it (longitudinal).

15-2 Types of Waves: Transverse and Longitudinal

Sound waves are **longitudinal** waves:

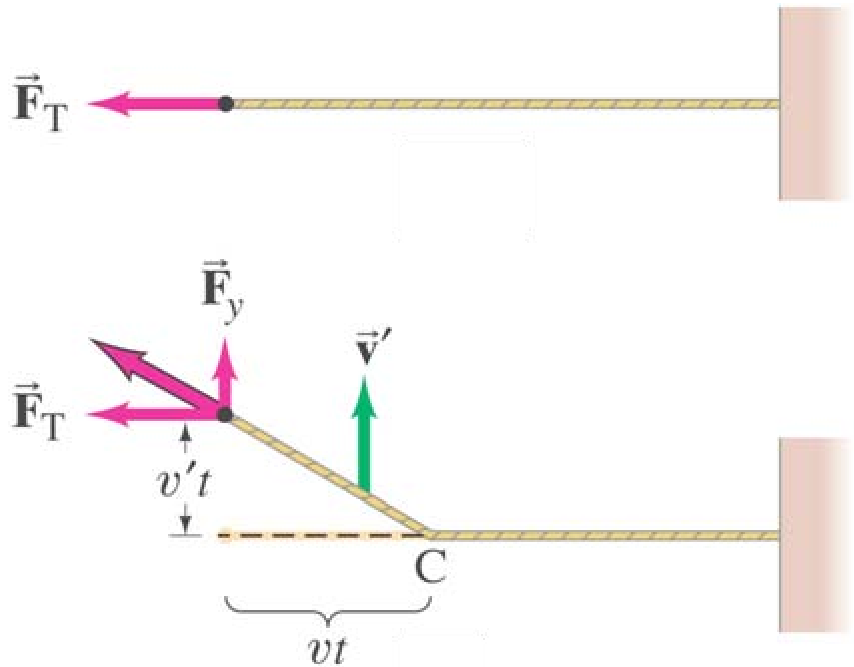


15-2 Types of Waves: Transverse and Longitudinal

The velocity of a transverse wave on a cord is given by:

$$v = \sqrt{\frac{F_T}{\mu}}.$$

As expected, the velocity increases when the tension increases, and decreases when the mass increases.



15-2 Types of Waves: Transverse and Longitudinal

Example 15-2: Pulse on a wire.

An 80.0-m-long, 2.10-mm-diameter copper wire is stretched between two poles. A bird lands at the center point of the wire, sending a small wave pulse out in both directions. The pulses reflect at the ends and arrive back at the bird's location 0.750 seconds after it landed. Determine the tension in the wire.

15-2 Types of Waves: Transverse and Longitudinal

The velocity of a longitudinal wave depends on the elastic restoring force of the medium and on the mass density.

$$v = \sqrt{\frac{E}{\rho}}$$

or

$$v = \sqrt{\frac{B}{\rho}}.$$

15-2 Types of Waves: Transverse and Longitudinal

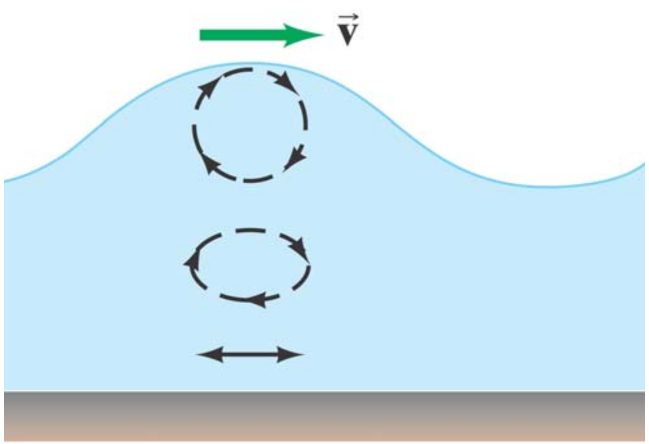
Example 15-3: Echolocation.

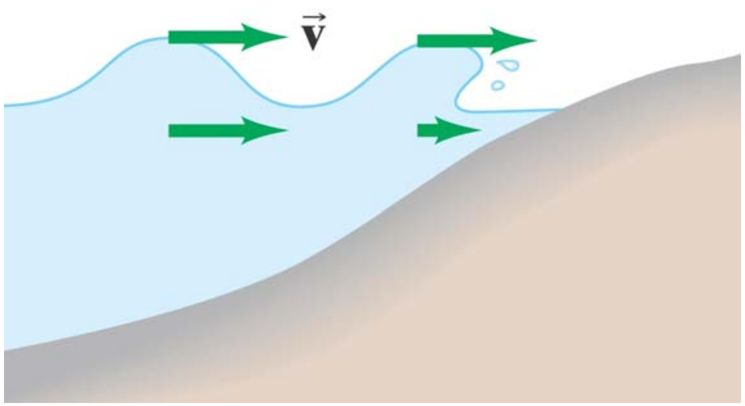
Echolocation is a form of sensory perception used by animals such as bats, toothed whales, and dolphins. The animal emits a pulse of sound (a longitudinal wave) which, after reflection from objects, returns and is detected by the animal. Echolocation waves can have frequencies of about 100,000 Hz. (a) Estimate the wavelength of a sea animal's echolocation wave. (b) If an obstacle is 100 m from the animal, how long after the animal emits a wave is its reflection detected?

15-2 Types of Waves: Transverse and Longitudinal

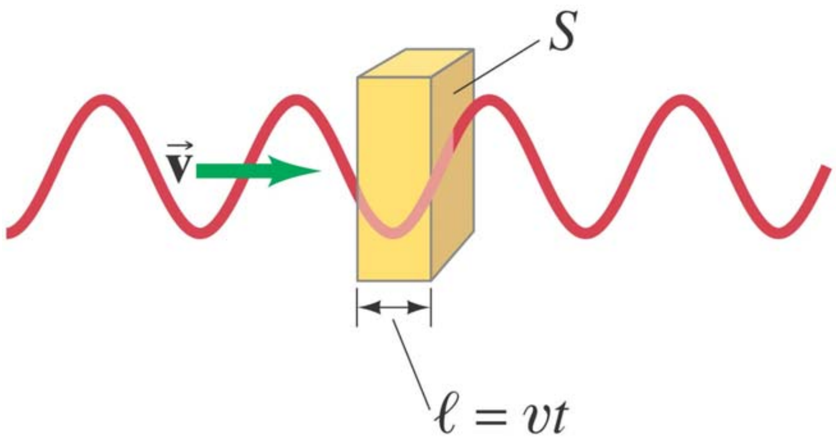
Earthquakes produce both **longitudinal** and **transverse** waves. Both types can travel through **solid** material, but only longitudinal waves can propagate through a **fluid**—in the transverse direction, a fluid has no **restoring force**.

Surface waves are waves that travel along the **boundary** between two media.





15-3 Energy Transported by Waves



By looking at the **energy** of a particle of matter in the medium of a wave, we find:

$$E = \tfrac{1}{2}kA^2 = 2\pi^2 m f^2 A^2.$$

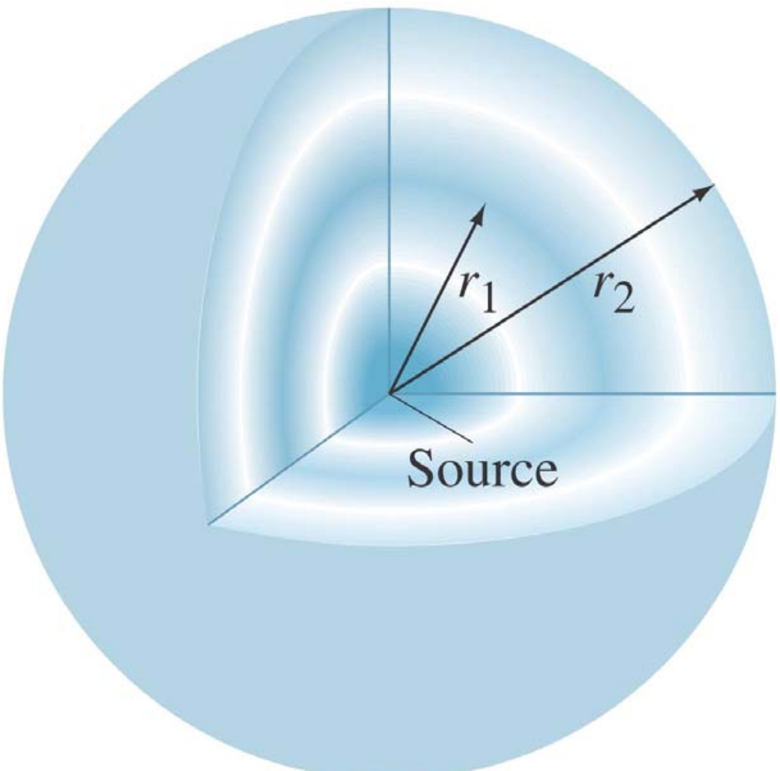
Then, assuming the **entire** medium has the same **density**, we find:

$$I = \frac{\overline{P}}{S} = 2\pi^2 v\rho f^2 A^2.$$

Therefore, the **intensity** is proportional to the square of the **frequency** and to the square of the **amplitude**.

15-3 Energy Transported by Waves

If a wave is able to spread out three-dimensionally from its source, and the medium is uniform, the wave is **spherical**.



Just from geometrical considerations, as long as the power output is **constant**, we see:

$$I \propto \frac{1}{r^2}.$$

15-3 Energy Transported by Waves.

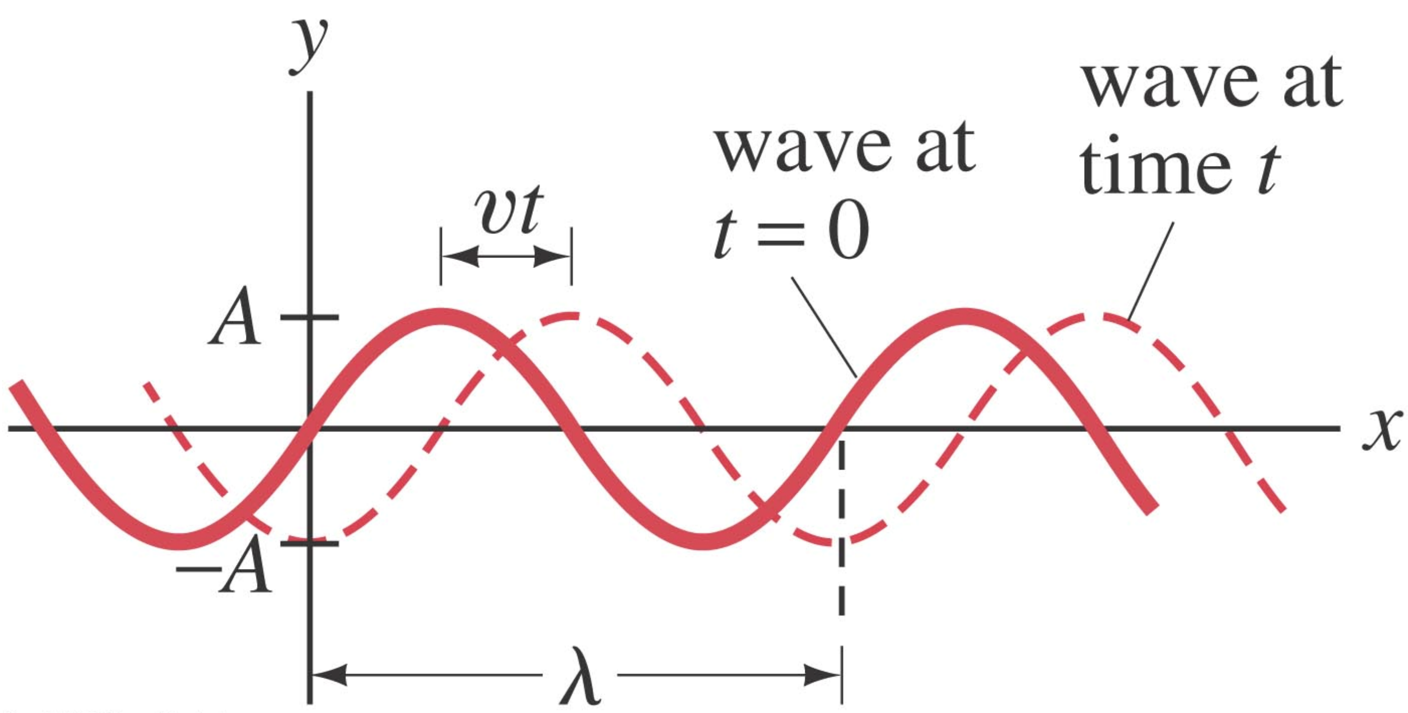
Example 15-4: Earthquake intensity.

The intensity of an earthquake P wave traveling through the Earth and detected 100 km from the source is 1.0×10^6 W/m^2. What is the intensity of that wave if detected 400 km from the source?

15-4 Mathematical Representation of a Traveling Wave

Suppose the shape of a wave is given by:

$$D(x) = A \sin \frac{2\pi}{\lambda} x.$$



15-4 Mathematical Representation of a Traveling Wave

After a time *t*, the wave crest has traveled a distance *vt*, so we write:

$$D(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right].$$

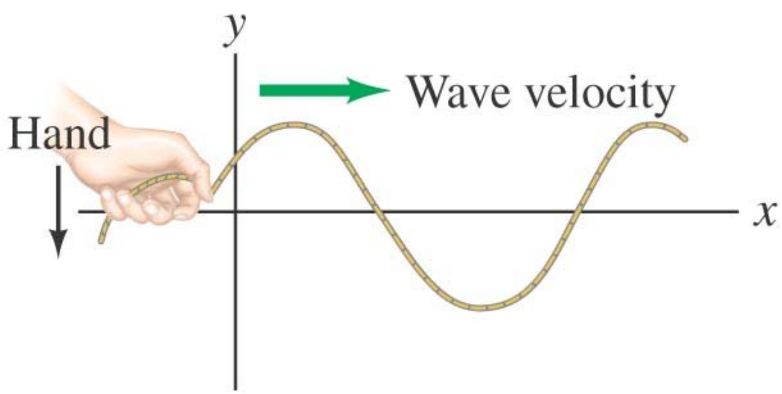
Or: $D(x,t) = A\sin(kx - \omega t),$

with $\omega = 2\pi f$, $k = \frac{2\pi}{\lambda}.$

15-4 Mathematical Representation of a Traveling Wave

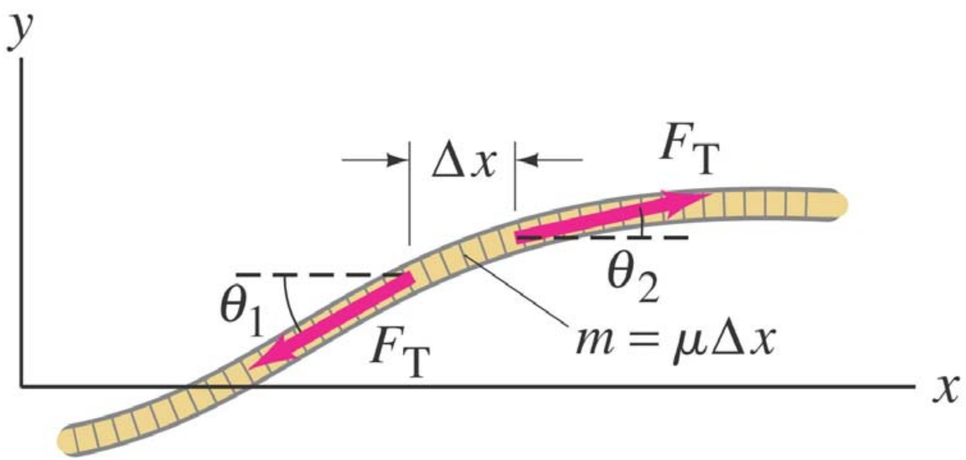
Example 15-5: A traveling wave.

The left-hand end of a long horizontal stretched cord oscillates transversely in SHM with frequency f = 250 Hz and amplitude 2.6 cm. The cord is under a tension of 140 N and has a linear density μ = 0.12 kg/m. At t = 0, the end of the cord has an upward displacement of 1.6 cm and is falling. Determine (a) the wavelength of waves produced and (b) the equation for the traveling wave.



15-5 The Wave Equation

Look at a segment of string under tension:



Newton's second law gives:

$$\Sigma F_y = m a_y$$

$$F_T \sin\theta_2 - F_T \sin\theta_1 = (\mu\,\Delta x)\,\frac{\partial^2 D}{\partial t^2}.$$

15-5 The Wave Equation

Assuming small angles, and taking the limit $\Delta x \rightarrow 0$, gives (after some manipulation):

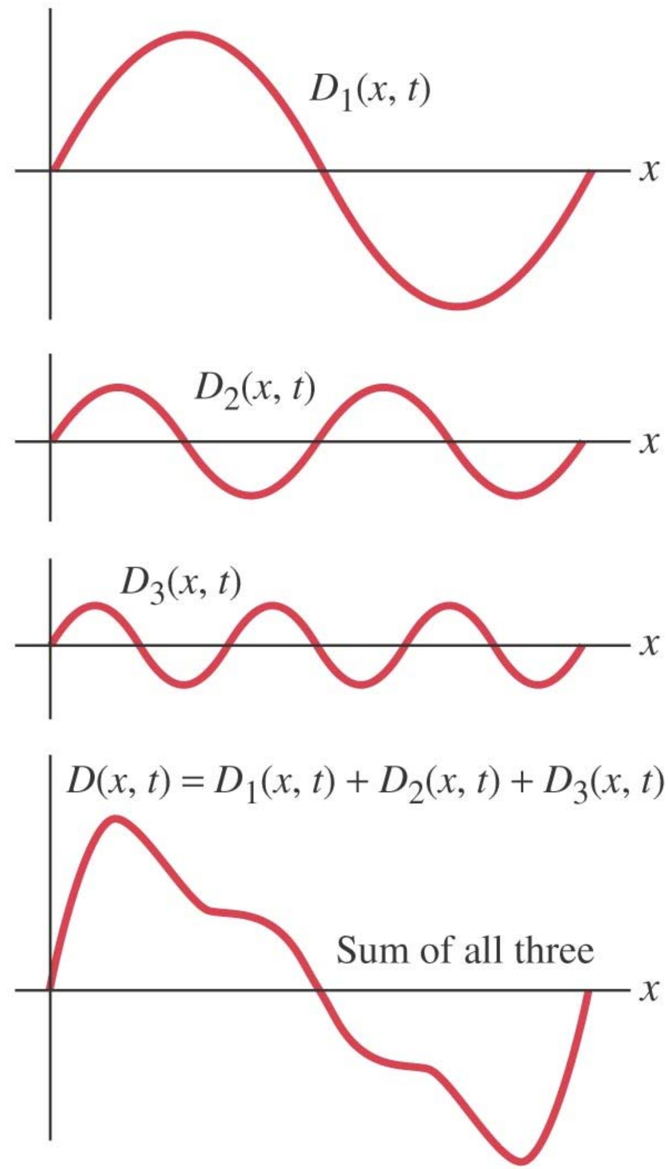
$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}.$$

This is the one-dimensional wave equation; it is a linear second-order partial differential equation in x and t. Its solutions are sinusoidal waves.

15-6 The Principle of Superposition

Superposition: The displacement at any point is the vector sum of the displacements of all waves passing through that point at that instant.

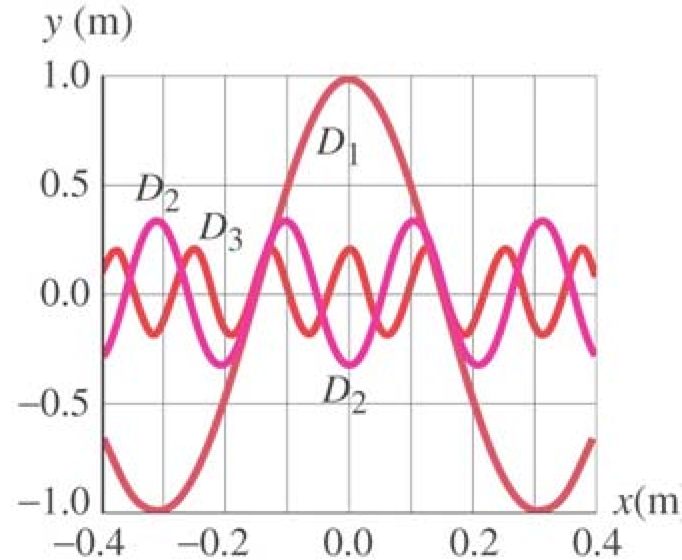
Fourier's theorem: Any complex periodic wave can be written as the sum of sinusoidal waves of different amplitudes, frequencies, and phases.

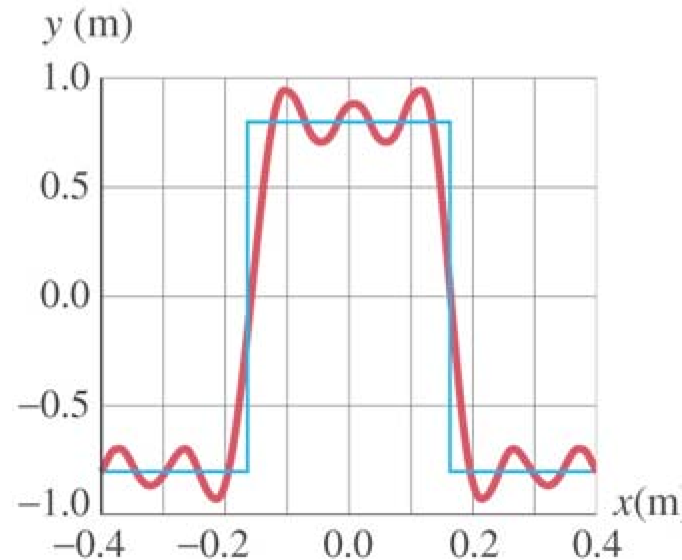


15-6 The Principle of Superposition

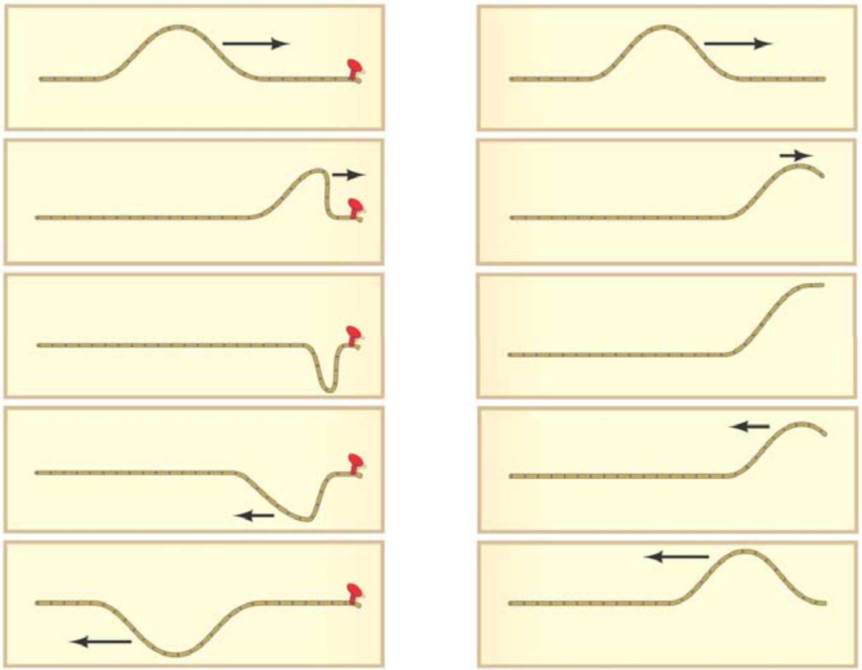
Conceptual Example 15-7: Making a square wave.

At $t = 0$, three waves are given by $D_1 = A \cos kx$, $D_2 = -{}^1/_3 A \cos 3kx$, and $D_3 = {}^1/_5 A \cos 5kx$, where $A = 1.0$ m and $k = 10\ \text{m}^{-1}$. Plot the sum of the three waves from $x = -0.4$ m to $+0.4$ m. (These three waves are the first three Fourier components of a "square wave.")





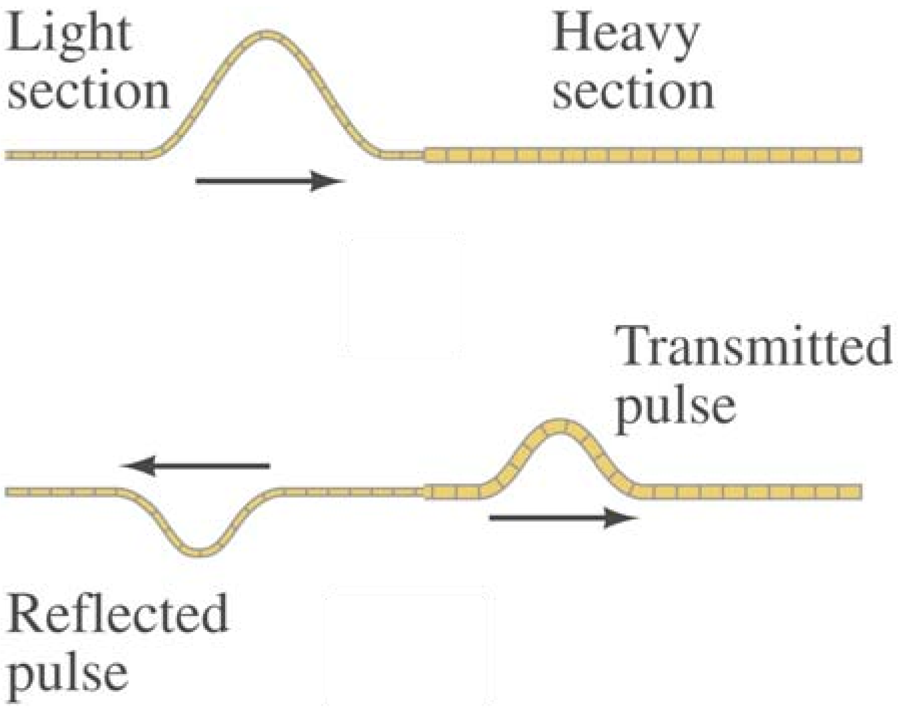
15-7 Reflection and Transmission



A wave reaching the end of its medium, but where the medium is still **free** to move, will be **reflected** (b), and its reflection will be **upright**.

A wave hitting an **obstacle** will be **reflected** (a), and its reflection will be **inverted**.

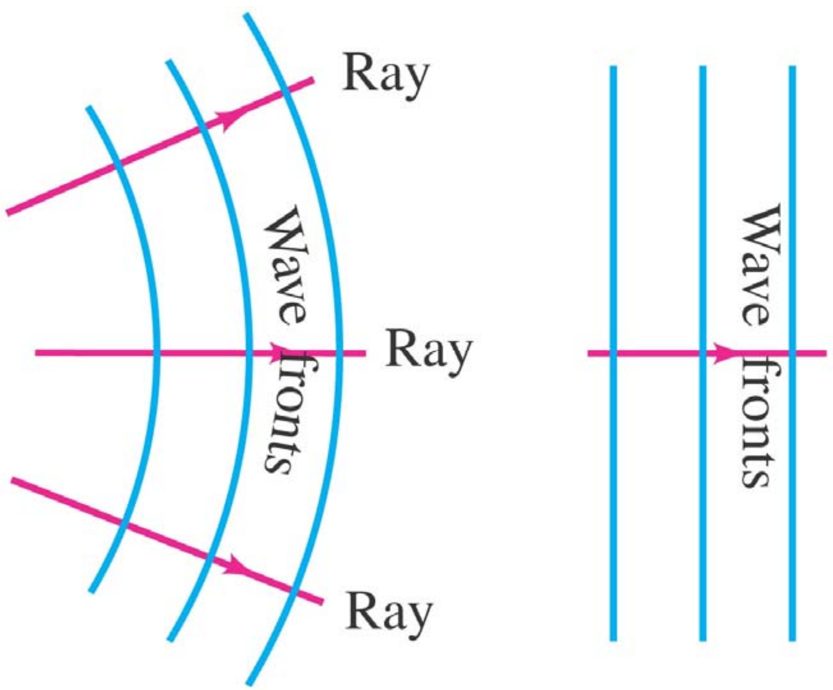
15-7 Reflection and Transmission



A wave encountering a denser medium will be partly reflected and partly transmitted; if the wave speed is less in the denser medium, the wavelength will be shorter.

15-7 Reflection and Transmission

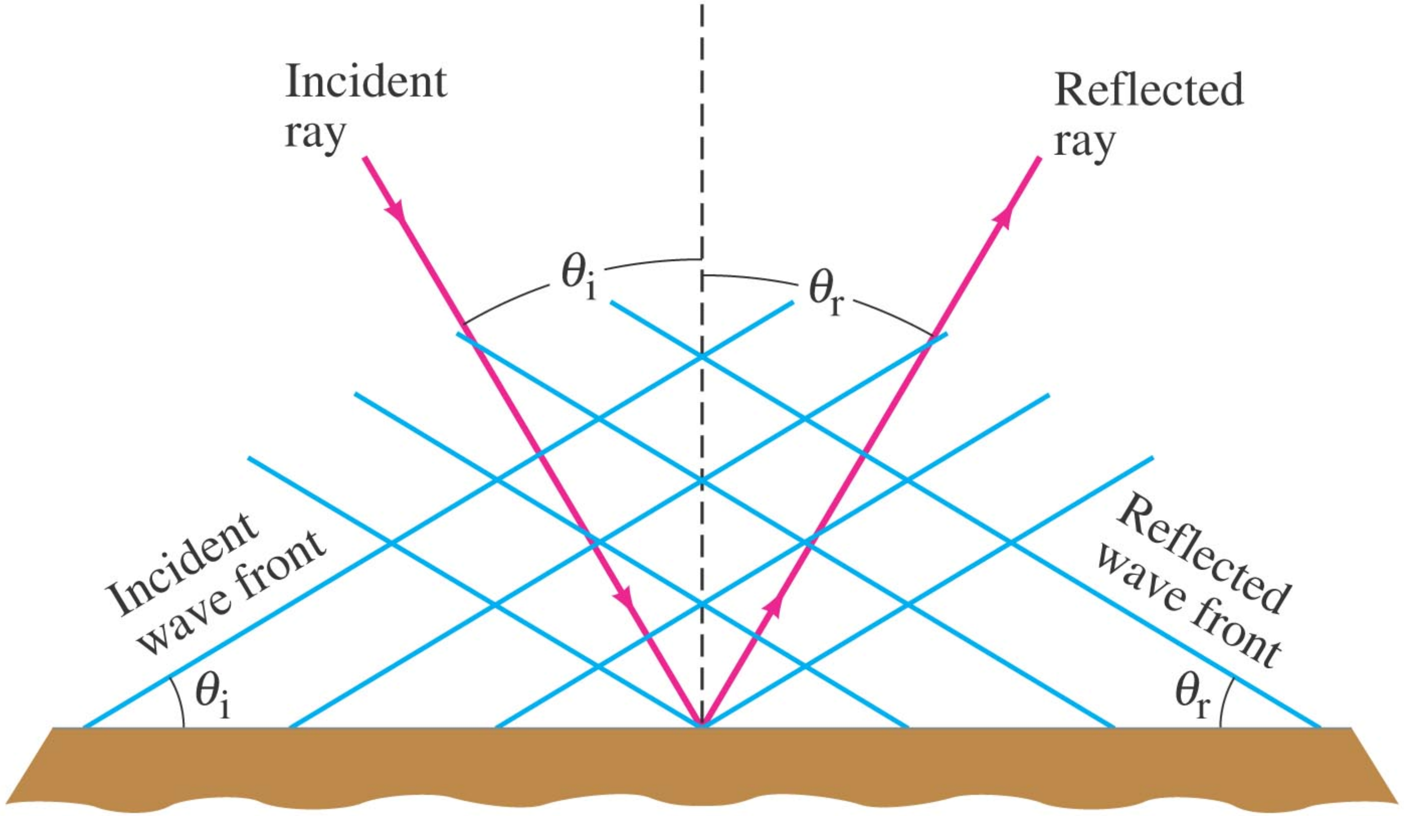
Two- or three-dimensional waves can be represented by **wave fronts**, which are curves of surfaces where all the waves have the same **phase**.



Lines **perpendicular** to the wave fronts are called **rays**; they point in the direction of **propagation** of the wave.

15-7 Reflection and Transmission

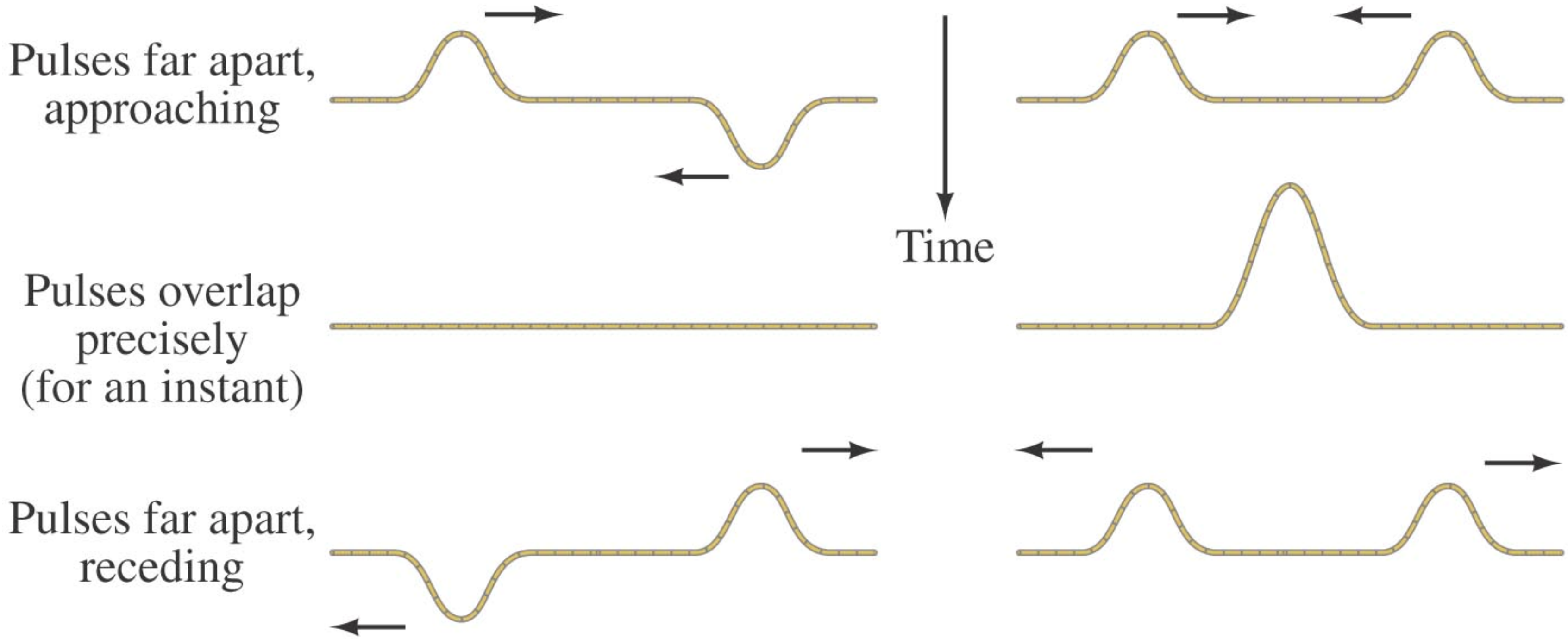
The law of reflection: the angle of incidence equals the angle of reflection.



15-8 Interference

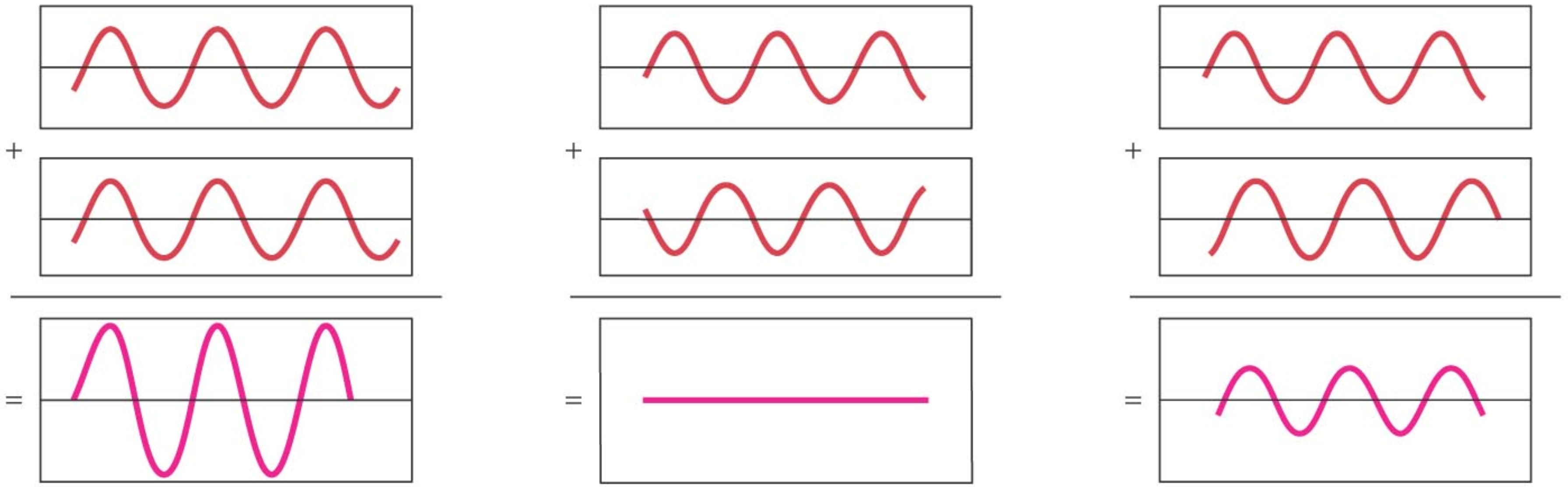
The **superposition** principle says that when two waves pass through the same point, the **displacement** is the arithmetic **sum** of the individual displacements.

In the figure below, (a) exhibits **destructive** interference and (b) exhibits **constructive** interference.

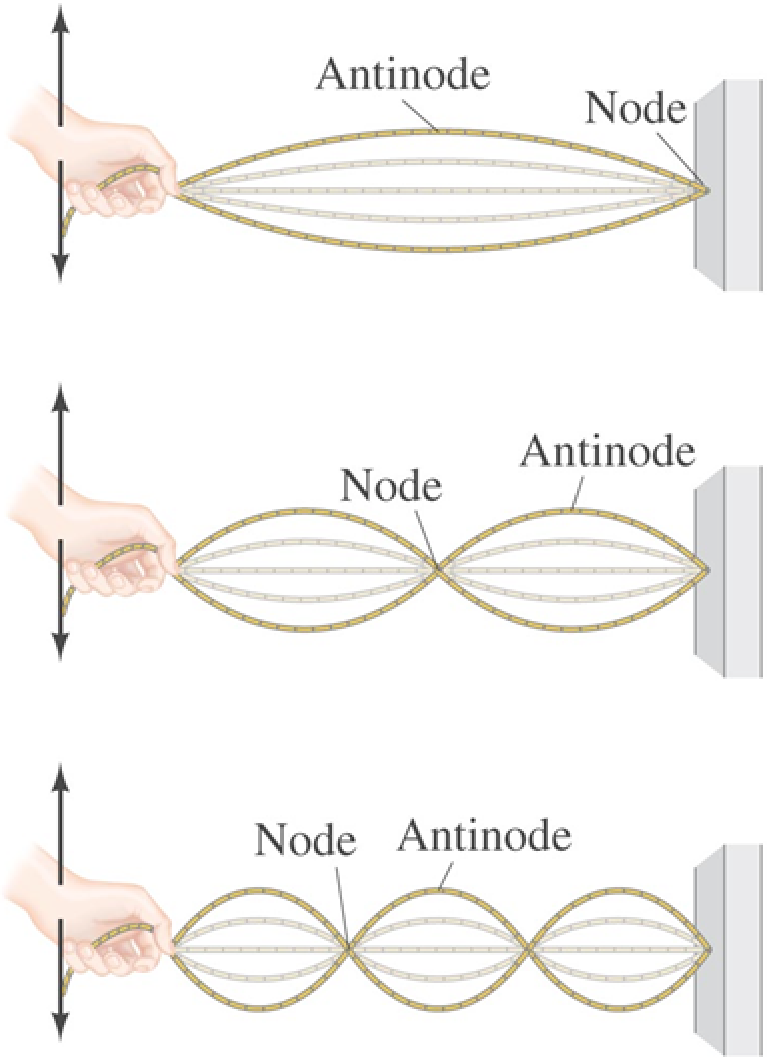


15-8 Interference

These graphs show the sum of two waves. In (a) they add **constructively**; in (b) they add **destructively**; and in (c) they add **partially destructively**.

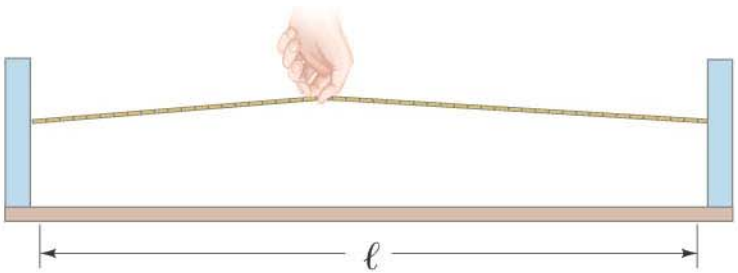


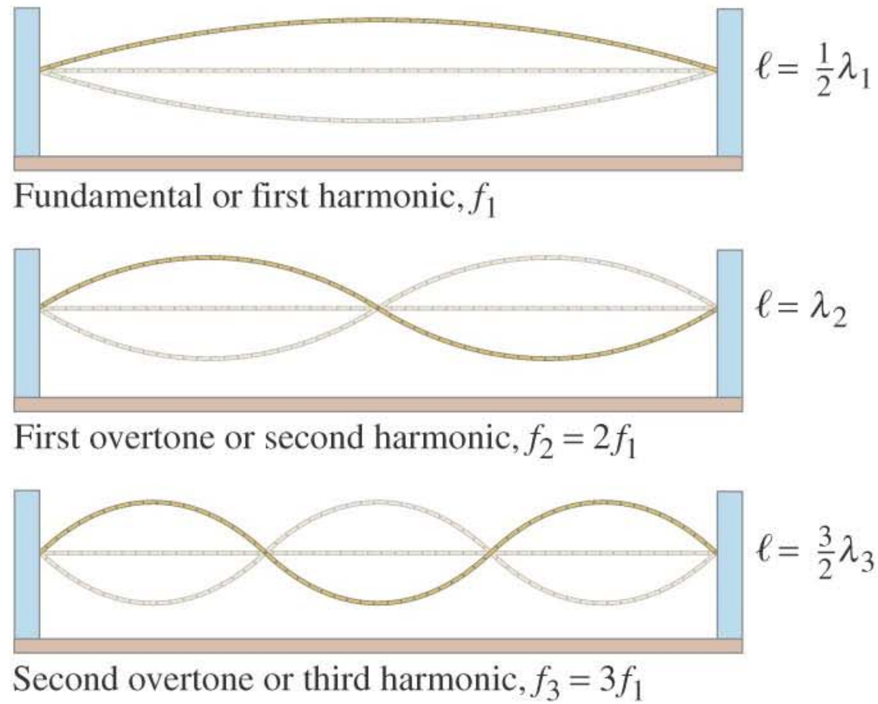
15-9 Standing Waves; Resonance



Standing waves occur when both ends of a string are **fixed**. In that case, only waves which are **motionless** at the ends of the string can persist. There are **nodes**, where the amplitude is always zero, and **antinodes**, where the amplitude varies from zero to the maximum value.

15-9 Standing Waves; Resonance





The **frequencies** of the standing waves on a particular string are called **resonant** frequencies.

They are also referred to as the **fundamental** and **harmonics**.

15-9 Standing Waves; Resonance

The **wavelengths** and **frequencies** of standing waves are:

$$\lambda_n = \frac{2\ell}{n}, \qquad n = 1, 2, 3, \cdots ,$$

and

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2\ell} = nf_1, \qquad n = 1, 2, 3, \cdots .$$

15-9 Standing Waves; Resonance

Example 15-8: Piano string.

A piano string is 1.10 m long and has a mass of 9.00 g. (a) How much tension must the string be under if it is to vibrate at a fundamental frequency of 131 Hz? (b) What are the frequencies of the first four harmonics?

15-9 Standing Waves; Resonance

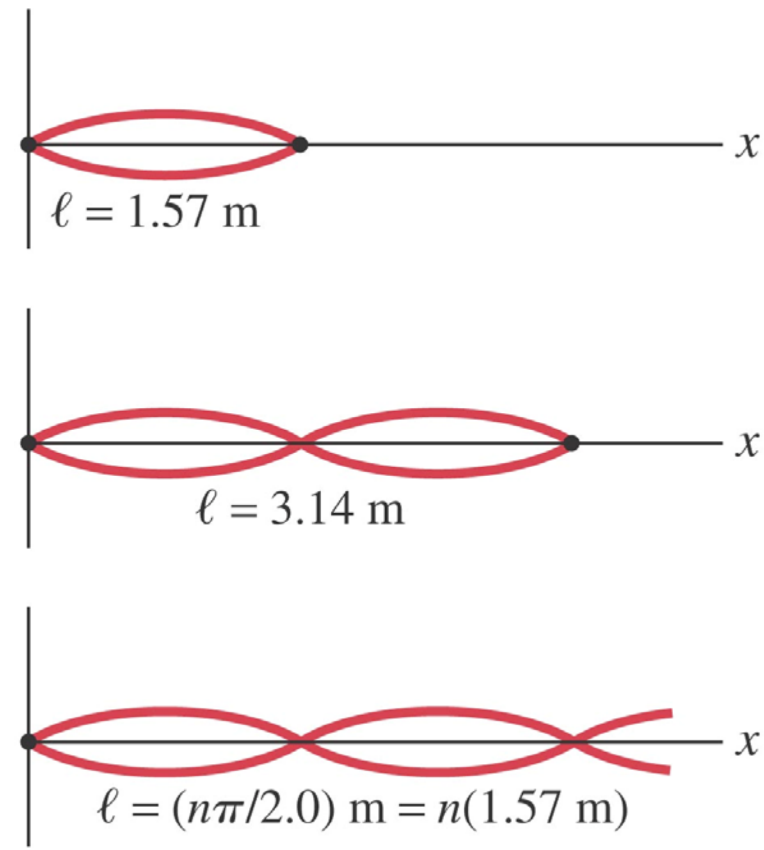
Example 15-9: Wave forms.

Two waves traveling in opposite directions on a string fixed at $x = 0$ are described by the functions

$D_1 = (0.20\ \text{m})\sin(2.0x - 4.0t)$ and

$D_2 = (0.20\text{m})\sin(2.0x + 4.0t)$

(where x is in m, t is in s), and they produce a standing wave pattern. Determine (a) the function for the standing wave, (b) the maximum amplitude at $x = 0.45$ m, (c) where the other end is fixed ($x > 0$), (d) the maximum amplitude, and where it occurs.

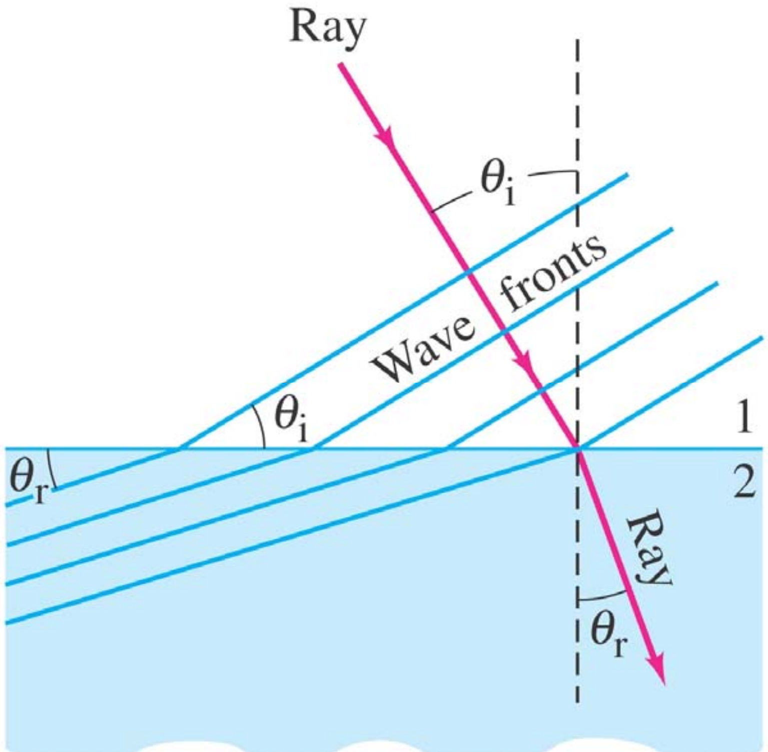


15-10 Refraction

If the wave enters a medium where the wave **speed** is different, it will be **refracted—**its wave fronts and rays will change direction.

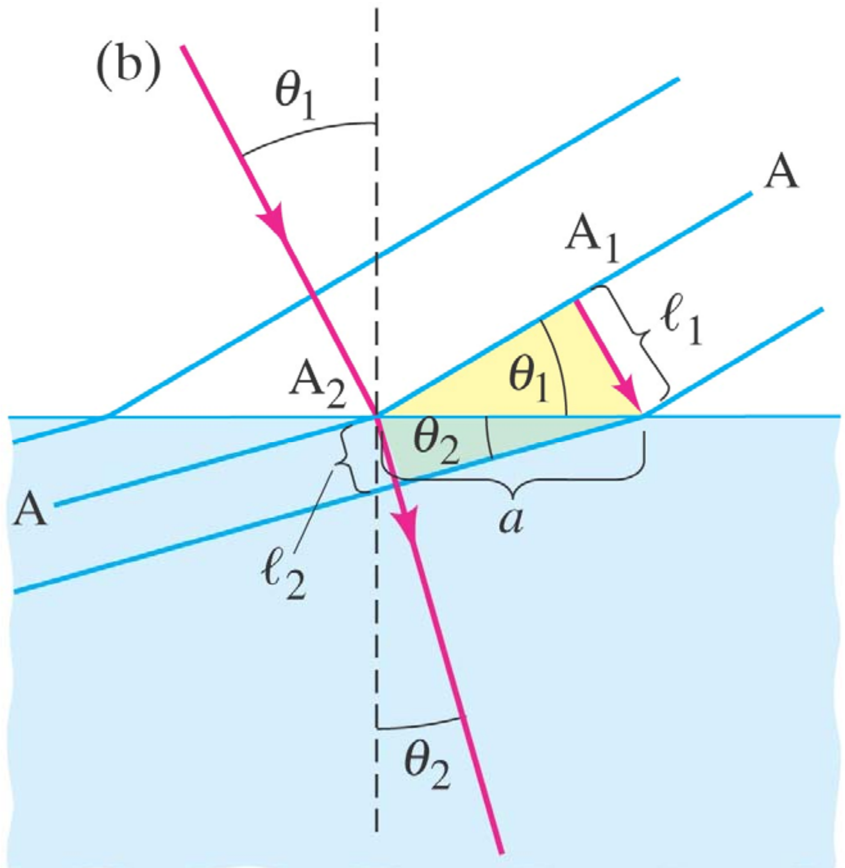
We can calculate the **angle** of refraction, which depends on both **wave speeds**:

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}.$$



15-10 Refraction

The **law of refraction** works both ways—a wave going from a **slower** medium to a **faster** one would follow the red line in the other direction.

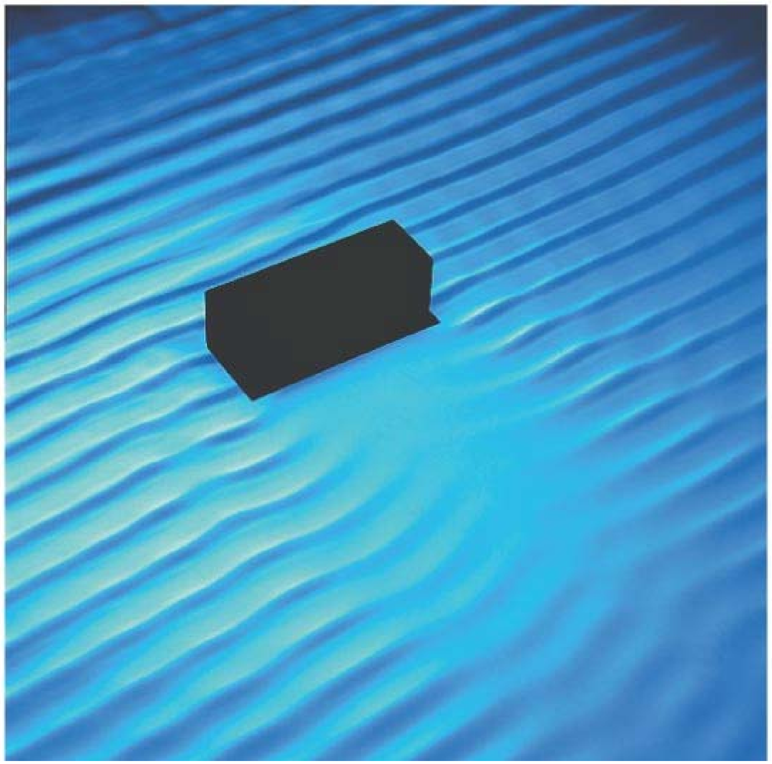


15-10 Refraction

Example 15-10: Refraction of an earthquake wave.

An earthquake P wave passes across a boundary in rock where its velocity increases from 6.5 km/s to 8.0 km/s. If it strikes this boundary at 30°, what is the angle of refraction?

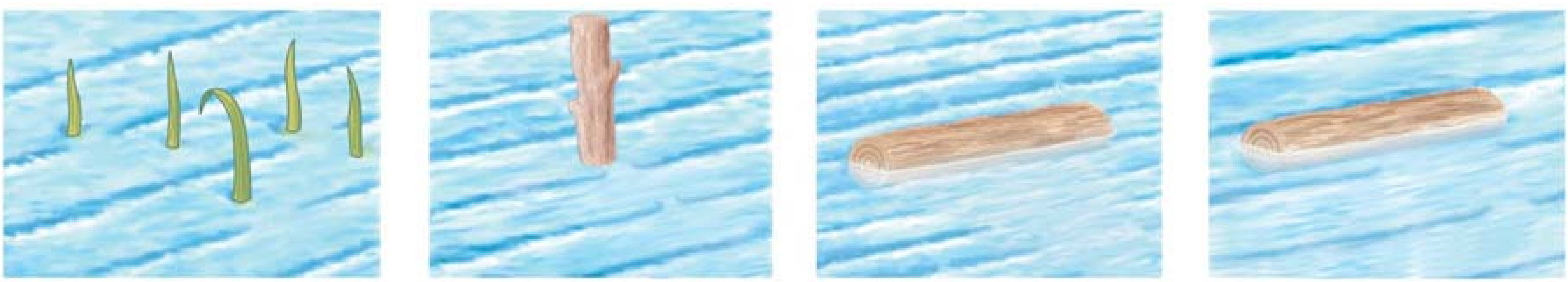
15-11 Diffraction



When waves encounter an **obstacle**, they bend around it, leaving a "shadow region." This is called **diffraction**.

15-11 Diffraction

The amount of **diffraction** depends on the size of the obstacle compared to the **wavelength**. If the obstacle is much **smaller** than the wavelength, the wave is barely affected (a). If the object is **comparable** to, or **larger** than, the wavelength, diffraction is much more significant (b, c, d).



Water waves passing blades of grass

Stick in water

Short-wavelength waves passing log

Long-wavelength waves passing log

Summary of Chapter 15

- Vibrating objects are sources of waves, which may be either pulses or continuous.
- Wavelength: distance between successive crests
- Frequency: number of crests that pass a given point per unit time
- Amplitude: maximum height of crest
- Wave velocity: $v = \lambda f$

Summary of Chapter 15

- **Transverse wave: oscillations perpendicular to direction of wave motion**

- **Longitudinal wave: oscillations parallel to direction of wave motion**

- **Intensity: energy per unit time crossing unit area (W/m^2):**

$$I \propto \frac{1}{r^2}$$

- **Angle of reflection is equal to angle of incidence**

Summary of Chapter 15

- When two waves pass through the same region of space, they interfere. Interference may be either constructive or destructive.
- Standing waves can be produced on a string with both ends fixed. The waves that persist are at the resonant frequencies.
- Nodes occur where there is no motion; antinodes where the amplitude is maximum.
- Waves refract when entering a medium of different wave speed, and diffract around obstacles.