#420275

Topic: Waves on a String

A string of mass 2.50 kg is under a tension of 200 N the length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string how long does he disturbance take to reach the other end ?

Solution

M = 2.50 kg

T = 200 N

l = 20.0 m

Mass per unit length, $\mu = M/l = 2.50/20 = 0.125 Kg\, m^{-1}$

The velocity (v) of the transverse wave in the string is given by the relation:

$v = \sqrt{T/\mu}$

$= \sqrt{200/0.125} = \sqrt{1600} = 40\ m/s$

∴ Time taken by the disturbance to reach the other end,

$t = l/v = 20/40 = 0.5s$

#420283

Topic: Speed of Sound

A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the

tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at $20^\circ C$(v = 343 m)s^{-1}:

Solution

Length of the steel wire, $l = 12m$

Mass of the steel wire, $m = 2.10kg$

Velocity of the transverse wave, $v = 343m/s$

Mass per unit length, $\mu = m/l = 2.10/12 = 0.175kg\, m^{-1}$

For Tension T, velocity of the transverse wave can be obtained using the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$\therefore\ T = v^2\mu$

$= (343)^2 \times 0.175 = 20588.575 \simeq 2.06 \times 10^4 N.$

#420302

Topic: Speed of Sound

Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air

(a) is independent of pressure,

(b) increases with temperature,

(c) Increases with humidity .

Solution

(a) Take the relation:

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{....(i)}$$

where,

Density, ρ = Mass/Volume = M/V

M = Molecular weight of the gas

V = Volume of the gas

Hence, equation **(i)** reduces to:

$v = \sqrt{\frac{\gamma PV}{m}}$ ....**(ii)**

Now from the ideal gas equation for n = 1:

PV = RT

For constant T, PV = Constant

Since both M and γ are constants, v = Constant

Hence, at a constant temperature, the speed of sound in a gaseous medium is independent of the change in the pressure of the gas.

(b) Take the relation:

$v = \sqrt{\frac{\gamma P}{\rho}}$ ....**(i)**

For one mole of any ideal gas, the equation can be written as:

PV = RT

P = RT/V ....**(ii)**

Substituting equation **(ii)** in equation **(i)**, we get:

$v = \sqrt{\frac{\gamma RT}{P\rho}} = \sqrt{\frac{\gamma RT}{m}}$ .....**(iii)**

where,

mass, M = ρV is a constant

γ and R are also constants

We conclude from equation **(iii)** that $V \propto \sqrt{T}$

Hence, the speed of sound in a gas is directly proportional to the square root of the temperature of the gaseous medium, i.e., the speed of the sound increases with an increase in the temperature of the gaseous medium and vice versa.

(c) Let V_m and V_d be the speed of sound in moist air and dry air respectively.

Let ρ_m and ρ_d be the densities of the moist air and dry air respectively.

Take the relation :

$v = \sqrt{\frac{\gamma \rho}{\rho}}$

Hence , the speed of sound in most air is

$v_m = \sqrt{\frac{\gamma \rho}{\rho_m}}$ ... (i)

And the speed of sound in dry air is:

$v_d = \sqrt{\frac{\gamma \rho}{\rho_d}}$ ... (ii)

On dividing equations (i) and (ii), we get:

$\frac{v_m}{v_d} = \sqrt{\frac{\gamma \rho}{\rho_m} \times \frac{\rho_d}{\gamma \rho}} = \frac{\rho_d}{\rho_m}$

However, the presence of water vapour reduces the density of air, i.e.,

$\rho_d < \rho_m$

$\therefore V_m > V_d$

Hence, the speed of sound in mois air is greater than it is in dry air. Thus, in gaseous medium, the speed of sound increases with humidity.

#420322

Topic: Wave Equation

You have learnt that a travelling wave in one dimension is represented by a function y = f (x,t) where x and t must appear in the combination x - v t or x + v t i.e. $y = f(x \pm vt)$. Is the converse true ? Examine if the following functions for y can possibly represent a travelling wave :

(a) $(x + vt)^2$

(b) $\log\left[(x + vt)/x_o\right]$

(c) 1/(x+vt)

Solution

No, the converse is not true. The basic requirements for a wave function to represent a travelling wave is that for all values of x and t, wave function must have finite value.

Out of the given functions for y, no one satisfies this condition. Therefore, none can represent a travelling wave.

#420332

Topic: Reflection, Transmission and Echo

A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340 ms^{-1} and in water 1486 m s^{-1}.

Solution

(a) Frequency of the ultrasonic sound, $f = 1000 kHz = 10^6 Hz$

Speed of sound in air, $V_a = 340 m/s$

The wavelength (λ_r) of the reflected sound is given by the relation:

$\lambda_r = V/f$

$= 340/10^6 = 3.4 \times 10^{-4} m.$

(b) Frequency of the ultrasonic sound, $f = 1000 kHz = 10^6 Hz$

Speed of sound in water, $V_w = 1486\ m/s$

The wavelength of the transmitted sound is given as:

$\lambda_r = 1486/10^6 = 1.49 \times 10^{-3} m.$

#420340

Topic: Introduction to Sound Waves

A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km s^{-1} ? The operating frequency of the scanner is 4.2 MHz.

Solution

The wavelength can be given by

$$\lambda = \frac{c}{v} = \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4.04 \times 10^{-4} m$$

#420369

Topic: Organ Pipes

A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?

Solution

Length of the narrow bore, L= 1 m = 100 cm

Length of the mercury thread, l= 76 cm

Length of the air column between mercury and the closed end, l_a= 15 cm

Since the bore is held vertically in air with the open end at the bottom, the mercury length that occupies the air space is: 100 – (76 + 15) = 9 cm

Hence, the total length of the air column = 15 + 9 = 24 cm

Let h cm of mercury flow out as a result of atmospheric pressure.

∴Length of the air column in the bore = 24 + h cm

And, length of the mercury column = 76 – h cm

Initial pressure, P_1 = 76 cm of mercury

Initial volume, V_1 = 15cm^3

Final pressure, P_2= 76 – (76 – h) = h cm of mercury

Final volume, V_2= (24 + h) cm^3

Temperature remains constant throughout the process.

$\therefore P_1V_1 = P_2V_2$

$76 \times 15 = h(24 + h)$

$h^2 + 24h - 1140 = 0$

$h = 23.8, \ -47.8\ cm$

Height cannot be negative. Hence, 23.8 cm of mercury will flow out from the bore and 52.2 cm of mercury will remain in it. The length of the air column will be 24 + 23.8 = 47.8 cm.

#420379

Topic: Wave Equation

A transverse harmonic wave on a string is described by

$y(x, t) = 3.0 sin(36t + 0.018x + \pi/4)$

where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave?

If it is travelling what are the speed and direction of its propagation?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

Solution

(a) The equation of progressive wave travelling from right to left is given by the displacement function:

$y(x, t) = a\,sin(\omega t + kx + \phi)$ ... (i)

The given equation is:

$y(x, t) = 3.0\,sin(36t + 0.018x + \frac{\pi}{4})$ ...(ii)

On comparing both the equations, we find that equation (ii) represents a travelling wave, propgating from right to left.

Now using equations (i) and (ii), we can write:

$\omega = 36$ rad/s and $k =$ 0.018 m^{-1}

We know that:

$v = \omega / 2\pi$ and $\lambda = 2\pi / k$

Also,

$v = f\lambda$

$\therefore\ v = (\omega / 2\pi) \times (2\pi / k) = \omega / k$

$= 36 / 0.018 = 2000\ cm / s = 20\ m / s$

Hence, the speed of the given travelling wave is 20 m/s.

(b) Amplitude of the given wave, $a = 3\ cm$

Frequency of the given wave:

$f = \omega / 2\pi = 36 / 2 \times 3.14 = 573 Hz$

(c) On comparing equations (i) and (ii), we find that the intial phase angle, $\phi = \pi / 4$

(d) The distance between two successive crests (or troughs) is equal to the wavelength of the wave.

Wavelength is given by the relation: $k = 2\pi / \lambda$

$\therefore\ \lambda = 2\pi / k = 2 \times 3.14 / 0.018 = 348.89\ cm = 3.49\ m$

#420385
Topic: Wave Equation

Exercise:

[A transverse harmonic wave on a string is described by

$y(x, t) = 3.0 sin(36t + 0.018x + \pi / 4)$

where x and y are in cm and t in s. The positive direction of x is from left to right.]

For the wave described in the above Exercise, plot the displacement (y) versus (t) graphs for x=0, 2 and 4 cm. What are the shapes of these of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

Solution

All the waves have different phases.

The given transverse harmonic wave is:

$y(x, t) = 3.0 sin\left(36t + 0.018x + \frac{\pi}{4}\right)$ ...(i)

For $x = 0$, the equation reduces to:

$y(0, t) = 3.0\ sin\left(36t + \frac{\pi}{4}\right)$

Also,

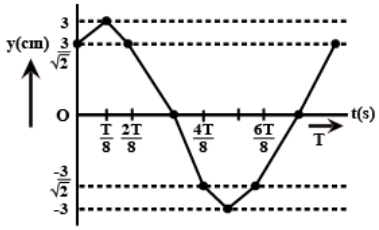
$\omega = 2\pi / t = 36\ rad / s^{-1}$

$\therefore\ t = \pi / 18\ s$

Now, plotting y vs. t graphs using the different values of t, as listed in the given table

t (s)	0 & T/8	2T/8 & 3T/8	4T/8 & 5T/8	6T/8 & 7T/8
y (cm)	$3\sqrt{2}$ & 3	$3\sqrt{2}$ & 0	$-3\sqrt{2}$ & -3	$-3\sqrt{2}$ & 0

For $x = 0$, $x = 2$, and $x = 4$, the phases of the three waves will get changed. This is because amplitude and frequency are invariant for any change in x. The y-t plots of the three waves are shown in the given figure.



#420399

Topic: Wave Equation

For the travelling harmonic wave

y(x,t)=2.0 cos 2π (10t-0.0080 x+0.35)

where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4 m,

(b) 0.5 m,

(c) $\lambda / 2$,

(d) $3\lambda / 4$,

Solution

Equation for a travelling harmonic wave is given as:

$y(x, t) = 2.0\ cos\, 2\pi(10t - 0.0080x + 0.35)$

$= 2.0\ cos(20\pi t - 0.016\pi x + 0.70\pi)$

Where,

Propagation constant, $k = 0.0160\pi$

Amplitude, $a = 2\ cm$

Angular frequency, $\omega = 20\pi\ rad/s$

Phase difference is given by the relation:

$\phi = kx = 2\pi/\lambda$

(a) For $\Delta x = 4m = 400cm$

$\Delta\phi = 0.016\pi \times 400 = 6.4\pi\ rad$

(b) For $\Delta x = 0.5m = 50cm$

$\Delta\phi = 0.016\pi \times 50 = 0.8\pi\ rad$

(c) For $\Delta x = \lambda/2$

$\Delta\phi = 2\pi/\lambda \times \lambda/2 = \pi\ rad$

(d) For $\Delta x = 3\lambda/4$

$\Delta\phi = 2\pi/\lambda \times 3\lambda/4 = 1.5\pi\ rad$.

#420413
Topic: Standing Waves

Exercise:

[The transverse displacement of a string (clamped at its both ends) is given by

$y(x, t) = 0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\ \pi t)$

where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.]

(i) For the wave on a string described in Exercise do all the points on the string oscillate with the same (a) frequency (b) phase (c) amplitude ? Explain your answers (ii) what is the amplitude of a point 0.375 m away from one end ?

Solution

(i) All the points on the string except the nodes have the same frequency and phase. However, amplitude at a point on the wave is a function of the position of the point.

(ii) $y(x, t) = 0.06\ sin\left(\frac{2\pi}{3}x\right)cos(120\ \pi t)$

The amplitude at $x = (0.375\ m)$ is

$A = 0.06\ sin\frac{2\pi}{3}x \times 1$

$= 0.06\ sin\frac{2\pi}{3} \times 0.375$

$= 0.06\ sin\frac{\pi}{4} = \frac{0.06}{\sqrt{2}} = 0.042m$

#420424
Topic: Standing Waves

Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave (ii) a stationary wave or (iii) none at all:

(a) $y = 2cos(3x)\sin(10t)$

(b) $y = 2\sqrt{x - vt}$

(c) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

(d) $y = cosx\ sint + cos2x\ sin2t$

Solution

(a) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation.

(b) The given equation does not contain any harmonic term. Therefore, it does not represent either a travelling wave or a stationary wave.

(c) The given equation represents a travelling wave as the harmonic terms kx and ωt are in the combination of $kx - \omega t$.

(d) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation. This equation actually represents the superposition of two stationary waves.

#420432
Topic: Modes of Vibration

A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is $3.5 \times 10^{-2}\ kg$ and its linear mass density is $4.0 \times 10^{-2}\ kgm^{-1}$. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string ?

Solution

(a) Mass of the wire, $m = 3.5 \times 10^{-2}kg$

Linear mass density, $\mu = m/l = 4.0 \times 10^{-2}kg\,m^{-1}$

Frequency of vibration, $f = 45Hz$

$\therefore$ Length of the wire, $l = \frac{m}{\mu} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = 0.875m$

The wavelength of the stationary wave (λ) is related to the length of the wire by the relation:

$\lambda = 2l/n$

where,

$n =$ Number of nodes in the wire

For fundamental node, $n = 1$:

$\lambda = 2l$

$\lambda = 2 \times 0.875 = 1.75m$

The speed of the transverse wave in the string is given as:

$v = f\lambda = 45 \times 1.75 = 78.75m/s$

(b) The tension produced in the string is given by the relation:

$T = v^2\mu$

$= (78.75)^2 \times 4.0 \times 10^{-2} = 248.06\ N$

#420440
Topic: Organ Pipes

A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effect may be neglected.

Solution

Frequency of the turning fork, $f = 340\ Hz$

Since the given pipe is attached with a piston at one end, it will behave as a pipe with one end closed and the other end open, as shown in the given figure. Such a system produces odd harmonics. The fundamental note in a closed pipe is given by the relation:

$l_1 = \lambda/4$

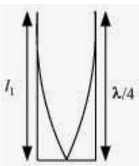
where,

length of pipe, $l_1 = 25.5 cm = 0.255 m$

$\lambda = 4l_1 = 4 \times 0.255 = 1.02\ m$

The speed of the sound is given by the relation:

$v = f\lambda = 340 \times 1.02 = 346.8\ m/s$



#420442

Topic: Organ Pipes

A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source ? Will the same source be in resonance with the pipe if both ends are open ? (speed of sound in air is 340 m s^{-1})

Solution

Length of the pipe, $l = 20\ cm = 0.2\ m$

Source frequency $= n^{th}$ normal mode of frequency, $f_n = 430 Hz$

Speed of sound, v = 340 m/s

In a closed pipe, the n^{th} normal mode of frequency is given by the relation:

$f = (2n-1)\frac{v}{4l} \qquad n \in \{1, 2, 3, \dots\}$

$430 = (2n-1)\frac{340}{4 \times 0.2}$

$2n - 1 = \frac{430 \times 4 \times 0.2}{340} = 1.01$

$2n = 2.01$

$n \approx 1$

Hence, the first mode of vibration frequency is resonantly excited by the given source.

In a pipe open at both ends, the nth mode of vibration frequency is given by the relation:

$f_n = nv/2l$

$n = 2lf_n/v \approx 0.5$

The same source will not be in resonance with the same pipe open at both ends.

#420445

Topic: Beats

Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz what is the frequency of B ?

Solution

Frequency of string A, $f_A = 324 Hz$

Frequency of string $B = f_B$

Beat's frequency, $n = 6 Hz$

Beat's Frequency is given as:

$n = \left| f_A - f_B \right|$

$6 = |324 - f_B|$

$f_B = 330 Hz$ or $318 Hz$

Frequency decreases with a decrease in the tension in a string. This is because frequency is directly proportional to the square root of tension. It is given as:

$f \propto \sqrt{T}$

Hence, the beat frequency cannot be 330 Hz

$\therefore f_B = 318 Hz$

#420447

Topic: Introduction to Sound Waves

Explain why (or how):

(a) in a sound wave, a displacement node is a pressure antinode and vice versa,

(b) bats can ascertain distances directions, nature and sizes of the obstacles without any "eyes"

(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,

(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and

(e) the shape of a pulse gets distorted during propagation in a dispersive medium.

Solution

(a) A node (N) is a point where the amplitude of vibration is the minimum and pressure is the maximum.

An antinode (A) is a point where the amplitude of vibration is the maximum and pressure is the minimum.

Therefore, a displacement node is nothing but a pressure antinode, and vice versa.

(b) Bats emit ultrasonic waves of large frequencies. When these waves are reflected from the obstacles in their path, they give them the idea about the distance, direction, size and nature of the obstacle.

(c) The overtones produced by a sitar and a violin, and the strengths of these overtones, are different. Hence, one can distinguish between the notes produced by a sitar and a violin even if they have the same frequency of vibration.

(d) This is because solids have both, the elasticity of volume and elasticity of shape, whereas gases have only the volume elasticity.

(e) A sound pulse is a combination of waves of different wavelength. As waves of different λ travel in a dispersive medium with different velocities, therefore, the shape of the pulse gets distorted.

#420449

Topic: Doppler Effect

A train standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10 $m_s{}^{-1}$, (b) recedes from the platform with a speed of 10 $m_s{}^{-1}$? (ii) what is the speed of sound in each case? The speed of sound in still air can be taken as 340 m $_s{}^{-1}$

Solution

(i) (a)Frequency of the whistle, $v = 400 Hz$

Speed of the train, $v_T = 10\ m/s$

Speed of sound, $v = 340\ m/s$

The apparent frequency (v') of the whistle as the train approaches the platform is given by the

relation:

$$v' = \left(\frac{v}{v - v_r}\right)v$$

$$= \left(\frac{340}{340 - 10}\right) \times 400 = 412.12 Hz$$

(b) The apparent frequency (v') of the whistle as the train recedes from the platform is given by the relation:

$$v'' = \left(\frac{v}{v + v_r}\right)v$$

$$= \left(\frac{340}{340 + 10}\right) \times 400 = 388.57\ Hz$$

(ii) The apparent change in the frequency of sound is caused by the relative motions of the source and the observer. These relative motions produce no effect on the speed of sound. Therefore, the speed of sound in air in both the cases remains the same, i.e., 340 m/s.

#420451
Topic: Doppler Effect

A train, standing in a station yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with a speed of 10$m_s{}^{-1}$. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform ? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10$m_s{}^{-1}$? The speed of sound in still air can be taken as 340 m $_s{}^{-1}$

Solution

For the stationary observer:

Frequency of the sound produced by the whistle, $v = 400Hz$

Speed of sound $= 340\ m/s$

Velocity of the wind, $v = 10\ m/s$

As there is no relative motion between the source and the observer, the frequency of the sound heard by the observer will be the same as that produced by the source, i.e., $400Hz$.

The wind is blowing toward the observer. Hence, the effective speed of the sound increases by 10 units, i.e.,

Effective speed of the sound, $v_e = 340 + 10 = 350\ m/s$

The wavelength (λ) of the sound heard by the observer is given by the relation:

$\lambda = v_e / v = 350/400 = 0.875\ m$

For the running observer:

Velocity of the observer, $v_o = 10\ m/s$

The observer is moving toward the source. As a result of the relative motions of the source and the observer, there is a change in frequency (v').

This is given by the relation:

$$v' = \left(\frac{v + v_o}{v}\right)v$$

$$= \left(\frac{340 + 10}{340}\right) \times 400 = 411.76Hz$$

Since the air is still, the effective speed of sound $= 340 + 0 = 340m/s$

The source is at rest. Hence, the wavelength of the sound will not change, i.e., λ remains 0.875 m.

Hence, the given two situations are not exactly identical.

#420459

Topic: Introduction to Waves

A travelling harmonic wave on a string is described by

$y(x, t) = 7.5\sin(0.0050x + 12t + \pi/4)$

(a) What are the displacement and velocity of oscillation of a point at $x = 1$ cm and $t = 1$ s? Is this velocity equal to the velocity of wave propagation ?

(b) Locate the points of the string which have the same transverse displacements and velocity as the $x = 1$ cm point at $t = 2s$, 5 s and 11s

Solution

(a) The given harmonic wave is

$$y(x, t) = 7.5\ sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

For $x = 1\ cm$ and $t = 1s$,

$$y(1, 1) = 7.5\ sin\left(0.0050x + 12 + \frac{\pi}{4}\right)$$

$$= 7.5\ sin\left(12.0050 + \frac{\pi}{4}\right)$$

$$= 7.5\ sin\theta$$

Where,

$$\theta = 12.0050 + \frac{\pi}{4} = 12.0050 + \frac{3.14}{4} = 12.79rad$$

$$= \frac{180}{3.14} \times 12.79 = 732.81^o$$

$\therefore\ y = (1, 1) = 7.5\ sin(732.81^o)$

$= 7.5\ sin(90 \times 8 + 12.81^o) = 7.5\ sin 12.81^o$

$= 7.5 \times 0.2217$

$= 1.6629 = 1.663\ cm$

The velocity of the oscillation at a given point and times is given as:

$v = \frac{d}{dt}y(x, t) = \frac{d}{dt}\left[7.5\, sin\left(0.0050x + 12t + \frac{\pi}{4}\right)\right]$

$= 7.5 \times 12\, cos\left(0.0050x + 12t + \frac{\pi}{4}\right)$

At $x = 1\,cm$ and $t = 1s$

$v = y(1, 1) = 90\, cos\left(12.005 + \frac{\pi}{4}\right)$

$= 90\, coss(732.81^o) = 90\, cos(90 \times 8 + 12.81^o)$

$= 90\, cos(12.81^o)$

$= 90 \times 0.975 = 87.75 cm/s$

Now, the equation of a propagating wave is given by:

$y(x, t) = a\, sin(kx + wt + \phi)$

where,

$k = 2\pi/\lambda$

$\therefore \lambda = 2\pi/k$

and, $\omega = 2\pi v$

$\therefore v = \omega/2\pi$

Speed, $v = v\lambda = \omega/k$

Where,

$\omega = 12\, rad/s$

$k = 0.0050\, m^{-1}$

$\therefore v = 12/0.0050 = 2400\, cm/s$

Hence, the velocity of the wave oscillation at $x = 1\,cm$ and $t = 1s$ is not equal to the velocity of the wave propagation.

(b) Propagation constant is related to wavelength as:

$k = 2\pi/\lambda$

$\therefore \lambda = 2\pi/k = 2 \times 3.14/0.0050$

$= 1256\, cm = 12.56\, m$

Therefore, all the points at distance $n\lambda(n = \pm 1 \pm 2, \ldots$ and so on), i.e. $\pm 12.56\, m, \pm 25.12\, m, \ldots$ and so on for $x = 1\,cm$, will have the same displacement as the $x = 1\,cm$ points at $t = 2s$, $5s$ and $11s$.

#420463

Topic: Introduction to Sound Waves

A narrow sound pulse (for example, a short pip by a whistle) is sent across a

medium. (a) Does the pulse have a definite (i) frequency, (ii)wavelength, (iii) speed of propagation ? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz?

Solution

(a) A short pip be a whistle has neither a definite wavelength nor a definite frequency. However, its speed of propagation is fixed, being equal to speed of sound in air.

(b) No, frequency of the note produced by a whistle is not 1/20 = 0.05 Hz. Rather 0.05 Hz is the frequency of repetition of the short pip of the whistle.

#421120

Topic: Introduction to Sound Waves

One end of a long string of linear mass density 8.0 $\times 10^{-3}$ kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At t = 0, the left end (fork end) of the string x = 0 has zero transverse displacement (y = 0) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string :

Solution

The equation of a travelling wave propagating along the positivey-direction is given by the displacement equation:

$y(x, t) = a\,sin(wt - kx)$ ...(i)

Linear mass density, $\mu = 8.0 \times 10 - 3\,kg\,m^{-1}$

Frequency of the tuning fork, $v = 256Hz$

Amplitude of the wave, $a = 5.0cm = 0.05m$ ...(ii)

Mass of the pan, m $= 90kg$

Tension in the string, $T = mg = 909 \times 9.8 = 882\,N$

The velocity of the transverse wave v, is given by the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \frac{882}{8.0 \times 10^{-3}} = 332\,m/s$$

Angular Frequency, $\omega = 2\pi f$

$= 2 \times 3.14 \times 256$

$= 1608.5 = 1.6 \times 10^3\,rad/s$ ...(iii)

Wavelength, $\lambda = \frac{v}{f} = \frac{332}{256}m$

$\therefore$ Propagation constant, $k = \frac{2\pi}{\lambda}$

$$= \frac{2 \times 3.14}{\frac{332}{256}} = 4.84m^{-1}$$ ...(iv)

Substituting the values from equations (ii), (iii), and (iv) in equation (i), we get the displacement equation:

$y(x, t) = 0.05\,sin(1.6 \times 10^3 t - 4.84x)m.$

#421122

Topic: Doppler Effect

A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine ? Take the speed of sound in water to be 1450 m s^{-1}

Solution

Operating frequency of the SONAR system, $f = 40kHz$

Speed of the enemy submarine, $v_e = 360km/h = 100m/s$

Speed of sound in water, $v = 1450m/s$

The source is at rest and the observer (enemy submarine) is moving toward it. Hence, the apparent frequency (f') received and reflected by the submarine is given by the relation:

$$f' = \frac{c + v_r}{c - v_s}f$$

$$= \frac{1450 + 360}{1450} \times 4 \times 10^4 = 50\,KHz$$

#421124

Topic: Doppler Effect

A bat is flitting about in a cave navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall ?

Solution

Ultrasonic beep frequency emitted by the bat, $f = 40kHz$

Velocity of the bat, $v_b = 0.03v$

where, $v =$ velocity of sound in air

The apparent frequency of the sound striking the wall is given as:

$$f' = \left(\frac{v}{v - v_b}\right)f$$

$$= \left(\frac{v}{v - 0.03v}\right) \times 40$$

$$= \frac{40}{0.97}kHz$$

This frequency is reflected by the stationary wall (f_s) toward the bat.

The frequency (f'') of the received sound is given by the relation:

$$f'' = \left(\frac{v + v_b}{v}\right)f'$$

$$= \left(\frac{v + 0.03v}{v}\right) \times \frac{40}{0.97}$$

$$= \frac{1.03 \times 40}{0.97} = 42.47kHz$$

#423789

Topic: Standing Waves

The transverse displacement of a string (clamped at its both ends) is given by

$$y(x, t) = 0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\ \pi t)$$

where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.

Answer following :

(a) Does function represent a travelling wave or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave ?

(c) Determine the tension in the string.

Solution

(a)

Travelling wave is given by $y(x,t) = A\sin(\omega t \pm kx + \phi)$

Standing wave is given by $y(x,t) = A\sin(kx)\cos(\omega t)$

Hence, this is an example of standing wave.

(b)

$y(x,t) = 0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$

$y(x,t) = 0.03\sin\left(\frac{2\pi x}{3} + 120\pi t\right) + 0.03\sin\left(\frac{2\pi x}{3} - 120\pi t\right)$

Wavelength, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi/3} = 3\ m$

Frequency, $f = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = 60\ Hz$

Speed, $v = f\lambda = 180\ m/s$

(c)

Speed of a wave in a string is given by:

$v = \sqrt{T/\mu}$

$\implies T = v^2 m/l$

$T = \frac{180^2 \times 3 \times 10^{-2}}{1.5} = 648\ N$

#463005

Topic: Introduction to Sound Waves

A pendulum oscillates 40 times in 4 seconds. Find its time period and frequency.

Solution

Time period $= \frac{t}{\text{Number of oscillations in time t}} = \frac{4}{40} = 0.1\ s$

Frequency $= \frac{1}{\text{Time \ period}} = 10\ Hz$

#463006

Topic: Introduction to Sound Waves

The sound from a mosquito is produced when it vibrates its wings at an average rate of 500 vibrations per second. What is the time period of the vibration?

Solution

Time period $= \frac{1}{Frequency} = \frac{1}{500} = 0.002s$

#464574

Topic: Introduction to Sound Waves

Describe with the help of a diagram, how compressions and rarefactions are produced in air near a source of sound :

Solution

When a vibrating body swings forward, it pushes and compresses the air in front of it creating a region of high pressure. This region is called a compression. This compression starts to move away from the vibrating object.

When the vibrating body swings backwards, it creates a region of low pressure called rarefaction. As the object swings / oscillates back and forth rapidly, a series of compressions and rarefactions is created in the air. These make the sound wave that propagates through air.