ANSWERS WITH HINTS

1. (c) $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$

$$[P - \lambda I] = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} P_{11} - \lambda & P_{12} \\ P_{21} & P_{22} - \lambda \end{bmatrix}$$

Now, the characteristic equation will be,

$|P - \lambda I| = 0$

i.e. $(P_{11} - \lambda)(P_{22} - \lambda) - P_{21} P_{12}$

$= 0$ ...(i)

$\because$ $\lambda = 0$ is one of its eigen values

i.e. $\lambda = 0$ satisfies equation (i)

So, $(P_{11} - 0)(P_{22} - 0) - P_{21} P_{12} = 0$

$\Rightarrow P_{11} P_{22} - P_{21} P_{12} = 0$

2. (b) $4x + 2y = 7$ ...(i)

$2x + y = 6$ ...(ii)

Comparing the equations with

$a_1 x + b_1 y = c_1$

and $a_2 x + b_2 y = c_2$

we have

$a_1 = 4, b_1 = 2, c_1 = 7$

$a_2 = 2, b_2 = 1, c_2 = 6$

$\frac{a_1}{a_2} = \frac{4}{2} = 2$

$\frac{b_1}{b_2} = \frac{2}{1} = 2$

$\frac{c_1}{c_2} = \frac{7}{6}$

$\because$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence the system of linear equations have no solution.

IInd Method :

Multiplying equation (ii) by 2

$4x + 2y = 12$ ...(iii)

Now L. H. S. of equation (i) and (iii) are same but R. H. S. are different.

Hence, the system has no solution.

3. (a) $\sin z = 10$

$\because$ The Range of sin z is from – 1 to + 1

i.e. $-1 \le \sin z \le 1$

Hence sin z = 10 has no real or complex solution.

4. (a) $f(x) = \exp(x) + \exp(-x)$

$f'(x) = e^x - e^{-x}$

For critical points

$f'(x) = 0$

$e^x - e^{-x} = 0$

$\Rightarrow$ $e^x = e^{-x}$

$\Rightarrow$ $e^{2x} = 1$

$\Rightarrow$ $e^{2x} = e^{2\pi n i}$

$(\because e^{2\pi n i} = \cos 2\pi n + i \sin 2\pi n)$

$\Rightarrow$ $2x = 2\pi n i$

$n = 0, \pm 1, \pm 2$ ......

$\Rightarrow$ $x = \pi n i$

$n = 0, \pm 1, \pm 2$ ......

Now, $f'(x) = e^x + e^{-x}$

$f''(x)_{x = \pi n i} = e^{\pi n i} + e^{-\pi n i}$

$= 2 \cos n\pi \left(\because \frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta \right)$

$= 2(-1)^n, n = 0, \pm 1, \pm 2$ ......

$(\because \cos n\pi = (-1)^n)$

$= 2$ or -2 $n = 0, \pm 1, \pm 2$ ......

$\Rightarrow$ $f''(x) = 2$, n even

$= -2$, n odd

$f''(x) = 2$ = positive for n even

Hence f (x) has minima at x

$= \pi n i$ for even n

or at $x = 2\pi m i$

where $n = 2m$

$m = 0, \pm 1, \pm 2$ ......

Now minimum value of f (x)

at $x = 2\pi m i$

will be $f(x) = e^{2\pi m i} + e^{-2\pi m i}$

$= 2 \cos 2\pi m$

$m = 0, \pm 1, \pm 2$ ......

$= 2 \cdot 1 = 2$ $(\because \cos 2\pi m = 1)$

5. (a) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots$

Expansion of sin x has all the terms having odd powers

$\sin(x^3) = x^3 - \frac{(x^3)^3}{3!} + \frac{(x^3)^5}{5!} - \frac{(x^3)^7}{7!} + \ldots\ldots$

or $\sin(x^3) = x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \ldots\ldots$

has all the odd powered terms.

6. (b) $\frac{dx(t)}{dt} + 3x(t) = 0$

$\frac{dx}{dt} = -3x$

$\int \frac{dx}{x} = -\int 3\, dt$

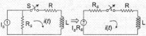
$\Rightarrow$ $\log x = -3t + \log c$

$\Rightarrow \qquad \frac{x}{c} = e^{-3t}$

$\Rightarrow \qquad x = ce^{-3t}$

Hence $x = 2e^{-3t}$ will be the solution of the above differential equation.

7. (c)
8. (b)



...(1)

By source Transformation

from (1)

As the switch is closed at $t = 0^+$

$\Rightarrow \qquad I(0^+) = 0 = i(0^-) \qquad ...(2)$

Apply KVL in *ckt* (1) after closing the switch

$\Rightarrow I_s R_s - i(t) R_s - R\,i(t) - \frac{L\,di(t)}{dt} = 0$

$\Rightarrow \qquad \frac{L\,di}{dt} = I_s R_s - (R_s + R)\,i(t)$

Using (2)

$\Rightarrow \qquad \frac{di(0^+)}{dt} = I_s \frac{R_s}{L}$

Hence alternative (b) is the correct choice.

9. (c) Causal system → output depends on only present and past values of I/P.

Given that input of a continuous time system is denoted by $x(t)$ while output of a continuous time system is denoted by $y(t)$.

- System → $y(t) = x(t-2) + x(t+4)$ is a non-causal system because for any value of t we get future value of output.
- System → $y(t) = (t-4)\,x(t+1)$ is also a non-causal system.
- System → $y(t) = (t+4)\,x(t-1)$ is a causal system because for any value of t we get past value of output.
- System → $y(t) = (t+5)\,x(t+5)$ is also a non-causal system.

Hence alternative (c) is the correct choice.

10. (d) $n(t) = \exp(\alpha t)\,u(t) + \exp(\beta t)\,u(-t)$

$= h_1(t) + h_2(t)$

We know that any system is stable only when

$$\boxed{h(t) < \infty}$$

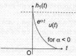
i.e. Impulse response is finite for given Input.

Here $\quad$ Let $h(t) = e^{\alpha t}\,u(t) + e^{\beta h}\,u(-t)$

$= h_1(t)\,t\,h_2(t)$

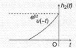
Here $h_1(t)$ will be finite only when $\alpha < 0$

i.e. α will be – ve.



and $h_2(t)$ will be finite only when $\beta > 0$

i.e. β will be + ve.



Alternative Method

The impulse response $h(t)$ of a linear time invariant continuous time system is given by

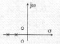
$$h(t) = e^{\alpha t}\,u(t) + e^{\beta t}\,u(-t),$$

where $u(t)$ denotes the unit step function and α and β are real constants.

This system is stable when α is negative and β is positive. Since first part of impulse response will be BIBO (bounded input bounded output) if and only if α is negative and the second part of impulse response will be BIBO if and only if B is positive.

Hence alternative (d) is the correct choice.

11. (d)



Since (i) for Ist order high pass filter all poles and zero cones on real axis and same is for low pass filter.

So it cannot be a high pass on low pass filter.

Also from its transfer function and from its T. function we can see that it is a notch filter.

12. (c) As for a IInd order underdamped system the characteristic equation → $s^2 + 2s\omega n\,\&\, + \omega_n^2 = 0$

and $\qquad s = -s\omega_n \pm j\,\omega_n\sqrt{1 - s^2} \qquad ...(1)$

Since given that all the 3 systems have same % overshoot

$\Rightarrow \qquad M = e^{-\pi s/\sqrt{1-s^2}}$

So s → same for all 3 systems.

from eq. (i) It is clear that at $\omega_n = 0$,

$s = -0 \pm j o$

which only satisfying the IIIrd option *i.e.* (c).

13. (d) In p-n junction, there is no any channel length type property or structures does not exist. It comes in MOSFET.

14. (a) As Boron is trivalent impurity and by doping with Boron we are increasing hole concentration.

So it will be a P^+ substrate on heavy doping with boron.

15. (c) For a Hertz dipole antenna, the half power beam width (HPBW) in the E-plane is 90°.

16. (d) We know that the Maxwell equations are given as

(i) $\nabla \times \vec{H} = \vec{D} \times \vec{J}$

(ii) $\nabla \times \vec{E} = -\vec{B}$

(iii) $\nabla \cdot \vec{E} = \dfrac{\delta}{E}$

(iv) $\nabla \cdot \vec{B} = 0$

For static and magnetic fields in an inhomogeneous source-free medium

(i) $\nabla \times \vec{H} = \vec{J}$

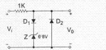
(ii) $\nabla \times \vec{E} = 0$ (Given in options)

(iii) $\nabla \cdot \vec{E} = \dfrac{\delta}{E}$

(iv) $\nabla \cdot \vec{B} = 0$ (Given in options)

Hence alternative (d) is the correct choice.

17. (c)



$V_z = 6{\cdot}8$ V given

(i) First when I/P voltage have + ve half cycle then D_2 is open circuit, D_1 is forward biased & the zener diode is in reverse biased.

So $V_0 = 0{\cdot}7 + 6{\cdot}8\text{ V} = 7{\cdot}5\text{ V}$

(ii) For – ve half cycle, the D_1 is open circuit. So the voltage across the D_2 will be the O/P voltage and here $V_0 = 0{\cdot}7V$.

So maximum and minimum O/P voltage are (7·5 V, 0·7V).

18. (c) A silicon wafer has 100 nm of oxide on it and is inserted in a furnace at a temperature above 1000°C for further oxidation in dry oxygen. The oxidation rate slows down as the oxide grows.

19. (b) As given, $I_D = K[V_{GS} - V_T]^2$

since, $g_m = \dfrac{\partial I_D}{\partial V_{GS}}$

or $g_m = 2K[V_{GS} - V_T]$

20. (a) Since for A.M. signal → Modulation index

$\mu = \dfrac{A_m}{A_c}$ and $\mu \le 1$ for demodulation.

Here AM signal in standard form.

$$= A_C\left[1 + \frac{2}{A_C}\cos\omega_m t\right]\cos\omega_{Ct}$$

where, $\mu = \dfrac{2}{A_C}$

A_m = Amplitude of modulating signal

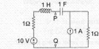
A_C = Amplitude of carrier signal.

For proper demodulation $\rightarrow \dfrac{2}{A_C} \le 1$ ...(A)

So from eq. (A)

Minimum value of $A_C = 2$

21. (a)

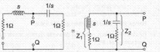


Finding Z_{th} between P & Q—

→ Shorting all independent voltage sources.

→ Open circuit all independent current sources.

→ Apply S-domain *i.e.* taking Laplace in the circuit.



Z_1 and Z_2 are in parallel.

and $Z_1 = s + 1$

and $Z_2 = (1 + 1/s) = \left(\dfrac{s+1}{s}\right)$

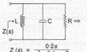
$\Rightarrow$ $Z_{th} = Z_1 /\!/ Z_2$

$$= \frac{(s+1)(s+1)}{s\left[(s+1) + \dfrac{(s+1)}{s}\right]}$$

$$= \frac{s^2 + 2s + 1}{s\left[\dfrac{s^2 + 2s + 1}{s}\right]} = 1$$

so, $Z_{th} = 1$

22. (d)



Given $Z(s) = \dfrac{0{\cdot}2s}{s^2 + 0{\cdot}1s + 2}$

$\Rightarrow$ $Y(s) = \dfrac{s^2 + 0{\cdot}1s + 2}{0{\cdot}2s}$

$Y(s) = 5s + 0{\cdot}5 + \dfrac{1}{0{\cdot}1s}$ ...(1)

From circuit we have

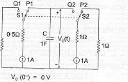
$$Y(s) = \frac{1}{R} + \frac{1}{Ls} + Cs \qquad \ldots(2)$$

Comparing (1) and (2)

$\Rightarrow$ $R = 2\ \Omega, L = 0{\cdot}1\ H$

and $C = 5\ F$

23. (a)



Given, $V_c(0^-) = 0\ V$

and $u(t)$ is a unit step function

For $2nT \le t < (2n+1)T, (n = 0, 1, 2, \ldots\ldots)$

S1 to P1 and S2 to P2

For $(2n+1) \le t < (2n+2)T, (n = 0, 1, 2, \ldots\ldots)$

S1 to Q1 and S2 to Q2

when $0 < t < T,$

$$V_c(t) = \frac{1}{C}\int_0^t i\, dt$$

or $V_c(t) = t\ (\because C = 1\ F \text{ and } i = 1A)$

when $T < t < 2T,$

$$V_c(t) = \int_0^T dt - \int_T^t dt$$

$$= T - (t - T) = (2T - t)$$

when $2T < t < 3T,$

$$V_c(t) = [V_c]_{2T} + \int_{2T}^t dt$$

or $V_c(t) = 0 + \int_{2T}^t dt = (t - 2T)$

Similarly—

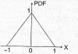
when $3T < t < 4T,$

$$V_c(t) = (4T - t)$$

Therefore the voltage $V_c(t)$ across the capacitor is given by $V_c(t) = \sum_{n=0}^{\infty} (-1)^n tu(t - nT)$

Hence alternative (a) is the correct choice.

24. (a) Given → PDF of a random variable X is



Let $p_X(x)$ is the PDF of random variable X.

then $\text{CDF} = F_X(x)$

$$= \int_{-\infty}^{x} p_X(x)\, dx \qquad \ldots(1)$$

since $\int_{-\infty}^{\infty} p_X(x)\, dx = 1 \qquad \ldots(2)$

and $F_X(x) \ge 0$

and $F_X(\infty) = 1$

and $F_X(-\infty) = 0$

Also $F_X(x)$ is a non decreasing function.

→ (D) can easily be eliminated as it is the derivative of given PDF.

→ (C) also can be eliminated as here CDF is becoming > 1.

From (A) and (D) we can easily find out that (A) will be the CDF of given PDF.

25. (c) $x = e^{-x}$

$\Rightarrow$ $f(x) = e^{-x} - x = 0$

Using Newton-Raphson Method

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$f'(x) = -e^{-x} - 1$$

$$x_{k+1} = x_k - \frac{e^{-x_k} - x_k}{-e^{-x_k} - 1} = x_k + \frac{e^{-x_k} - x_k}{e^{-x_k} + 1}$$

or $$x_{k+1} = \frac{x_k e^{-x_k} + x_k + e^{-x_k} - x_k}{e^{-x_k} + 1}$$

$$x_{k+1} = \frac{x_k e^{-x_k} + e^{-x_k}}{e^{-x_k} + 1}$$

or $$x_{k+1} = \frac{e^{-x_k}(x_{k+1})}{e^{-x_k} + 1}$$

and $$x_{n+1} = (1 + x_n)\frac{e^{-x_n}}{1 + e^{-x_n}}$$

26. (a) $$f(z) = \frac{1}{(z+2)^2 (z-2)^2}$$

Residue of $f(z)$ at $z = 2$

$\because z = 2$ is a pole of order 2.

So,

$$[\text{Res } f(z)]_{z=2} = \lim_{z \to 2} \frac{1}{(2-1)!} \frac{d}{dz}(z-2)^2$$

$$= \lim_{z \to 2} \frac{1}{1!} \frac{d}{dz}(z-2)^2 \frac{1}{(z+2)^2 (z-2)^2}$$

$$= \lim_{z \to 2} \frac{d}{dz}\left(\frac{1}{z+2}\right)^2 = \lim_{z \to 2} -\frac{2}{(z+2)^3}$$

$$= -\frac{2}{(4)^3} = -\frac{2}{64} = -\frac{1}{32}$$

Hence option (a) is correct.

Note : If $z = a$ is pole of order m of function $f(z)$

Then $$[\text{Res } f(z)]_{z=a} = \lim_{z \to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} (z-a)^m f(z)$$

27. (d) Given, $P = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

The value of e^P is given by relation

$$e^P = L^{-1}[(SI - P)^{-1}],$$

where I is the identity matrix

Now, $$[SI - P] = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

and $$[SI - P]^{-1} = \frac{1}{(S^2 + 3s + 2)}\begin{bmatrix} S+3 & +1 \\ -2 & S \end{bmatrix}$$

or $$[SI - P]^{-1} = \frac{1}{(S+1)(S+2)}\begin{bmatrix} S+3 & +1 \\ -2 & S \end{bmatrix}$$

$$= \begin{bmatrix} \frac{S+3}{(S+1)(S+2)} & \frac{+1}{(S+1)(S+2)} \\ \frac{-2}{(S+1)(S+2)} & \frac{S}{(S+1)(S+2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{(S+1)} - \frac{1}{(S+2)} & \frac{1}{(S+1)} - \frac{1}{(S+2)} \\ \frac{-2}{(S+1)} + \frac{2}{(S+2)} & \frac{2}{(S+2)} - \frac{1}{(S+1)} \end{bmatrix}$$

$$e^P = L^{-1}[SI - P]^{-1}$$

or $$e^P = L^{-1}\begin{bmatrix} \frac{2}{(S+1)} - \frac{1}{(S+2)} & \frac{1}{(S+1)} - \frac{1}{(S+2)} \\ \frac{-2}{(S+1)} + \frac{2}{(S+2)} & \frac{2}{(S+2)} - \frac{1}{(S+1)} \end{bmatrix}$$

or $$e^P = \begin{bmatrix} 2e^{-1} - e^{-2} & e^{-1} - e^{-2} \\ -2e^{-1} + 2e^{-2} & 2e^{-2} - e^{-1} \end{bmatrix}$$

Hence alternative (d) is the correct choice.

28. (b) Let $f(x) = \exp(x) + \sin(x)$

As Taylor series relates the value of the function and its higher order derivates.

As Taylor series about a point $x = a$ given by

$$f(x) = f(a) + \frac{(x-a)f'(a)}{\underline{|1}} + \frac{(x-a)^2 f''(a)}{\underline{|2}} + \frac{(x-a)^3 f'''(a)}{\underline{|3}} + \ldots\ldots$$

We have to find out the coefficient of $(x - \pi)^2$ in Taylor series about the point $x = \pi$

$$\Rightarrow \quad f'(x) = e^x + \cos x$$

$$\Rightarrow \quad f''(x) = e^x - \sin x$$

so coefficient of $(x - a)^2$

$$= \frac{(e^\pi - \sin \pi)}{\underline{|2}}$$

$$= \frac{e^\pi}{2} - 0 = 0{\cdot}5 \exp(\pi)$$

29. (a) Given PDF of random variable X is

$$P_X(x) = M e^{-2|x|} + N e^{-3|x|}$$

since, we know that

$$\int_{-\infty}^{\infty} P_X(x) = 1 \qquad \ldots(1)$$

so using eq. (1)

$$\Rightarrow \int_{-\infty}^{\infty} [Me^{-2|x|} + Ne^{-3|x|}]\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} M e^{-2|x|}\,dx + \int Ne^{-3|x|}\,dx = 1$$

Break the limits

As $-|x| = +x$ for $-\infty \le x \le 0$

$= -x$ for $0 \le x \le \infty$

$$\Rightarrow \int_{-\infty}^{0} M e^{+2x}\,dx + \int_0^{\infty} M e^{-2x}\,dx + \int_{-\infty}^{0} Ne^{+3x}\,dx + \int_0^{\infty} Ne^{-3x}\,dx = 1$$

(the four integrals being I_1, I_2, I_3, I_4)

$$\Rightarrow \quad I_1 = \int_{-\infty}^{0} M e^{2x}\,dx = \frac{M}{2},$$

$$I_2 = \frac{M}{2},$$

$$I_3 = \frac{N}{3},$$

$$I_4 = \frac{N}{3}$$

So $$\frac{2M}{2} + \frac{2}{3}N = 1$$

$$\Rightarrow \quad M + \frac{2}{3}N = 1$$

30. (a) $g(x, y) = 4x^3 + 10y^4$

Equation of the straight line joining (0, 0) to (1, 2)

$$y - 0 = \frac{2-0}{1-0}(x - 0)$$

$$\left(\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right)$$

$$\Rightarrow \quad y = 2(x - 0)$$

$$\Rightarrow \quad g(x, y) = 4x^3 + 10[2(x-0)]^4$$

$$g(x) = 4x^3 + 160x^4$$

$\because$ x varies from 0 to 1

$$\int_0^1 g(x)\,dx = \int_0^1 (4x^3 + 160x^4)\,dx$$

$$= \left[\frac{4x^4}{4} + \frac{160x^5}{5}\right]_0^1$$

$= [x^4 + 32x^5]_0^1$

$= 1 + 32 = 33$

31. (d) Poles $\rightarrow s = -2$, $s = -4$ and Zero $\rightarrow s = -1$

so the transfer function

$$\text{T. F.} = \frac{(s+1)}{(s+2)(s+4)}$$

$$\Rightarrow \quad H(s) = \frac{(s+1)}{(s+2)(s+4)} \quad \ldots(1)$$

Now Impulse Response

$$Y(s) = H(s) \times \text{L·T}[S(t)]$$

or $\quad Y(s) = H(s)$

Now taking inverse Laplace of (1)

$\Rightarrow \text{L·T}^{-1}[H(s)]$

$$\Rightarrow \quad \frac{s+1}{(s+2)(s+4)} = \frac{A}{(s+2)} + \frac{B}{(s+4)}$$

$$\Rightarrow \quad A = \left.\frac{s+1}{(s+4)}\right|_{S=-2} = \frac{-1}{2}$$

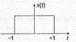
and $\quad B = \left.\frac{(s+1)}{(s+2)}\right|_{S=-4} = \frac{3}{2}$

$$\Rightarrow \quad H(s) = \frac{-1}{2}\left[\frac{1}{(s+2)}\right] + \frac{3}{2}\left[\frac{1}{(s+4)}\right]$$

so $\quad h(t) = [-0{\cdot}5\, e^{-2t} + 1{\cdot}5\, e^{-4t}]$

32. (a) $x(t) = \begin{cases} 1 & \text{for } -1 \le t \le +1 \\ 0 & \text{otherwise} \end{cases}$

$\Rightarrow$



Here $x(t)$ is a Gate function.

where width of this Gate function = 2

since $\quad$ F·T of $x(t) = s \sin c\left(\frac{\omega s}{2}\right)$

$$= \frac{s \sin\left(\frac{\omega s}{2}\right)}{\left(\frac{\omega s}{2}\right)}$$

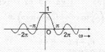
$$\Rightarrow \quad \sin C\left(\frac{\omega s}{2}\right) = 0 \text{ when } \frac{\omega s}{2} = \pm n\pi$$

Here $s = 2 \Rightarrow \omega = \pm n\pi$

except at $n = 0$, where it is indeterminate.

so, $\sin C\left(\frac{\omega s}{2}\right) = 0$ for $\omega = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\ldots$

and $\quad \sin C(0) = 1$ by L'Hopital Rule



From options it is clear that it is zero here only at π, 2π.

33. (d) Given →

$h[0] = 1$, $h[1] = -1$, $h[2] = 2$, $h[3] = 0$ ……

and $x[0] = 1$, $x[1] = 0$, $x[2] = 1$, $x[3] = 0$ ……

I/P → Sequence

Convolution

so $\quad y[n] = \text{O/P} = h[n] * x[n]$

$x[n]$ \ $y[n]$	1	−1	2	0			
1	1	−1	2	0			
0		0	0	0	0		
1			1	−1	2	0	
0				0	0	0	0
$y[n]$	1,	−1,	3,	−1	2,	0	0

So number of non zero terms in $y[n] = 5$

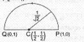
and $\quad y[2] = 3$

34. (b) P (1, 0) and Q (0, 1)

$$\text{Length PQ} = \sqrt{(1-0)^2 + (0-1)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

(∵ Distance between P (x_1, y_1) and Q (x_2, y_2) is given by $\sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$



and centre is given by $\left(\frac{1+0}{2}, \frac{0+1}{2}\right)$ i.e. $\left(\frac{1}{2}, \frac{1}{2}\right)$

and $\quad$ radius is $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Along the semicircular path θ varies from 0 to π

Let $x = \frac{1}{\sqrt{2}}\cos\theta \therefore dx = -\frac{1}{\sqrt{2}}\sin\theta\, d\theta$

$y = \frac{1}{\sqrt{2}}\sin\theta \therefore dy = \frac{1}{\sqrt{2}}\cos\theta\, d\theta$

$\Rightarrow 2\int_{p}^{Q} (x\,dx + y\,dy) =$

$$2\int_0^{\pi}\left[\frac{1}{\sqrt{2}}\cos\theta\cdot\left(-\frac{1}{\sqrt{2}}\sin\theta\right)+\frac{1}{\sqrt{2}}\sin\theta\left(\frac{1}{\sqrt{2}\cos\theta}\right)\right]d\theta$$

$$= 2\int_{\theta}^{\pi}\left(-\frac{1}{2}\sin\theta\cos\theta+\frac{1}{2}\sin\theta\cos\theta\right)d\theta$$

$= 0$

35. (a) $R_1 \to y(t) = t^2\, x(t)$

- Check for linearity

$y_1(t) = t^2 x_1(t)$ and $y_2(t) = t^2 x_2(t)$

where $x(t) = [x_1(t) + x_2(t)]\, t^2$

$\Rightarrow$ $y(t) = t^2 x_1(t) + t^2 x_2(t)$

So it is linear.

- Check for time invariance

Put $t = t - t_0$

$y_1(t - t_0) = (t - t_0)^2 \cdot x(t - t_0)$ ...(1)

Delaying I/P then see the change.

$\Rightarrow$ $y_2(t - t_0) = t^2\, x(t - t_0)$ ...(2)

Since $y_1(t) \neq y_2(t)$

So it is time variant. *i.e.* (P_1, R_1)

$R\,4 : y(t) = x(t - 5)$

We can see directly that it is linear and time invariant. *i.e.* (P_3, R_4)

Hence alternative (a) is the correct choice

36. (a) For *n* symbols probability

$$P = \frac{1}{\eta}$$

$\therefore$ $$\text{Entropy} = \sum_{k=0}^{n} P_k \log_2 \frac{1}{P_k}$$

$$= \log_2\left(\frac{1}{P}\right) = \log_2(n)$$

Thus, entropy of the source as a function of *n* increases as log *n*.

Hence alternative (a) is the correct choice.

37. (a) Given, $y(n) = \frac{1}{N}\sum_{r=0}^{n-1} x(r)\, x(n + r)$ ...(A)

The above equation (A) is auto-correlation function.

Hence $y(n) = r_{XX}(n)$

i.e. $r_{XX}(n) \xrightarrow[\text{N-point}]{\text{DFT}} |X(k)|^2$.

38. (d) $$P = \frac{25}{S^2 + 25}$$

$\to$ It is purely sinusoidal, undamped system.

i.e. $P \to 3$

$$Q = \frac{36}{s^2 + 20s + 36}$$

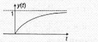
Characteristic eqn. is $s^2 + 20s + 36 = 0$

$\Rightarrow$ $\omega_n^2 = 36 \Rightarrow \omega_n = 6$

So $2\xi\omega_n = 20$

$\Rightarrow$ $\xi = \frac{20}{12} = \frac{5}{3} = 1{\cdot}6$

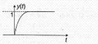
It is a slugish system *i.e.* $Q \to 4$



$$R = \frac{36}{s^2 + 12s + 36}$$

gives $\xi = 1 \to$ critically damped

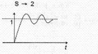
i.e. $R \to 1$



Similarly $$S = \frac{49}{s^2 + 7s + 49}$$

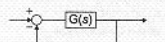
gives $\xi = 0{\cdot}5 \to$ under damped system

i.e. $S \to 2$



Hence alternative (d) is the correct choice

39. (c) $$G(s) = \frac{s + 8}{s^2 + \alpha s - 4}$$



Here $H(s) = 1$

so characteristic eqn. : $1 + G(s)\,H(s) = 0$

$\Rightarrow$ $1 + \frac{s + 8}{s^2 + \alpha s - 4} = 0$

$\Rightarrow$ $s^2 + \alpha s - 4 + s + 8 = 0$

$\Rightarrow$ $s^2 + s[\alpha + 1] + 4 = 0$

Using Routh Hurwitz Criteria

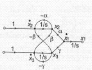
$\Rightarrow$

s^2	1	4
s^1	$(\alpha + 1)$	0
s^0	4	0

the system will be stable if $(\alpha + 1) > 0$

or $\alpha > -1$

40. (d)



From given figure

$$\dot{x}_1 = \alpha x_2 + \gamma x_3 \quad ...(i)$$

$$\dot{x}_2 = \mu_1 - \alpha x_2 - \beta x_3 \quad ...(ii)$$

$$\dot{x}_3 = u_2 - \gamma x_3 + \beta x_2 \quad ...(iii)$$

from equation (i), (ii) and (iii)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & \alpha & \gamma \\ 0 & -\alpha & -\beta \\ 0 & \beta & -\gamma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Depending upon position of x_1, x_2 and x_3 and u_1 & u_2, the rows and columns may interchange but the corresponding entries will remain the same. Hence alternative (d) is the correct choice.

41. (c) $1 + G(x) = 0$

or $$1 + \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3} = 0$$

or $$s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 13 = 0$$

Constructing Routh-array, we have

s^5	1	3	5
s^4	2	6	3
s^3	$0 \to \in$	$\frac{7}{2}$	0
s^2	$\frac{6\in - 7}{\in}$	3	0
s^1	$\frac{7}{2}$	0	
s^0	3		

As $\in \to 0$

$$\lim_{\in \to 0} \left(\frac{6\in - 7}{t}\right) = -ve$$

Since two sign changes, so two positive poles in RHS plane.

42. (a) Given that at $\omega = 0$,

frequency response = 5

i.e. K = 5

Peak (max.) Resonance

$$= \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{10}{\sqrt{3}}$$

$$\omega_r = \omega_n\sqrt{1 - 2\xi^2}$$

$$= 5\sqrt{2} \text{ red/sec}$$

Let the transfer function second order system is

$$\text{T.F.} = \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Now, $$\frac{10}{\sqrt{3}} = \frac{5}{2\xi\sqrt{1-\xi^2}}$$

$$\frac{100}{3} = \frac{25}{4\xi^2(1-\xi^2)}$$

or $$16\xi^2 - 16\xi^4 = 3$$

or $$16\xi^4 - 16\xi^2 + 3 = 0$$

or $$16p^2 - 16p + 3 = 0$$

$$p = \frac{16 \pm \sqrt{256 - 4 \times 16 \times 3}}{2 \times 16}$$

or $$p = \frac{16 \pm 8}{32}$$

or $$p = \frac{1}{4} \text{ and } \frac{3}{4}$$

or $$\xi = \frac{1}{2}$$

and $$\xi = \frac{\sqrt{3}}{2} \text{ (Not possible)}$$

Now, $$\omega_r = \omega_n\sqrt{1 - 2\xi^2} = 5\sqrt{2}$$

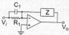
gives $$\omega_n = \frac{5\sqrt{2}}{\sqrt{1 - 2\left(\frac{1}{2}\right)^2}} = 10$$

Therefore, $$\text{T.F.} = \frac{5 \times \omega_n^2}{s^2 + 2\xi\,\omega_n s + \omega_n^2}$$

$$= \frac{500}{s^2 + 20s + 100}$$

Hence alternative (a) is the correct choice.

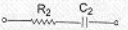
43. (b)



We know that $$\frac{V_o}{V_i} = \frac{-Z}{Z_1}$$

where, $$Z = R_1 // \frac{1}{C_1 s} = \frac{R_1}{R_1C_1s + 1}$$

Q : Here, $$Z = R_2 + \frac{1}{sC_2} = \frac{1 + sR_2C_2}{sC_2}$$



Now, $$\frac{Z}{Z_1} = \frac{1 + sR_2C_2}{sC_2} \times \frac{1}{\frac{R_1}{R_1C_1s + 1}}$$

or $$\frac{Z}{Z_1} = \frac{(1 + sR_1C_1)(1 + sR_2C_2)}{sR_1C_2}$$

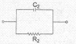
It is given that circuit element is Z satisfy the condition

$$R_2C_2 > R_1C_1$$

Therefore, $\frac{Z}{Z_1} = \frac{(1+sR_1C_1)(1+sR_2C_2)}{sR_1C_2}$

$= $ PID Controller

R : Here $Z = R_2 // \frac{1}{C_2 s} = \frac{R_2}{R_2C_2s+1}$



Now, $\frac{Z}{Z_1} = \frac{R_1(R_2C_2s+1)}{(R_1C_1s+1)R_2}$

$= \frac{(1+sR_2C_2)}{(1+sR_1C_1)} \times \frac{R_2}{R_1}$

$\therefore$ $R_2C_2 > R_1C_1 \rightarrow$ Log compensator

Hence alternative (b) is the correct choice.

44. (b) From the current mirror analogy, we can easily see that here

$$I_X = I_{bias}$$

45. (c) Since, $I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS}-V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2\right]$

(In Linear Region.) ...(1)

Also here it is assumed that $V_{DS} \le (V_{GS} - V_{TH})$ ...(2)

For $V_{DS} = $ Constant

Now $g_m = \frac{\partial I_D}{\partial V_{GS}}$

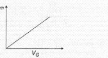
$= \mu_n C_{ox} \frac{W}{L} V_{DS}$

from (2)

$$g_m = \left(\mu_n C_{ox}\frac{W}{L}\right)(V_{GS} - V_{TH})$$

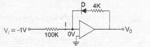
It means $g_m \propto V_{GS}$

$\Rightarrow$



46. (b) Given, $I = I_0\left(e^{\frac{V}{V_T}} - 1\right)$

where, $V_T = 25$ mV, $I_0 = 1$ μA



From figure

$$I = \frac{0-(V_i)}{100\ K} = \frac{1}{100\ K} = 10\ \mu A$$

Now from relation

$$I = I_0\left(e^{\frac{V}{V_T}} - 1\right)$$

we will calculate the voltage drop across diode. Let V

$$10\ \mu A = 1\ \mu A\left(e^{\frac{V}{25mV}} - 1\right)$$

or $V = 60$ mV $= V_0$ (say)

Voltage drop across 4K resistance

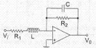
$V_R = I_0 \times R = 10 \times \mu A \times 4K = 40$ mV

Now, $V_0 = V_D + V_R = 60$ mV $+ 40$ mV

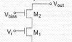
or $V_0 = 100$ mV $= 0{\cdot}1$ V

Hence alternative (b) is the correct choice.

47. (b) The OPAMP circuit shown below represents a low pass filter :



48. (c) Let the transconductance of NMOS transistor M_1 is g_{m1} and M_2 is g_{m2}.



Since both the transistor are connected in series, so equivalent transconductance is given by expression

$$\frac{1}{g_m} = \frac{1}{g_{m1}} + \frac{1}{g_{m2}}$$

or $g_m = \frac{g_{m1} \times g_{m2}}{g_{m1} + g_{m2}}$

As we know that, $g_m = \frac{\partial I_{out}}{\partial V_i}$

and $g_{m2} = \frac{\partial I_{out}}{\partial V_{bias}}$ ...(A)

$g_{m1} = \frac{\partial I_{out}}{\partial V_i}$ ...(B)

From relation (A) and (B) we conclude that $g_{m2} >> g_{m1}$

Because M_2 is always saturated due to bias voltage (*i.e.* $\frac{\partial I_{out}}{\partial V_{bias}}$ is very high as $\partial V_{bias} \approx 0$, which gives $g_{m2} \rightarrow \infty$) and g_{m1} changes according to V_i.

Therefore, $g_m = \frac{g_{m1}\, g_{m2}}{g_{m2}\left(1 + \frac{g_{m1}}{g_{m2}}\right)} \approx g_{m1}$

Thus, the equivalent g_m of pair is nearly equal to the g_m of transistor M_1.

49. (c) → LXI H, 30A0 H [Load register pair immediate]

H	L
30	AO

⇒ data stored in HL Pair after LXI operation.

→ DAD H [Add 16 bit data of the specified register to the contents of the HL register saved in the HL register.]

⇒ DAD operation →

$$\begin{array}{r} 3\,0\,A\,0 \\ +\ 3\,0\,A\,0 \\ \hline 6\,1\,4\,0 \end{array}$$

so

H	L
61	40

→ PCHL [Contents of the registers H & L are copied into the program counter].

Hence,

$$PC = 6140\text{ H}$$
$$HL = 6140\text{ H}$$

50. (b) As it is an astable multivibrator so here the voltage across the capacitor will vary between $\frac{V_{CC}}{3}$ to $2\,\frac{V_{CC}}{3}$.

$\to \frac{9}{3}$ to $2\left[\frac{9}{3}\right]$ *i.e.* from 3V to 6V

51. (c) Given → $N_A = 4 \times 10^{17}$ atoms/cm³,

[Acceptor ion concentration]

$n_i = 1{\cdot}5 \times 10^{10}$ cm⁻³

[Intrinsic carrier concentration.]

$$\frac{kT}{q} = 25\text{ mV}$$
$$kT = {\cdot}025\text{ eV}$$

Hole concentration,

$$p = 4 \times 10^{17}\text{ atoms/cm}^3$$

or $p = n_i\, e^{(E_i - E_F)/kT}$

or $\frac{p}{n_i} = e^{(E_i - E_F).kT}$

Take log on both side.

$$(E_i - E_F) = kT \ln \frac{p}{n_i}$$
$$(E_i - E_F) = 0{\cdot}025 \ln \frac{40}{15} \times 10^{17}$$
$$= 0{\cdot}025 \times 17{\cdot}09 = 0{\cdot}427\text{ eV}$$

52. (d) We know that

$$g_m = g_{m0}\left(1 - \frac{V_{GS}}{V_P}\right)$$

Given, $V_{GS} = -2$V and V_p is a particular value of V_{GS}

Also $V_p = \frac{q\,N_0\,W^2}{2\epsilon_0}$

where W = Width of the channel

i.e. $W \propto \sqrt{V_p}$

when width, W is doubled then

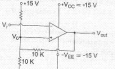
$$V_P = \sqrt{2}\,V_p$$

Let the initial transconductance be g_{m_i} and the modified transconductance be g_{m_m}. Then

$$\frac{g_{m_i}}{g_{m_m}} = \frac{1 - \left(\frac{2}{\sqrt{V_p}}\right)}{1 - \left(\frac{1}{2\sqrt{V_p}}\right)}$$

53. (c) Let the voltage at non-inverting termial be V_o

Given that the output of OP-AMP swings from + 15 V to − 15 V.



Case 1 : When V_{out} is + 15 V

$$\left(\frac{V_0 - V_{out}}{10}\right)2 + \frac{V_0 + 15}{10} = 0$$

or $$\left(\frac{V_0 - 15}{10}\right)2 + \frac{V_0 + 15}{10} = 0$$

or $V_0 = +5$ V

Case 2 : When V_{out} is − 15 V

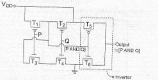
$$\frac{V_0 - V_{out}}{10} + \left(\frac{V_0 + 15}{10}\right)2 = 0$$

or $$\frac{V_0 - 15}{10} + \left(\frac{V_0 + 15}{10}\right)2 = 0$$

or $V_0 = -5$V

Thus the voltage at the non-inverting input swings between − 5 V to + 5 V.

54. (d)



T_1, T_2 → PMOS, T_3, T_4 → NMOS : Making NAND Gate

T_5 → PMOS, T_6 → NMOS : Making Inverter.

55. (a)

56. (a) $\rightarrow P = 11101101$

↑ Sign bit

● As sign bit is 1 so it is a – ve number in 2's complement form.

● To know its magnitude take again 2's complement of P.

i.e. $00010011 \leftarrow 2^{s}$ complement of P

so $P = -19$

$\rightarrow Q = 11100110$

● It is also a – ve number.

● Again taking its 2's complement to know its magnitude.

i.e. $00011010 \leftarrow$ *2's* complement of Q.

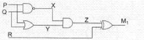
$Q = -26$

so $P - Q = -19 - (-26) = +7$

+7 → in 2's complement form [8 bit format]

$= 00000111$

57. (d)



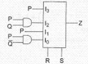
As $X = (P \cdot Q)' = (\overline{P} + \overline{Q})$

$Y = (P + Q)$

Now, $Z = (\overline{P} + \overline{Q})(P + Q) = P \text{ XOR } Q$

$M = R \text{ XOR } Z = (P \text{ XOR } Q) \text{ XOR } R$

58. (a)



From given circuit, we have

R S	Output
0 0	$I_0 = (P + \overline{Q})$
0 1	$I_1 = P$
1 0	$I_2 = (P \cdot Q)$
1 1	$I_3 = P$

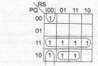
$\Rightarrow Z = \overline{R}\,\overline{S}\,[P + \overline{Q}] + \overline{R}SP + R\overline{S}\,(P \cdot Q) + RSP$

$= \overline{R}\,\overline{S}\,P + \overline{R}\,\overline{S}\,\overline{Q} + \underbrace{\overline{R}SP} + R\overline{S}PQ + \underbrace{RSP}$

$= \overline{R}\,\overline{S}P + \overline{R}\,\overline{S}\,\overline{Q} + SP[\overline{R} + R] + R\,\overline{S}QP$

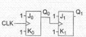
$= \overline{R}\,\overline{S}P + \overline{R}\,\overline{S}\,\overline{Q} + SP + R\,\overline{S}QP$

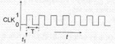
Using K-Map :



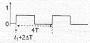
$Z = PQ + \overline{R}\,\overline{S}\,\overline{Q} + P\overline{Q}S$

59. (b)

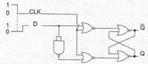




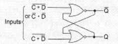
In the given figure J = K = 1 *i.e.* Flip-Flops are in toggle condition. We know that under toggle condition frequency of the output of the Flip flop is divided by factor 2. Since here two flip-flops are connected. So output frequency is, $f_0 = \frac{f_i}{4}$ or in other words time period $T_0 = 4\ T_i$. Also since ΔT is the propagation delay of 1 flip-flop, so output will be shifted by $t_1 + 2\Delta T$ time right side. Hence alternative (b) is the correct choice.



60. (c) The given circuit



The redrawn circuit is shown below :



When C goes from 1 → 0 then $\overline{C}$ becomes 1. From the inputs of the above circuit it is clear that output depends on D and $\overline{D}$

when $D = 0$, we get $Q = 1$

and when $D \to 1$, we get $Q \to 0$

Hence alternative (c) is the correct choice *i.e.*, Q goes to 1 at the clock transition and goes to 0 when D goes to 1.

61. (a) Broader dimension (a) = 4 cm

Smaller dimension (b) = 3 cm

$$TE_{11} \to TE_{mn}$$

$\Rightarrow$ $m = 1$

= no. of half wave variation along broader dimension

n = 1 = no of half wave variations along smaller dimension

Minimum operating frequency = Cut off frequency :

$$f_{min} = \frac{1}{2\sqrt{\mu\epsilon}} \times \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

or $$f_{min} = \frac{C}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

or $$f_{min} = \frac{3 \times 10^{10}}{2} \times \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{3}\right)^2}$$

where, $C = 3 \times 10^{10}$ cm/sec [velocity of light in space]

or $$f_{min} = \frac{3 \times 10^{10}}{2} \times \frac{5}{3 \times 4} = \frac{50}{8} \times 10^9 = 6{\cdot}25 \text{ GHz}$$

62. (d) Input impedence of a lossless transmission line is given by $Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan Bl}{Z_0 + jZ_L \tan Bl}\right]$

As given → line short circuited

$\Rightarrow$ $Z_L = 0$

$\Rightarrow$ $$Z_{in} = Z_0\left[\frac{0 + jZ_0 \tan Bl}{Z_0 + j \times 0}\right]$$

$$= Z_0 \frac{jZ_0 \tan Bl}{Z_0}$$

$$= jZ_0 \tan B = \text{Inductive}$$

63. (c)

Medium (1)	free space $\epsilon_0 = 1$
Medium (2)	thick dielectric slab $\epsilon_r = 9$

Since Reflection Coefficient

$\Rightarrow$ $$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

since $$\eta = \frac{1}{\sqrt{\epsilon}}$$

$\Rightarrow$ $$\eta_2 = \sqrt{\frac{1}{\epsilon_r}} = \frac{1}{3} \text{ and } \eta_1 = 1$$

so $$\rho = -\frac{1}{2}$$

$\Rightarrow$ $|\rho| = 0{\cdot}5$ → Magnitude

64. (d) In single mode optical fiber, the frequency of limiting mode increases as radius decreases. The radius and frequency are related by expression

$$r \propto \frac{1}{f}$$

where, r = radius and f = frequency

So, if the radius is doubled, the frequency of propagating mode gets halved *i.e.* wavelength is doubled. Hence alternative (d) is the correct choice.

65. (d) The gain of parabolic dish antenna is given by expression

Given, D = 1m = 100 cm, η = 70% = 0·7, f = 20 GHz

Now, $$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{20 \times 10^9} = 1{\cdot}5 \text{ cm.}$$

66. (c)

67. (a) Here given → Each bit is repeated 3 times to transmit

i.e. $n = 3$

As in Binary symmetric channel, an error will occur, if (M + 1) bits out of n = 2M + 1 bits will be received incorrectly

$\Rightarrow$ $$M = \frac{n-1}{2} = 1$$

Now, probability of error

$$p_e = \sum_{i=M+1}^{n} {}^nC_i\, p^i\, [1-p]^{n-i}$$

$${}^nC_i = \frac{\lfloor n}{\lfloor i \lfloor n-i}, \; i = 1 + 1 = 2, n = 3$$

$\Rightarrow$ $p_e = {}^3C_2 P^2 [1-p]^{3-2} + {}^3C_3\, p^3 [1-p]^{3-3}$

or $p_e = 3p^2 [1-p] + 1 \cdot p^3 \cdot 1$

or $p_e = 3p^2 [1-p] + p^3$

Hence alternative (a) is the correct choice.

68. (c) Mini Bandwidth required for Transmission of this TDM

= 2 [Signal having maximum frequency component]

= 2 × 3 W = 6 W

69. (d) Given frequency modulated signal

$$S(t) = 10 \cos[2\pi \times 10^5 t + 5 \sin(2\pi \times 1500\, t) + 7{\cdot}5 \sin(2\pi \times 1000\, t)]$$

we know that the equation of FM is given by

$$S(t) = A_c \cos\left[\omega_c t + \frac{K_m A_m}{\omega_m} \sin \omega_m t\right]$$

$$\frac{K_{f_1} A_{m1}}{\omega_{m_1}} = 5$$

or $K_{f_1} A_{m1} = 5 \times \omega_{m_1} = 5 \times 2\pi \times 1500$

$= 15000\,\pi$ and

$$\frac{K_{f2} A_{m2}}{\omega_{m_2}} = 7{\cdot}5$$

or $K_{f2} A_{m2} = 7{\cdot}5 \times \omega m_2 = 7{\cdot}5 \times 2\pi \times 1000 = 15000\,\pi$

Since, $\Delta\omega = K_f A_m$ *i.e.* frequency deviation is same in both cases, so

Modulation index,

$$\beta = \frac{\Delta\omega}{|\omega_m|_{max}} = \frac{15000\,\pi}{2\pi \times 1500} = 5$$

Hence alternative (d) is the correct choice.

70. (b) We know that AM is expressed as

$$S_{AM}(t) = A_C[1 + \mu \cos \omega_m t] \cos \omega_c t$$

whereas the angle-modulated signal is expressed as

$$S_\phi(t) = A_c \cos[\omega_c t + \phi(t)] \text{ where}$$

$$\phi(t) = K_p m(t) = K_p A_m \cos \omega_m t \text{ for PM}$$

and $$\frac{d\phi(t)}{dt} = K_f m(t) = \frac{K_f A_m}{\omega_m} \sin \omega_m t \text{ for FM}$$

The given signal

$$X = \cos \omega_c t + 0{\cdot}5 \cos \omega_m t \sin \omega_c t$$

Thus, we conclude that the given signal is AM only.

71. (b) Maximum modulating freq. present in the signal

$$= 4 \text{ kHz}$$

No of bits required to represent each sample

$$= 8 \text{ bits}$$

So number of levels $= 2^8 = 256$

So Bit rate $= nm f_s$

n = no of bits required to represent each sample

m = no of signals multiplexed

f_s = sampling frequency

$$\text{Bandwidth} = \frac{1}{2} \text{ Bit rate}$$

And, Bit rate $= 8 \times 1 \times 8 = 64$ kbps

$$\text{So, Bandwidth} = \frac{1}{2} 64 = 32 \text{ kHz}$$

72. (c) $S/N_{(dB)} = 1{\cdot}76 + 6n$

or $S/N_{(dB)} = 1{\cdot}76 + 6 \times 8 = 49{\cdot}76$

where, n = number of bits

Here close option $\approx$ 48 dB.

73. (b) Quantation noise $(N_q) = \frac{S^2}{12}$

$$\delta = \text{step size} = \frac{2V_{max}}{2^n}$$

V_{max} = Maximum amplitude [voltage level) of the signal

$n \rightarrow$ number of bits/sample

$$\Rightarrow \quad N_q = \left[\frac{2V_{max}}{2^n}\right]^2 \times \frac{1}{12}$$

$$= \frac{4\,V_{max}^2}{2^{2n}} \times \frac{1}{12}$$

$$\Rightarrow \quad N_q \propto \frac{1}{2^{2n}}$$

$$\Rightarrow \quad N_q = K\left[\frac{1}{2^{2n}}\right] \qquad \ldots(1)$$

k = constant

As N_q is to be reduced by $\frac{1}{4}$

So finding n for $\left(\frac{N_q}{4}\right)$

$$\frac{1}{4} = \frac{1}{2^{2n}} \Rightarrow \frac{1}{2^2} = \frac{1}{2^{2n}}$$

or $2 = 2n$ gives $n = 1$

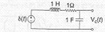
$\Rightarrow n$ has to be increase by 1 bit.

So New $n = 8 + 1 = 9$ bits/samples

So number of quantization levels

$$= 2^n = 2^9 = 512$$

74. (d) The given series RLC circuit



From given circuit

$$V_C(s) = \frac{1/s\ \delta(s)}{\frac{1}{s} + s + 1} = \frac{1}{s^2 + s + 1} \times 1$$

$$\{\because \delta(s) = 1\}$$

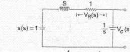
or $$V_C(s) = \frac{2}{\sqrt{3}}\left[\frac{\sqrt{3}/2}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\right]$$

Taking inverse Laplace transform to obtain $V_c(t)$

$$V_c(t) = \frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

Hence alternative (d) is the correct choice.

75. (b)



$$V_R(s) = \frac{1}{1 + s + \frac{1}{s}} = \frac{s}{s^2 + s + 1}$$

or $$V_R(s) = \frac{s}{\left(s + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

or $$V_R(s) = \frac{s + \frac{1}{2} - \frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

or $$V_R(s) = \frac{\left(s + \frac{1}{2}\right)}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$-\frac{1}{2} \cdot \frac{1}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

or $$V_R(s) = \frac{\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$-\frac{1}{\sqrt{3}}\frac{(\sqrt{3}/2)}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

Taking inverse Laplace transform, we get

$$V_R(t) = e^{-\frac{t}{2}}\cos\frac{\sqrt{3}}{2}t - \frac{1}{\sqrt{3}}e^{-\frac{t}{2}}\sin\frac{\sqrt{3}}{2}t$$

or $$V_R(t) = e^{-\frac{t}{2}}\left[\cos\frac{\sqrt{3}}{2}t - \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right]$$

Hence alternative (b) is the correct choice.

76. (c) Given (1) S_1-Open, S_2-closed then

A_1 = 0A, V_1 = 4·5 V,

A_2 = 1A, V_2 = 1·5 V

(2) S_1-closed, S_2-open then

A_1 = 4A, V_1 = 6V

A_2 = 0A, V_2 = 6V

As Z-Parameter-given by

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \qquad ...(1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \qquad ...(2)$$

→ When port 2 is open and port 1 is closed

⇒ $$Z_{11} = \frac{V_1}{I_1} = \frac{6}{4} = 1·5, \text{ and}$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{6}{4} = 1·5$$

⇒ When port 1 is open and port 2 is closed

⇒ $$Z_{12} = \frac{V_1}{I_2} = \frac{4·5}{1} = 4·5$$

$$Z_{22} = \frac{V_2}{I_2} = 1·5$$

so Z-Matrix = $\begin{bmatrix} 1·5 & 4·5 \\ 1·5 & 1·5 \end{bmatrix}$

77. (a) *h*-parameter is given by

$$V_1 = h_{11}I_1 + h_{12}V_2 \qquad ...(5)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \qquad ...(6)$$

Here *h*-parameter cannot be find out directly from given conditions. So convert Z-parameter eqn. into *h*-parameter eqns.

⇒ from eq^n (1) and (2)

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \qquad ...(3)$$

and $$V_2 = Z_{21}I_1 + Z_{22}I_2 \qquad ...(4)$$

from (4) $I_2 = V_2 - Z_{21}I_1$. Put in eq. (3)

⇒ $$V_1 = \frac{[Z_{11}Z_{22} - Z_{12}Z_{21}]}{Z_{22}}I_1 + \frac{Z_{12}}{Z_{22}}V_2$$

Compare this eq. with (5)

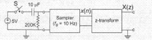
⇒ $$h_{11} = \frac{V_1}{I_1} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}} = -3,$$

$$h_{12} = \frac{V_1}{V_2} = \frac{Z_{12}}{Z_{22}} = 3$$

Similarly we can find out h_{21} and h_{22}

As by seeing h_{12} and h_{11} only option (a) is matching.

78. (b) The given network

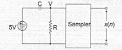


Given time constant,

$$\tau = RC = 200 \times 10^3 \times 10 \times 10^{-6}$$

$$= 2 \text{ sec.}$$

In order to calculate samples *x* (*n*). The redrawn network of the given network is shown below :



Given, sampling frequency = 10 Hz

The mark voltage V is given by

$$V = V_o e^{\frac{-t}{RC}} = 5e^{\frac{-t}{2}}$$

and the samples are given by *x* (*n*)

$$x(n) = 5e^{\frac{-t}{2\times 10}}$$

or $$x(n) = 5e^{-0·05n}$$

Hence alternative (b) is the correct choice.

79. (c) $x(n) = 5e^{-0·05n}\,u(n)$ is a causal signal

Z-transform of the causal signal

$$X(Z) = Z\{x(n)\}$$

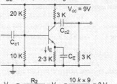
or $$X(Z) = Z\{5e^{-0·05n}(un)\}$$

or $$X(Z) = 5\cdot\frac{1}{1-e^{-0·05}Z^{-1}};\ |Z| > e^{-0·05}$$

or $$X(Z) = \frac{5\cdot Z}{Z - e^{-0·05}};\ |Z| > e^{-0·05}$$

Hence alternative (c) is the correct choice.

80. (a) V_{BE} = 0·7, from the figure



$$V_{th} = \frac{R_2}{R_1+R_2}\times V_{CC} = \frac{10\,k \times 9}{30\,k} = 3\text{ V}$$

As given β and capacitances are very large so no need to find out R_{th} and I_B

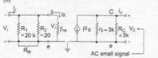
Use approximate analysis

$$V_{th} - V_{BE} - I_E R_E = 0$$

or $$I_E = \frac{V_{th} - V_{BE}}{R_E}$$

or $$I_E = \frac{3 - 0{\cdot}7}{2{\cdot}3\text{ K}} = \frac{2{\cdot}3}{2{\cdot}3\text{ K}} = 1 \text{ mA} \quad \dots(1)$$

81. (d)



As A_V [Voltage gain] $= \frac{V_o}{V_i}$

$$V_O = -(\beta I_B) \times [R_c \,//\, r_0]$$

and $$I_B = \frac{V_i}{\beta r_e},$$

$$V_o = -\beta\left[\frac{V_i}{\beta r_e}\right][r_o \parallel R_C]$$

$$A_v = \frac{V_o}{V_i} = -\left[\frac{R_C \,//\, r_o}{r_e}\right]$$

or $$A_v = \frac{-3 \times 10^3}{2 \times 25} = \frac{-1{\cdot}5 \times 1000}{25}$$

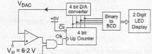
or $$A_v = -60$$

As $$r_e = \frac{25 \text{ mV}}{I_E} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25\ \Omega,$$

$$R_C \parallel r_o = \frac{9}{6}\text{ k}\Omega = \frac{3}{2}\text{ k}\Omega$$

As $$I_E = 1 \text{ mA from (1)}.$$

82. (d)



From given circuit, we conclude

If $V_{in} > V_{D/A}$ → output goes to 1 otherwise 0

Hence if $V_{D/A} < 6{\cdot}2$ V → output 1 and counter counts

And if $V_{D/A} > 6{\cdot}2$ V → output 0 and counter stops

$$V_{D/A} = \sum_{n=0}^{3} 2^{n-1}\, bn$$

or $$V_{D/A} = b_0\, 2^{-1} + b_1\, 2^0 + b_2\, 2^1 + b^3\, 2^2$$

Thus, for stable reading of the LED display

$$V_{D/A} > 6{\cdot}2 \text{ V}$$

or $b_0\, 2^{-1} + b_1\, 2^0 + b_2\, 2^1 + b^3\, 2^2 > 6{\cdot}2$ V

(a) $06 \to 0110 \to 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 3$ V *i.e.* $< 6{\cdot}2$ V → × (*i.e.* wrong)

(b) $07 \to 0111 \to 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1}$

$= 3{\cdot}5$ V *i.e.* $< 6{\cdot}2$ V → × (*i.e.* wrong)

(c) $12 \to 1100 \to 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1}$

$= 6$ V *i.e.* $< 6{\cdot}2$ V → × (*i.e.* wrong)

(d) $13 \to 1101 \to 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1}$

$= 6{\cdot}5$ V *i.e.* $> 6{\cdot}2$ V → √ (*i.e.* correct)

Hence alternative (d) is the correct choice.

83. (b) The magnitude of error voltage

$$= V_{D/A(max)} - V_{in} = 6{\cdot}5 \text{ V} - 6{\cdot}2 \text{ V} = 0{\cdot}3 \text{ V}$$

84. (c) $$h(t) = e^{-2t} \cdot u(t)$$

As $$F{\cdot}T\,[h(t)] = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{+\infty} e^{-2t} \cdot u(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^{\infty} e^{-2t}\, e^{-j\omega t}\, dt = \int_0^{\infty} e^{-(2+j\omega)t}\, dt$$

$$= \frac{-1}{2+j\omega}\left[e^{-(2+j\omega)t}\right]_0^{\infty} = -\frac{1}{2+j\omega}\left[e^{-\infty} - e^{-0}\right]$$

$$= \frac{-1}{2+j\omega}\left[\frac{1}{e^{\infty}} - \frac{1}{e^0}\right] = +\frac{1}{2+j\omega}$$

85. (d) Given $$H(\omega) = \frac{1}{2+j\omega}$$

or $$H(s) = \frac{1}{2+s}$$

If $$x(t) = 2\cos 2t$$

then $$X(s) = \frac{2 \cdot s}{s^2+4} = \frac{2s}{s^2+4}$$

Now from relation

$$Y(s) = H(s) \times X(s)$$

$$Y(s) = \frac{1}{2+s} \times \frac{2s}{s^2+4}$$

Using partial fraction method

$$Y(s) = \frac{A}{2+s} + \frac{(Bs+C)2s}{s^2+4}$$

gives $$Y(s) = -\frac{1}{2(s+2)} + \frac{1(s+2)}{2(s^2+4)}$$

Now output $Y(t)$ can be determine by using inverse laplace transform. Therefore

$$Y(t) = -\frac{1}{2}e^{-2t} + \frac{1}{2}[\cos 2t + \sin 2t]$$

↑

Transient

or $$Y(t) = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\cos 2t + \frac{1}{\sqrt{2}}\sin 2t\right]$$

or $$Y(t) = \frac{1}{\sqrt{2}}\left[\cos\frac{\pi}{4}\cos 2t + \sin\frac{\pi}{4}\sin 2t\right]$$

or $$Y(t) = \frac{1}{\sqrt{2}}\cos\left[2t - \frac{\pi}{4}\right]$$

or $$Y(t) = 2^{-0{\cdot}5}\cos[2t - 0{\cdot}25\,\pi]$$

Hence alternative (d) is the correct choice.