MATHEMATICS *for* Class XII

CBSE - 2010
Delhi

Max. Time : 3 hrs. **Max. Marks : 100**

General Instructions :

(i) All questions are compulsory.

(ii) The question paper consists of 34 questions divided into 4 sections — A, B and C. **Section A** comprises of **ten questions** of **01 mark** each, **Section B** comprises of **twelve questions** of **04 marks** each, **Section C** comprises of **seven questions** of **06 marks** each.

(iii) Use of calculators is not permitted. However you may ask for mathematical tables.

SET – I

SECTION 'A'

Q1. What is the range of the function $f(x)=\frac{|x-1|}{(x-1)}$?

Q2. What is the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

Q3. If $A=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$, then for what value of α is A an identity matrix ?

Q4. What is the vlaue of the determinant $\begin{vmatrix}0 & 2 & 0\\ 2 & 3 & 4\\ 4 & 5 & 6\end{vmatrix}$?

Q5. Evaluate $\int\frac{\log x}{x}dx$

Q6. What is the degree of the following differential equation ?

$$5x\left(\frac{dy}{dx}\right)^2-\frac{d^2y}{dx^2}-6y=\log x$$

Q7. Write a vector of magnitude 15 units in the direction of vector $\hat{i}-2\hat{j}+2\hat{k}$.

Q8. Write the vector equation of the following line :

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$$

Q9. If $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}3 & 1\\ 2 & 5\end{pmatrix}=\begin{pmatrix}7 & 11\\ k & 23\end{pmatrix}$, then write the value of k.

Q10. What is the cosine of the angle which the vector $\sqrt{2}\hat{i}+\hat{j}+\hat{k}$ makes with y-axis ?

SECTION 'B'

Q11. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probabilities that a candidate would get fou or moer correct answers just by guessing ?

Q12. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors arew $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ respec-tively, externally in the ratio 1 : 2. Also show that P is the mid point of the line segment RQ.

Q13. Find the Cartesian equation of the plane passing through the points A (0, 0, 0) and B (3, –1, 2) and parallel to line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$.

Q14. Using elementary row operations, find the inverse of the following matrix :

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

Q15. Let Z be the set of all integers and R be the relation on Z defined as R = {(a, b) : a, b $\in$ Z , and (a –b) is divisible by 5}. Prove that R is an equivalence relation.

Q16. Prove the following : $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x\in(0,1)$

OR

Prove the following :

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$$

Q17. Show that the function defined as follows, is continuous x = 2, but not differentiable thareat

$$f(x)=\begin{cases} 3x-2 & ,\ 0<x\le 1 \\ 2x^2-x & ,\ 1<x\le 2 \\ 5x-4 & ,\ x>2 \end{cases}$$

OR

Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$

Q18. Evaluate $\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$

OR

Evalaute $\int\frac{1-x^2}{x(1-2x)}dx$

Q19. Evaluate $\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$

Q20. Find the points on the curve $y = x^2$ at which the slope of the tangent is equal to the y-coordinate of the point.

Q21. Find the general solution of the differential equation $x\log x.\frac{dy}{dx}+y=\frac{2}{x}.\log x$

OR

Find the particular solution of the differential equation satisfying the given conditions :

$\frac{dy}{dx}=y\tan x$, given that y = 1 when x = 0.

Q22. Find the particular solution of the differential equation satisfying the given conditions :

$$x^2dy+(xhy+y^2)dx=0$$

SECTION 'C'

Q23. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours

available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs. 190, find the number of rings and chains that should manufactured per day, so as to earn he maximum profit. Make it as an LPP and solve it graphically.

Q24. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards aer drawn at random and are found to both clubs. Find the probability of the lost card being of clubs.

OR

From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

Q25. The point A(4, 5, 10), B(2, 3, 4) and C(1, 2, –1) are three vertices of a parallelogram ABCD. Find the vectors equations of the sides AB and BC and also find the coordinates of point D.

Q26. Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line x = 4y – 2.

OR

Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

Q27. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

Q28. Find the values of x for which $f(x) = [x(x-2)]^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to x-axis.

Q29. Using properties of determinants, show the following :

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

SET – II

Q7. Find the minor of the element of second row, and third column (a_{23}) in the following determinant :

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

Q11. Find all points of discontinuouity of f, where f is defined as follows :

$$f(x) = \begin{cases} |x|+3 & , \quad x \le -3 \\ -2x & , \quad -3 < x < 3 \\ 6x+2 & , \quad x \ge 3 \end{cases}$$

OR

Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{1/x}$.

Q14. Let $*$ be a binary operation on Q defined by $a * b = \frac{3ab}{5}$ show that $*$ is commutative as well as associative. Also find its identity element, if it exists.

Q18. Evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$

Q20. Find the equations of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line x + 14y + 4 = 0.

Q23. Evaluate $\int_1^3 (3x^2 + 2x)\,dx$

OR

Using integration, find the area of the following region : $\left\{(x,y): \frac{x^2}{9} + \frac{y^2}{4} \le 1 \le \frac{x}{3} + \frac{y}{2}\right\}$

Q29. Write the vector equations of the following lines and hence determine the distance between them :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}; \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

SET – III

Q1. Find the principal value of $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$

Q9. If A is asquare matrix of order 3 and | 3A | = K|A|, then write the value of K.

Q11. There are two Bags, Bag I and Bag II. Bag I contains 4 white and 3 red balls while another bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from Bag I.

Q14. Prove that : $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$

OR

If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ find the value of x.

Q17. Show that the relation S in the set R of real numbers defined as S = {(a, b) : a,b $\in$ R and $a \le b^3$} is neither reflexive, nor symmetric nor transitive.

Q19. Find the equation of tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$, at the point where it cuts the x-axis.

Q23. Find the intervals in which the function f given by $f(x) = \sin x - \cos x,\ 0 \le x \le 2x$ is strictly increasing or strictly decreasing.

Q24. Evaluate $\int_1^4 (x^2 - x)\,dx$ as limit of sums.

OR

Using integration find the area of the following region : $\{(x, y): |x-1| \le y \le \sqrt{5 - x^2}\}$

ANSWERS

SET-I

1. {–1, 1} **2.** $\frac{-\pi}{3}$ **3.** $\alpha = 0°$ **4.** 8 **5.** $\frac{(\log x)^2}{2} + c$ **6.** Degree = 1

7. $5\hat{i} - 10\hat{j} + 10\hat{k}$ **8.** $(5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} - 2\hat{k})$ **9.** k = 17 **10.** $\frac{1}{2}$ **11.** $\frac{11}{343}$

12. $2\vec{a} + \vec{b}$ **13.** x – 19y – 11z = 0 **14.** $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ **17. OR** $\frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$

18. $e^x \cot 2x + c$ **OR** $\frac{1}{2}x + \log|x| - \frac{3}{4}\log|1-2x| + c$ **19.** $2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$ **20.** (0, 0), (3, 27)

21. – 2 logx – 2 + Cx **OR** y = sec x **22.** $y + 2x = 3x^2 y$ **23.** Max. profit is rs. 5440 when x = 8, y = 16

24. $\frac{11}{50}$ **OR**

x	0	1	2
P(x)	$\frac{14}{30}$	$\frac{14}{30}$	$\frac{2}{30}$

25. (3, 4,5) **26.** $\frac{9}{8}$ sq. units **OR** $\frac{\pi}{2}(\pi - 2)$ **28.** (0, 0), (1, 1), (2, 0)

SET-II

7. 13 **11. OR** $(\cos x)^x[\log x - x\tan x] - (\sin x)^{1/x}\left[\frac{\cot x}{x} - \frac{\log\tan x}{x^2}\right]$ **l8.** π **20.** x + 14y + 86 = 0

23. 34 **OR** $3\left(\frac{\pi}{2} - 1\right)$ sq. units **29.** $\frac{\sqrt{293}}{7}$ units

SET-III

1. $\frac{\pi}{2}$ **9.** K = 27 **11.** $\frac{40}{61}$ **14. OR** $x = \pm\frac{1}{\sqrt{2}}$ **19.** x – 20y – 7 = 0

23. inc. $\left[0, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$, dec. $\left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$ **24.** $\frac{27}{2}$ **OR** $-\frac{1}{2} + \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right)$ sq. units