Qn No	Sub Qn	Value points	Score	Total
1	(i)	$2A = \begin{bmatrix} 6 & -8 \\ 2 & 4 \end{bmatrix}$	$\frac{1}{2}$	
		$2A+B = \begin{bmatrix} 6 & -8 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$		1
		$= \begin{bmatrix} 8 & -5 \\ 3 & 8 \end{bmatrix}$	$\frac{1}{2}$	
	ii	$AB = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$	$\frac{1}{2}$	
		$= \begin{bmatrix} 6-4 & 9-16 \\ 2+2 & 3+8 \end{bmatrix}$	$\frac{1}{2}$	2
		$= \begin{bmatrix} 2 & -7 \\ 4 & 11 \end{bmatrix}$	1	
2	(i)	$A_{31} = -8$, $A_{32} = 2$, $A_{33} = 6$.	2	2
	(ii)	$\Delta = a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$	$\frac{1}{2}$	
		$= 0 \times -8 + 1 \times 2 + 2 \times 6$		1.
		$= 2 + 12 = 14$	$\frac{1}{2}$	
3.	(i)	$2 * 3 = 2+3+1 = 6$.	1	1
	(ii)	$a * e = a = e * a$	$\frac{1}{2}$	
		$a+e+1 = a$.		
		$e = a-a-1$		
		$= -1$.	$\frac{1}{2}$	2
		$e * a = a \Rightarrow e+a+1 = a$.	$\frac{1}{2}$	
		$\Rightarrow e = a-a-1$		
		$\Rightarrow e = -1$.	$\frac{1}{2}$	

Total 12 pages.

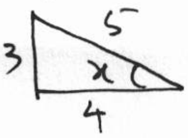
Qn No	Sub qn			
4.	(i)	$f(1) = 2$	1	1
	(ii)	$\underset{x\to 1}{Lt} f(x) = \underset{x\to 1}{Lt} \frac{x^2-1}{x-1}$	$\frac{1}{2}$	
		$= \underset{x\to 1}{Lt} \frac{(x+1)(x-1)}{x-1}$	$\frac{1}{2}$	
		$= \underset{x\to 1}{Lt} (x+1) = 2$	$\frac{1}{2}$	2
		$\underset{x\to 1}{Lt} f(x) = f(1)$.	$\frac{1}{2}$	
		$\therefore$ f is Continuous at $x=1$		
5	(i)	$\int \frac{1}{x} dx = \log\lvert x\rvert + c$.	1	1
	(ii)	$\int \frac{(\log x)^2}{x} dx$.		
		$\log x = t$.		
		$\frac{1}{x} dx = dt$.	$\frac{1}{2}$	
		$= \int t^2 dt = \frac{t^3}{3}$	1	2
		$= \frac{(\log x)^3}{3} + c$.	$\frac{1}{2}$	
6	(i)	$\vec{a}\cdot\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k})\cdot(3\hat{i} - 2\hat{j} + \hat{k})$ 1	$\frac{1}{2}$	1
		$= 3 + 4 + 3 = 10$.	$\frac{1}{2}$	
	(ii)	$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\cdot\lvert\vec{b}\rvert}$.	$\frac{1}{2}$	
		$\lvert\vec{a}\rvert = \sqrt{1+4+9} = \sqrt{14}$	$\frac{1}{2}$	
		$\lvert\vec{b}\rvert = \sqrt{9+4+1} = \sqrt{14}$.		

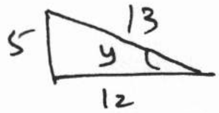
		$\therefore \cos\theta = \dfrac{10}{\sqrt{14}\cdot\sqrt{14}}$ $= \dfrac{10}{14} = \dfrac{5}{7}$. $\theta = \cos^{-1}(5/7)$	$\frac{1}{2}$ $\frac{1}{2}$	2
7	(i)	2, 3, 5	1	1
	(ii)	$\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c}$ Eqn of a line passing through $(-2, 4, -5)$ is $\dfrac{x+2}{a} = \dfrac{y-4}{b} = \dfrac{z+5}{c}$. Since it is parallel to the given line $a = 2,\ b = 3,\ c = 5$. $\therefore$ Eqn is $\dfrac{x+2}{2} = \dfrac{y-4}{3} = \dfrac{z+5}{5}$.	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
8	(i)	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ $4x_1 + 3 = 4x_2 + 3$ $4x_1 = 4x_2$ $x_1 = x_2$ $\therefore f$ is one-one.	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	2
	(ii)	Let $y = 4x+3$ $y-3 = 4x$ $x = \dfrac{y-3}{4} \in \mathbb{R},\ \forall y \in \mathbb{R}$, f is onto	$\frac{1}{2}$ $\frac{1}{2}$	1
	iii	$f^{-1}(11) = \dfrac{11-3}{4} = \dfrac{8}{4} = 2$	1	1

Qn No	Sub Qn		Score	Total
9	(i)	$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ (A)	1	1

(ii)

$\cos^{-1}\frac{4}{5} = x, \quad \cos^{-1}\frac{12}{13} = y \Rightarrow \cos x = \frac{4}{5}, \quad \cos y = \frac{12}{13}$ — ½





½

$\sin x = 3/5 \qquad \sin y = \frac{5}{13}$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$ — ½ — Total: 3

$= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}$ — ½

$= \frac{48}{65} - \frac{15}{65} = \frac{33}{65}$ — ½

$x+y = \cos^{-1}\frac{33}{65}$

ie, $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$ — ½

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10. (i)

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Score: 1 — Total: 1

(ii)

$$\Delta = a+b+c \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$ — $\frac{1}{2}$

$C_1 \to C_1 - C_2, \qquad C_2 \to C_2 - C_3$

$$\Delta = (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b+c+a & -b-c-a & 2b \\ 0 & c+a+b & c-a-b \end{vmatrix}$$ — 1 — Total: 3

$$= (a+b+c) \times \begin{vmatrix} b+c+a & -b-c-a \\ 0 & c+a+b \end{vmatrix}$$ — 1

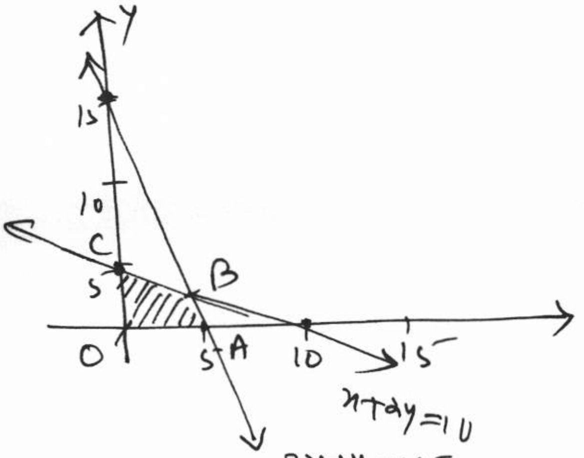
$= (a+b+c)(a+b+c)^2 = (a+b+c)^3$ — $\frac{1}{2}$

Qn No	Sub Qn		Score	Total
11	(i)	$fog(x) = f(g(x))$	$\frac{1}{2}$	
		$= f(\cos x)$	$\frac{1}{2}$	2
		$= (\cos x)^2 = \cos^2 x$	1	
	(ii)	$\frac{d}{dx}(fog)(x) = \frac{d}{dx}(\cos^2 x)$	$\frac{1}{2}$	
		$= 2\cos x \times -\sin x$	$\frac{1}{2}+\frac{1}{2}$	2
		$= -\sin 2x$	$\frac{1}{2}$	
12	(i)	$\int_0^a f(a-x)\,dx$ (B)	1	1
	(ii)	Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos^3 x\,dx}{\cos^3 x + \sin^3 x} \longrightarrow (1)$		
		Also, $I = \int_0^{\pi/2} \frac{\cos^3(\frac{\pi}{2}-x)\,dx}{\cos^3(\frac{\pi}{2}-x) + \sin^3(\frac{\pi}{2}-x)}$	$\frac{1}{2}$	
		$= \int_0^{\pi/2} \frac{\sin^3 x\,dx}{\sin^3 x + \cos^3 x.} \longrightarrow (2)$	$\frac{1}{2}$	
		$(1) + (2) \Rightarrow 2I = \int_0^{\pi/2} \frac{\cos^3 x + \sin^3 x}{\cos^3 x + \sin^3 x}\,dx$	1	3
		$= \int_0^{\pi/2} 1\,dx$		
		$= (x)_0^{\frac{\pi}{2}} = \frac{\pi}{2}$	$\frac{1}{2}$	
		$\therefore I = \frac{\pi}{4}$	$\frac{1}{2}$	

Qn No	Sub Qn No		Score	Total
13	(i)	$\int_a^b y\,dx$ (C)	1	1
	(ii)	Area $= \int_a^b y\,dx$	$\frac{1}{2}$	3
		$= \int_1^4 \sqrt{4x}\,dx.$	$\frac{1}{2}$	
		$= 2\int_1^4 \sqrt{x}\,dx.$		
		$= 2 \times \int_1^4 x^{1/2}\,dx$	$\frac{1}{2}$	
		$= 2 \times \left[\frac{x^{3/2}}{3/2}\right]_1^4$	$\frac{1}{2}$	
		$= 2 \times \frac{2}{3}\left[4^{3/2} - 1^{3/2}\right]$	$\frac{1}{2}$	
		$= \frac{4}{3}\left[8-1\right] = \frac{4}{3} \times 7 = \frac{28}{3}$	$\frac{1}{2}$	
14		Eqn of the plane is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$	$\frac{1}{2}$	4
		ie. $\begin{vmatrix} x-1 & y-1 & z-1 \\ -1 & 0 & 1 \\ 2 & -2 & 3 \end{vmatrix} = 0.$	$1\frac{1}{2}$	
		$(x-1)(2) - (y-1)(-5) + (z-1)(2) = 0$	1	
		$2x - 2 + 5y - 5 + 2z - 2 = 0$		
		$2x + 5y + 2z = 9$	$\frac{1}{2} + \frac{1}{2}$	

Qn No	Sub Q.N		Score	Total

15



Score: 2

Corner points	Value of $Z = 3x+2y$
$O(0,0)$	$Z = 0$
$A(5,0)$	$Z = 3\times5+0 = 15$
$B(4,3)$	$Z = 3\times4+2\times3 = 18.$
$C(0,5)$	$Z = 0+10 = 10.$

Maximum value of $Z = 18$ at $B(4,3)$

Score: 2 — Total: 4

16. (i)

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

$P = \frac{1}{x}, \quad Q = x^2$ — $\frac{1}{2}$

$I.F = e^{\int p\,dx}$ — $\frac{1}{2}$

$= e^{\int \frac{1}{x}dx} = e^{\log x}$ — $\frac{1}{2}$

$= x.$ — $\frac{1}{2}$

Total: 2

(ii) Solution is $y \cdot IF = \int Q \cdot IF\,dx.$ — $\frac{1}{2}$

$y \cdot x = \int x^2 \cdot x\,dx$ — 1

$= \int x^3\,dx$

$xy = \frac{x^4}{4} + C$ — $\frac{1}{2}$

Total: 2

Qn No.	Sub qn		Score	Total
17	(i)	$P(H) = 1/3 \qquad P(W) = \frac{1}{5} \qquad P(H') = 1 - \frac{1}{3} = 2/3$ $P(H \cap W') + P(H' \cap W) \qquad P(W') = 1 - 1/5 = \frac{4}{5}.$	$\frac{1}{2}$	
		$= P(H).\ P(W') + P(H').\ P(W).$ $= 1/3 \cdot \frac{4}{5} + 2/3 \cdot \frac{1}{5} = \frac{4}{15} + \frac{2}{15} = \frac{6}{15} = 2/5$	$\frac{1}{2}$	1
	(ii)	$P(H' \cap W') = P(H').\ P(W')$ $= 2/3 \cdot \frac{4}{5} = \frac{8}{15}$	1	1
	(iii)	$P(H \cap W) = P(H).\ P(W) = \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$	1	1
	(iv)	P(atleast one of them gets the job) = 1 — P(none of them gets the job) $= 1 - 8/15 = 7/15.$	1	1
18.	(i)	$A = \begin{bmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \\ 7 & 5 & 3 \end{bmatrix}$	3	3
	(ii)	$A' = \begin{bmatrix} 1 & 4 & 7 \\ -1 & 2 & 5 \\ -3 & 0 & 3 \end{bmatrix}$	$\frac{1}{2}$	
		$A + A' = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$	$\frac{1}{2}$	
		$P = \frac{1}{2}(A + A') = \begin{bmatrix} 1 & 3/2 & 2 \\ 3/2 & 2 & 5/2 \\ 2 & 5/2 & 3 \end{bmatrix}$	$\frac{1}{2}$	

$$A - A' = \begin{bmatrix} 0 & -5 & -10 \\ 5 & 0 & -5 \\ 10 & 5 & 0 \end{bmatrix}$$ $\frac{1}{2}$

$$Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & -5/2 & -5 \\ 5/2 & 0 & -5/2 \\ 5 & 5/2 & 0 \end{bmatrix}$$ $\frac{1}{2}$ 3

$$A = P + Q$$

$$= \begin{bmatrix} 1 & 3/2 & 2 \\ 3/2 & 2 & 5/2 \\ 2 & 5/2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -5 \\ 5/2 & 0 & -5/2 \\ 5 & 5/2 & 0 \end{bmatrix}$$ $\frac{1}{2}$

19

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$ $\frac{1}{2}$

$$X = A^{-1} \cdot B.$$ $\frac{1}{2}$

$$A^{-1} = \frac{\operatorname{adj} A}{|A|}.$$ $\frac{1}{2}$

$$A_{11} = 0 \quad A_{12} = 30 \quad A_{13} = -20$$
$$A_{21} = -5 \quad A_{22} = 0 \quad A_{23} = 10$$
$$A_{31} = 10 \quad A_{32} = -20 \quad A_{33} = 10$$ 1

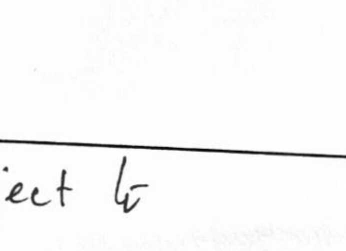
$$|A| = 50.$$ $\frac{1}{2}$

$$\operatorname{adj} A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$ 1

$$A^{-1} = \frac{1}{50} \times \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$ 1

$$X = \frac{1}{50} \times \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$ $\frac{1}{2}$

		$X = \frac{1}{50} \times \begin{bmatrix} -450+700 \\ 1800+{}^{-}1400 \\ -1200+900+700 \end{bmatrix}$ $= \frac{1}{50} \times \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$ $x=5, \quad y=8, \quad z=8.$	$\frac{1}{2}$	6.
20	(i)	$\frac{dx}{d\theta} = a(0 - \sin\theta) = -a\sin\theta.$ $\frac{dy}{d\theta} = a\cos\theta$	1 1	2
	(ii)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $= +\frac{a\cos\theta}{-a\sin\theta} = -\cot\theta$	1 1	2
	(iii)	$\frac{d^2y}{dx^2} = \frac{d}{dx}(-\cot\theta)$ $= \frac{d}{d\theta}(-\cot\theta) \cdot \frac{d\theta}{dx}$ $= \operatorname{cosec}^2\theta \cdot \frac{-1}{a\sin\theta} = \frac{-1}{a}\operatorname{cosec}^3\theta.$	$\frac{1}{2}$ 1 $\frac{1}{2}$	2
21.	(i)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\vec{i} - 3\vec{j} - 2\vec{k}.$ $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 0\vec{i} - 2\vec{j} + \vec{k}.$	1 1	2
	(ii)	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & -2 \\ 0 & -2 & 1 \end{vmatrix} = -7\vec{i} + \vec{j} + 2\vec{k}.$	2	2
	(iii)	Area of $\Delta ABC = \frac{1}{2} \times \lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert.$ $\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert = \sqrt{49+1+4} = \sqrt{54} = 3\sqrt{6}$ Area $= \frac{1}{2} \times 3\sqrt{6}$	$\frac{1}{2}$ 1 $\frac{1}{2}$	2

Qn No	Sub qn		Score	Total
22	(i)	$\Sigma P(x) = 1$. $k+2k+3k+4k = 1$. $10k = 1$ $k = 1/10$	$\frac{1}{2}$ $\frac{1}{2}$	1
	(ii)	$P(X<2) = P(X=0) + P(X=1)$ $= k + 2k = 3k$ $= 3/10$.	1 1	2
	(iii)	$E(x) = x_1 P_1 + x_2 P_2 + \dots + x_n P_n$. $= 0.k + 1\times 2k + 2\times 3k + 3\times 4k$. $= 2k + 6k + 12k$ $= 20k = 20 \times \frac{1}{10} = 2$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$	3.
23.	(i)	Let x be the number of item A and y be the number of item B. $Z = 40x + 25y$.	1	1
	(ii)	$300x + 150y \le 15000$. $x+y \le 80$ $x, y \ge 0$	1 $\frac{1}{2}$ $\frac{1}{2}$	2
	(iii)	Maximise $Z = 40x+25y$ Subject to the Constraints $2x+y \le 100$ $x+y \le 80$, $x, y \ge 0$. Corner points — Value of $Z = 40x+25y$ $O(0,0)$ — $Z = 0$ $A(50,0)$ — $Z = 1000$ $B(20,60)$ — $Z = 2300$ $C(0,80)$ — $Z = 2000$  Maximum profit $= 2300$ at $B(20,60)$	3	3

Qn No	Sub qn		Score	Total
24	(i)	$f'(a)$ (D)	1	1
	(ii)	$\frac{dy}{dx} = 3x^2 - 4$	1	1
	(iii)	$m = \left.\frac{dy}{dx}\right]_{(2,1)} = 3\times 2^2 - 4 = 8$	1	
		Eqn of tangent is $y - y_1 = \left.\frac{dy}{dx}\right]_{x_1,y_1} (x - x_1)$		
		$y - 1 = 8(x - 2)$		2
		$y - 1 = 8x - 16$		
		$8x - y = 15$	1	
	(iv)	Eqn of normal is		
		$y - y_1 = \frac{-1}{\left.\frac{dy}{dx}\right]_{x_1,y_1}} (x - x_1)$		
		$y - 1 = \frac{-1}{8}(x - 2)$	1	2
		$8y - 8 = -x + 2$		
		$x + 8y = 10.$	1	