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ESE – 2019
(MAINS)

Questions with Detailed Solutions

ELECTRICAL ENGINEERING

PAPER – II

All Queries related to **ESE - 2019 MAINS** Solutions are to be sent to the following email address
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ELECTRICAL ENGINEERING

ESE MAINS-2019 PAPER-2

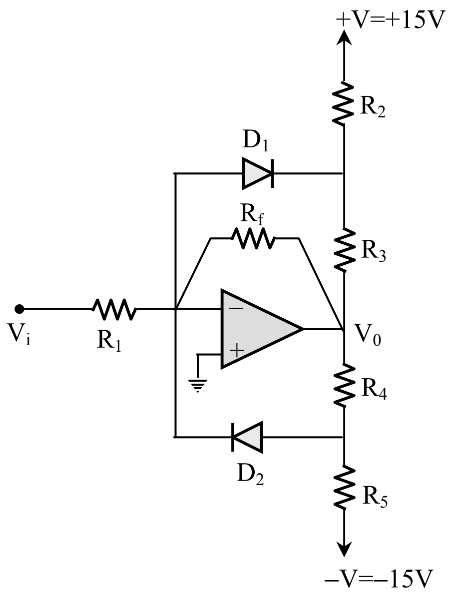
PAPER REVIEW

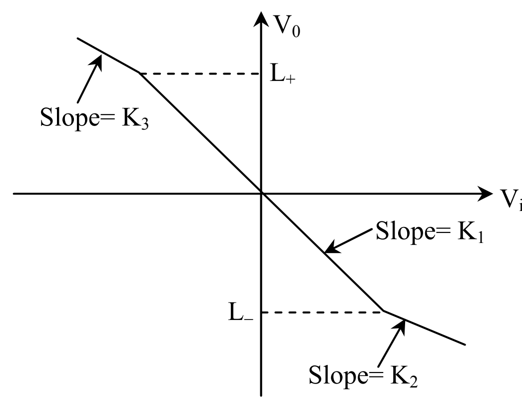
Overall paper was Moderate, in this paper in every subject some of the questions are easy and some are difficult and also subjects are not divided in section wise. Section-B is easy as compared to Section-A. So choosing three questions from Section-B will fetch you to score good marks.

Subjects	LEVEL	Marks
Analog & Digital Electronics	Moderate	64
Systems & Signal Processing	Moderate	52
Control Systems	Moderate	84
Electrical Machines	Hard	104
Power Systems	Moderate	92
Power Electronics	Moderate	84

SECTION - A

1(a) Determine the values of slope K_1, K_2, K_3 and the voltages L_+ and L_- for the amplifier and its transfer characteristics shown in the figure given below: **12**





(R_1 = 30 kΩ, $R_2 = R_5$ = 9 kΩ, $R_3 = R_4$ = 3 kΩ, R_f = 60 kΩ)

The diodes may be assumed to be ideal.

Sol: Case (i): For the small values of V_i i.e. close origin 'O'

Step: 1 As V_i & V_0 are small, the voltage at node 'A' is +Ve and voltage at node 'B' is –Ve. Therefore D_1 and D_2 will be OFF

$$\Rightarrow V_0 = -\frac{R_f}{R_1}V_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\therefore \text{ The slope, } K_1 = \frac{-R_f}{R_1} = -\frac{60k}{30k} = -2 \ldots\ldots\ldots\ (2)$$

Case (ii): For V_i = positive

Step 2: As V_i increases, D_2 remains OFF, but D_1 will be ON

$$L_- = -V\left[\frac{R_3}{R_2}\right] - V_{D_1}\left(1+\frac{R_3}{R_2}\right) \ldots\ldots\ldots\ (3)$$ [Using super position principle node 'A'

$= -15V\left[\frac{3k}{9k}\right] - 0$ ……….. (4) [$\because$ D_1 is ideal, $V_{D1} = 0$]

$L_{-} = -5V$

Step 3:

Slope, $K_2 = -\frac{R_f // R_3}{R_1}$ ……………. (6)

$-\frac{60K // 3K}{30K}$

$K_2 = -0.095238$

Case (iii)

Step 4: As V_i is –Ve, D_1 OFF D_2 will be ON

$L_{+} = V\left(\frac{R_4}{R_5}\right) + V_{D_2}\left(1 + \frac{R_4}{R_5}\right)$……… (9) [$\because$ Super position principle at]

Node (B)

$= +15V\left[\frac{3K}{9K}\right] + 0$ …………... (10) [$\because$ D2 is ideal, $V_{D2} = 0$]

$\therefore L_{+} = +5V$ …………………….. (11)

Step 5:

Slope, $K_3 = -\frac{R_f // R_4}{R_1}$ ………….. (12)

$= -\frac{60k // 3k}{30k}$ ………… (13)

$K_3 = -0.095238$ ………………….. (14)

1(b) Determine the total energy and average power of the following signal: **12**

$$x(t) = \begin{cases} 2 & -3 \le t \le 3 \\ 5-t & 3 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$$

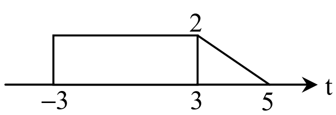
Sol: $x(t) = \begin{cases} 2 & -3 \le t \le 3 \\ 5-t & 3 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$

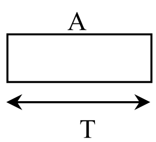
$$E_{x(t)} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-3}^{3} (2)^2 dt + \int_{3}^{5} (5-t)^2 dt$$

$$= 4(t)\Big|_{-3}^{3} + \left(\frac{(5-t)^3}{-3} \right)_3^5$$

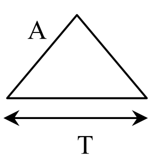
$$= 4(6) + \frac{8}{3} = 26.67 \text{ J}$$

Alternative method:





$E = A^2T$



$E = \frac{A^2T}{3}$

$$E = (2)^2(6) + \frac{(2)^2(2)}{3}$$

$$= 24 + \frac{8}{3}$$

$$= 24 + 2.67$$

$$= 26.67 \text{ Joules}$$

For an energy signal average power = 0

1(c) Show the permissible area for the poles of a second order system which must simultaneously meet the following criteria:

12

(i) Maximum percent overshoot ≤ 5%

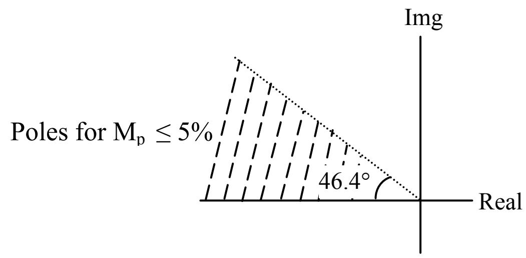
(ii) Settling time for 2% criterion ≤ 500 ms

Sol: $M_P = 5\% = 0.05$

$$M_P = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$\zeta = \sqrt{\frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}} = 0.69$$

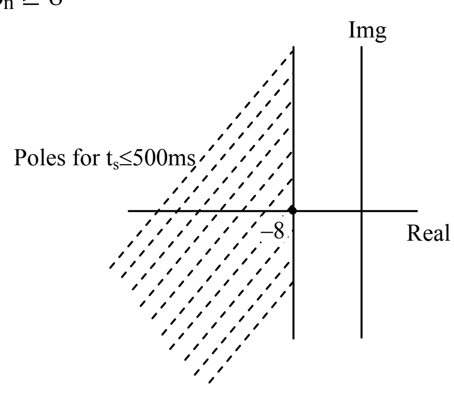
$\phi = \cos^{-1}\zeta = 46.4°$ ……….…….(1)



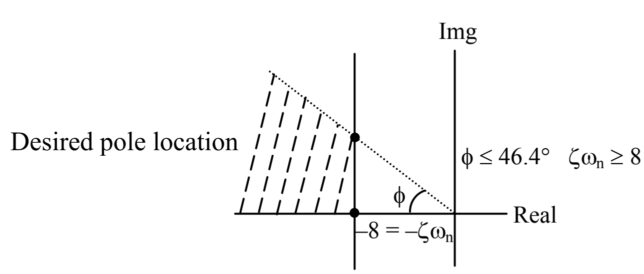
$$t_s \cong \frac{4}{\zeta\omega_n} \le 500\times10^{-3}$$

$$\zeta\,\omega_n \ge \frac{4}{500\times10^{-3}} = 8 \quad \text{…………(2)}$$

$$\zeta\omega_n \ge 8$$



From (1) & (2) poles should lie in the following region



1(d) A 1000 VA, 440/220 V single-phase two-winding transformer is connected as autotransformer to supply a load at 440 V from a supply voltage of 660 V ac mains. Draw the schematic diagram of the autotransformer with proper labelling. If the full load unity power factor (pf) efficiency of the two-winding transformer is 96.2%, what will be the full-load efficiency of the autotransformer at 0.85 pf lagging? Also find the maximum primary and secondary currents of the autotransformer. 12

Sol: VA rating of two winding transformer (kVA_{TW}) = 1000 VA

Voltage rating of two winding transformer = 440/220V

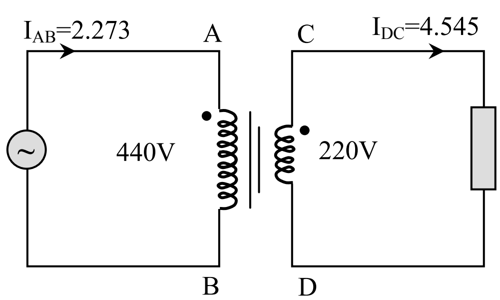
Voltage rating of auto transformer = 440/660 V

$\%\eta_{(\text{TW TF at FL \& UPF})} = 96.2\%$

$\%\eta_{\text{Auto TF at new FL \& 0.85 lag}} = ?$

Maximum primary current of auto transformer, $I_{1\max\,auto} = ?$

Maximum secondary current of auto transformer $I_{2\max\,auto} = ?$



Current rating of winding 'AB' = $\dfrac{\text{VA rating}}{\text{V rating}} = \dfrac{1000}{440} = 2.273$ A

Current rating of winding 'DC' = $\dfrac{\text{VA rating}}{\text{V rating}} = \dfrac{1000}{220} = 4.545$ A

In auto transformer to get voltage rating of 440/660V, two winding transformer should be connected in series additive polarity. Here two possibilities are there to get required voltage rating.

1. Connect A & D terminals with terminal 'B' is common to both primary and secondary.
2. Connect B & C terminals with terminal 'A' is common to both primary and secondary.

Here A & D terminals are connected with common 'B'

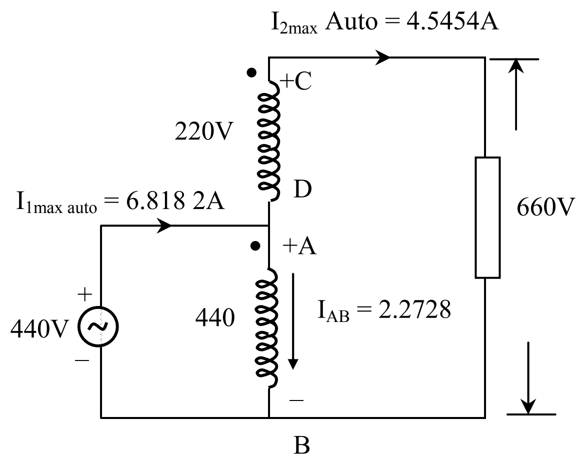


Fig. 440/660 V Auto transformer

$K_{auto} = \dfrac{LV}{HV} = \dfrac{440}{660} = 0.667$

Maximum kVA rating of auto transformer,

$$kVA_{auto\ max} = \left(\frac{1}{1-K_{auto}}\right) kVA_{TWTF}$$

$$= \left(\frac{1}{1-\frac{440}{660}}\right)\times 1000$$

$$= 3000\ VA$$

Maximum current drawn from supply, $I_{1max\ auto} = \dfrac{VA_{auto\,max}}{\text{Primary voltage rating of auto}}$

$$= \frac{3000}{440} = 6.8182\ A$$

Maximum current delivered by auto transformer, $I_{2\ auto\ max} = \dfrac{VA_{auto\ max}}{\text{Secondary voltage rating of auto}}$

$$= \frac{3000}{660} = 4.5454\ A$$

Apply KCL to know current in common winding of auto transformer = $I_{1\ max\ auto} - I_{2\ max\ auto}$

$$= 6.8182 - 4.5454$$

$$= 2.2728\ A$$

$$\eta_{(pu)\ TW\ TF\ FL\ \&\ UPF} = \frac{(VA)_{TW}\cos\theta_2}{(VA)_{TW}\cos\theta_2 + \text{Total loss in TWTF at FL}}$$

$$0.962 = \frac{1000 \times 1}{1000 + \text{Total loss in TW TF at FL}}$$

Total loss in two winding transformer at FL = $\frac{1000}{0.962} - 1000$ = 39.5 W

Total loss in two winding transformer at FL = Total loss in auto transformer at new FL = 39.5 W

$$\eta_{\text{auto TF at new FL 0.85 lag}} = \frac{(VA)_{auto}\cos\theta_2}{(VA)_{auto}\cos\theta_2 + \text{Total loss in auto at new FL}} \times 100$$

$$= \frac{3000 \times 0.85}{3000 \times 0.85 + 39.5} \times 100$$

$$= 98.47\%$$

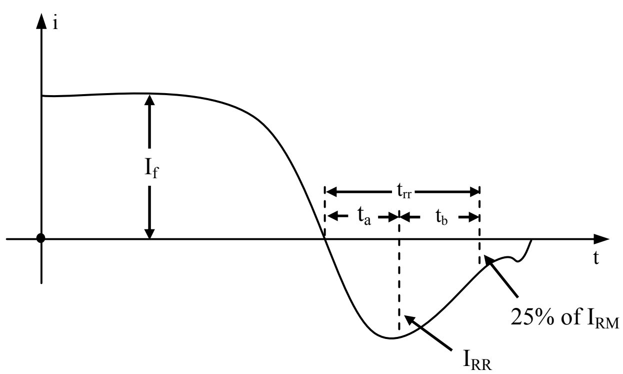
1(e) The reverse recovery time of a diode is t_{rr} = 6 μs, and the rate of fall of the diode current di/dt = 10 A/μs. If the softness factor SF = 0.5,

(i) Find the storage charge Q_{RR},

(ii) Find the peak reverse current I_{RR}, and

(iii) Draw the labelled reverse recovery characteristics. **12**

Sol: Reverse characteristics of diode can be drawn as follows



I_{RR} = maximum value of reverse current

t_{rr} = Reverse recovery time

$$t_{rr} = t_a + t_b$$

During time "t_a", the function get's recovered and during time t_b the semiconductor layers gets recovered.

Given data:

$$t_{rr} = 6\ \mu s$$

$$\frac{di}{dt} = 10\,A/\mu s$$

S-factor = S.F = 0.5

$t_{rr} = t_a + t_b$

$$S = \frac{t_b}{t_a} \Rightarrow 0.5 = \frac{t_b}{t_a}$$

$t_b = 0.5\ t_a$

$t_{rr} = t_a + 0.5\ t_a$

$\Rightarrow 1.5\ t_a = t_{rr} = 6$

$$t_a = \frac{6}{1.5} = 4\ \mu s$$

Peak reverse current (I_{RR}) = $t_a . \frac{di}{dt}$

$I_{RR} = 4 \times 10 = 40$ A

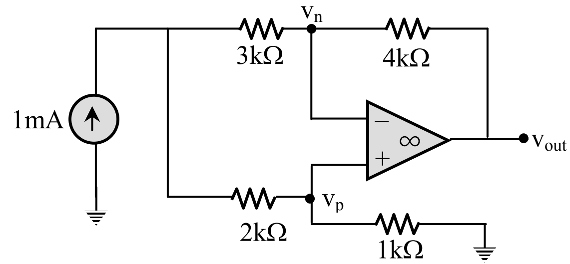
Storage charge (Q_{RR}) = $\frac{1}{2} I_{RR} \times t_{rr}$

$$= \frac{1}{2} \times 40 \times 6$$

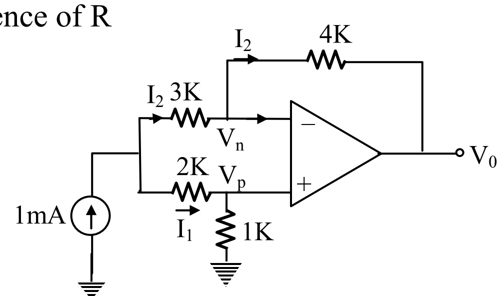
= 120 μc

= 120 μ coulombs

2(a) Determine the value of v_p, v_n and v_{out} in the circuit given below which uses an ideal operational amplifier. Find the resistance R that, when connected in parallel with the 1 mA source, will cause v_{out} to drop to half its value when R is not present. 20



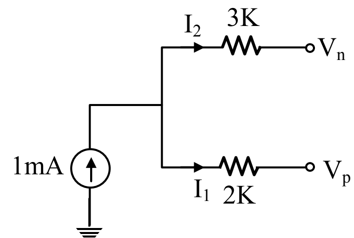
Sol: In absence of R



Since $V_d = 0$

$V_P = V_n$

$$I_1 = 10^{-3}\left(\frac{3K}{5K}\right) = 0.6 \text{ mA}$$



$I_2 = 0.4$ mA

$V_P = V_n = 10^3 . I_1 = 0.6V$

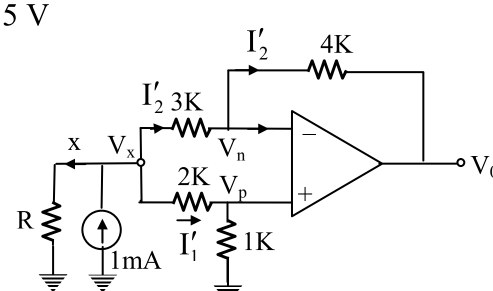
Then $V_n - V_0 = 4 \times 10^3\ I_2$

$$V_0 = V_n - 4(0.4)$$
$$= 0.6 - 1.6$$
$$= -1V$$
$$V_0 = -1V$$

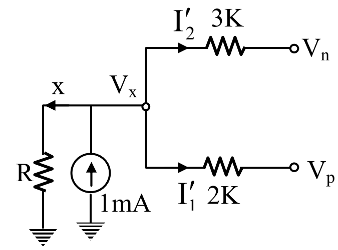
Now connect a resistor across 1 mA to get $V_0 = -0.5$ V

Since $V_d = 0$

$V_P = V_n$



$$I_1' = \left(10^{-3} - x\right)\left[\frac{3K}{5K}\right]$$



$I_1' = 0.6(10^{-3} - x)$

$I_2' = 0.4\ (10^{-3} – x)$

$V_P = 10^3 I_1'$

$= 10^3 \times 0.6\ (10^{-3} - x)$

$V_P = 0.6 - 600x = V_n$

Then $V_n - V_0 = 4 \times 10^3\ I_2'$

$V_n = V_0 + 4 \times 10^3 \times 0.4\ (10^{-3} - x)$

$= –0.5 + 1.6 - 1.6 \times 10^3 x$

$V_n = 1.1 - 1600\ x$

But $V_n = V_P$

$1.1 - 1600\ x = 0.6 - 600x$

$1.1 - 0.6 = 1600\ x - 600x$

$$\Rightarrow\ x = \frac{0.5}{1000} = 0.5\ \text{mA}$$

$V_P = 0.6 - 0.6 \times 0.5$

$V_P = 0.3\ V = V_n$

$V_x - V_P = 2\times 10^3\ I_1'$

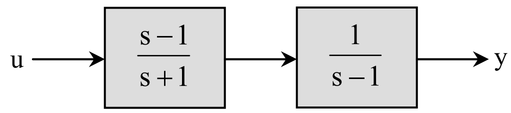
$V_x = V_P + 2\times10^3 \times 0.6\ (10^{-3} - 0.5 \times 10^{-3})$

$= 0.3 + 1.2 \times 0.5$

$= 0.9\ V$

$$\text{Then } R = \frac{V_x}{x} = \frac{0.9}{0.5\times10^{-3}} = 1.8\ k\Omega$$

2(b) Check the controllability and observability of the system shown in the figure given below. *u* is the input and *y* is the output. **20**



Sol: From the given block diagram

$$TF = \frac{Y(s)}{U(s)} = \frac{s-1}{(s+1)(s-1)} = \frac{s-1}{s^2-1}$$

$$\frac{Y(s)}{U(s)}\frac{G(s)}{G(s)} = \frac{s-1}{s^2-1}$$

Let $\frac{Y(s)}{G(s)} = s-1$

$Y(s) = sG(s) - G(s)$

$$y(t) = \frac{dg(t)}{dt} - g(t) \text{ ---------- (1)}$$

Let $\frac{G(s)}{U(s)} = \frac{1}{s^2-1}$

$s^2 G(s) - G(s) = U(s)$

$$\frac{d^2g(t)}{dt^2} - g(t) = u(t) \text{----------- (2)}$$

Let the state variables are

$x_1(t) = g(t)$ ---------------- (3)

$x_2(t) = \dot{g}(t)$ ---------------- (4)

Differentiating the above equations

$\dot{x}_1 = \dot{g}(t) = x_2(t)$ ------------ (5)

$\dot{x}_2 = \ddot{g}(t)$ ------------ (6)

using equation (2) in (6)

$\dot{x}_2 = g(t) + u(t)$

$\dot{x}_2 = x_1(t) + u(t)$ --------- (7)

From (5) and (6) state equations

$$\begin{bmatrix}\dot{x}_1 \\ x_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}u$$

using equations (3) and (4) in (1)

$y(t) = x_2 - x_1$ --------- (8)

$\therefore$ Output equation $y = \begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Controllability matrix $M = \begin{bmatrix} B & AB \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$|M| = -1$ non zero

$\therefore$ Controllable

Observability matrix $N = \begin{bmatrix} C \\ CA \end{bmatrix}$

$$CA = \begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$N = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$|N| = 1 - 1 = 0$

$\therefore$ Observable

$\therefore$ System is controllable but not observable

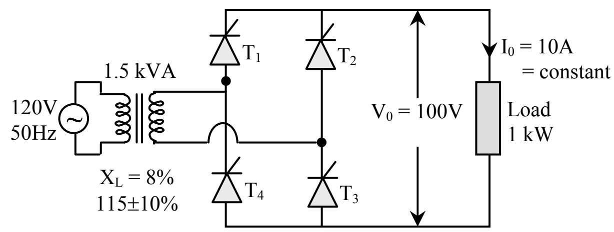
2 (c) A single-phase thyristor controlled bridge rectifier is supplying a dc load of 1 kW. A 1.5 kVA isolation transformer with a source side voltage rating of 120 V at 50 Hz is used. It has total leakage reactance of 8% based on its rating. The source voltage of nominally 115 V is in the range of ± 10%. Assuming load current is nearly constant, find

(i) The minimum turns ratio of the transformer, if the dc load voltage is to be regulated at constant value of 100 V,

(ii) The reduction in average load voltage due to commutation, and

(iii) The value of firing angle α when the source voltage is 115 + 10% V. **20**

Sol: (i)



Source voltage $V_S = 115 \pm 10\%$

$= 126.5$ V & 103.5 V

Output voltage $V_0 = \dfrac{2V_m}{\pi}\cos\alpha \qquad (\because I_0 = \text{constant})$

To get the minimum turns ratio $\cos\alpha$ should be maximum.

Therefore $\alpha = 0$

$$100 = \frac{2\times V_m}{\pi}$$

$\Rightarrow V_m = 50\pi$

$$\therefore \text{Turns ratio} = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{103.5\times\sqrt{2}}{50\pi} = 0.931$$

(ii) Reduction in average load voltage $= \dfrac{2\omega L_S}{\pi} I_0$

$X_L = \omega L_S = 8\% = 0.08$ pu

$$X_{pu} = X_{a.} \times \frac{MVA_b}{(kV_b)^2}$$

$kVA_b = 1.5$ kVA

$V_b = 120$ V

$$0.08 = X_a \times \frac{1.5}{1000\times\left(\frac{120}{1000}\right)^2}$$

$\Rightarrow X_a = 0.768\ \Omega = X_L = \omega L_S$

$$\therefore \text{Voltage drop} = \frac{2\times 0.768\times 10}{\pi} = 4.889 \text{ V}$$

(iii) $V_0 = \frac{2V_m}{\pi}\cos\alpha - \frac{2\omega L_S}{\pi} I_0$

$$100 = \frac{2\sqrt{2}\times 126.5}{\pi}\cos\alpha - 4.889$$

$$109.778 = \frac{2\sqrt{2}\times 126.5}{\pi}\cos\alpha$$

$$\cos\alpha = 0.920$$

$$\alpha = 22.93^{o}$$

3(a) A salient-pole synchronous motor (with negligible armature resistance and X_d = 23.2 Ω and X_q = 14.5 Ω./phase) can support a maximum load of 563 kW without field excitation.

This motor is now excited with nominal field current and the motor is loaded with a load torque of 3.82 kN-m. If the motor draws armature current at 0.8 power factor (leading), determine excitation emf and corresponding power angle (δ). **20**

Sol: Insufficient data in given question. If speed is given, the solution to this question is given below.

1. X_d = 23.2 Ω/ph. $\frac{1}{X_d}$ = 0.043, X_q = 14.5 Ω/ph, $\frac{1}{X_q}$ = 0.069

$$\frac{V^2}{2}\left(0.069 - 0.043\right) = \frac{563000}{3}$$

$$V^2\left(\frac{0.026}{2}\right) = \frac{563000}{3}$$

$$V^2 = \frac{2\times 563000}{3\times 0.026} \Rightarrow V = 3800 \text{ volts}$$

2. **Nominal field current:** What does it mean?

   It is assumed that nominal field current induces a E = V

   ∴ E = 3800 V

   This assumption cannot be valid since problem states that motor draws a leading current for which E > V.

3. Data about torque cannot be used since P & f are not given, we cannot find synchronous speed and hence we cannot find power developed. We can assume suitable values for P & f; say 4-poles & 50 Hz, and work out the problem.

Assuming P = 4, f = 50 Hz,

ω_s, synchronous speed (mech.rad/sec) = $\frac{4\pi f}{P} = \frac{4\pi \times 50}{4}$ = 50π rad/sec

∴ Mechanical power developed = (3820) 50π watts

= 6×10^5 watts (for all 3 phases)

= Power input to motor, since all losses are neglected.

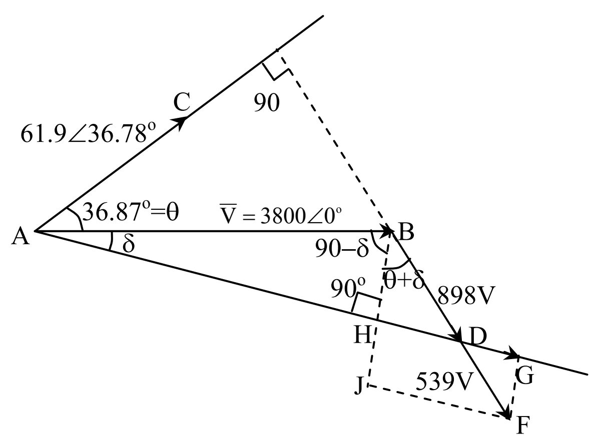
(resistance is given as zero, other losses are not mentioned)

Power input/ph, $2 \times 10^5 = 3800 \times I \times 0.85$

$$\Rightarrow \quad I = \frac{2\times10^5}{3800\times0.85} = 61.9 \text{ A}$$

∴ Armature current/ph = $61.9\angle36.87^o$ A

Phasor diagram:



AH = 3800cosδ

BH = 3800sinδ

Also BH = 898cos(θ + δ)

= 898cos(36.87° + δ)

3800sinδ = 898(0.8cosδ −0.6sinδ)

⇒ 0.8cosδ −0.6sinδ = 4.23sinδ

⇒ 4.83sinδ = 0.8cosδ

$$\Rightarrow \tan\delta = \frac{0.8}{4.83} = 0.1656$$

δ = 9.4° elec.

AH = 3749 V

HG = JF = 1437sin(36.87° + 9.4°) = 1038 V

E = 3749 + 1038 = **4786 V.**

3(b) (i) Fourier transform of a periodic signal is given as

$$X(j\omega) = j\delta\left(\omega - \frac{\pi}{3}\right) + 2\delta\left(\omega - \frac{\pi}{7}\right).$$

Determine the fundamental angular frequency and the Fourier series coefficients. Then determine the corresponding time signal.

(ii) Determine the Laplace transform and the ROC for the signal $x(t) = e^{at}\,u(t-k)$. **20**

Sol: (i) FT of a periodic signal

$$X(\omega) = j\delta\left(\omega - \frac{\pi}{3}\right) + 2\delta\left(\omega - \frac{\pi}{7}\right)$$

Fundamental frequency $\omega_0 = \text{GCD of } \frac{\pi}{3}, \frac{\pi}{7}$

$$= \frac{\pi}{21}$$

EFS $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

$\downarrow$ FT

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0) \quad \ldots\ldots\ldots\ldots\ldots (1)$$

$$X(\omega) = j\delta\left(\omega - \frac{7\pi}{21}\right) + 2\delta\left(\omega - \frac{3\pi}{21}\right)$$

$\downarrow$ 7^{th} harmonic $\quad$ $\downarrow$ 3^{rd} harmonic

By comparision with equation (1)

$2\pi C_7 = j \qquad 2\pi C_3 = 2$

$C_7 = \frac{j}{2\pi} \qquad C_3 = \frac{1}{\pi}$

$$\therefore\ x(t) = C_3 e^{j(3)(\pi/21)t} + C_7 e^{j(7)(\pi/21)t}$$

$$= \frac{1}{\pi} e^{j(\pi t/7)} + \frac{j}{2\pi} e^{j\pi t/3}$$

(ii) $x(t) = e^{at}\,u(t-k)$

$$x(t) = e^{a(t-k+k)}u(t-k)$$

$$= e^{ak}e^{a(t-k)}u(t-k) \qquad x(t-t_0) \xleftrightarrow{LT} e^{-st_0}X(s) \text{ with ROC} = R$$

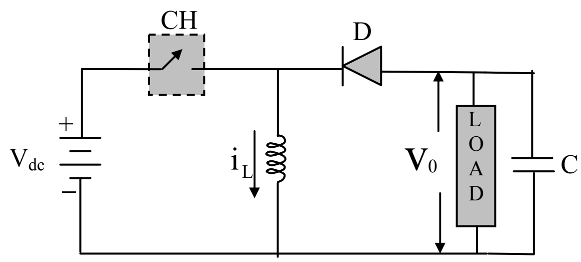
$$LT[x(t)] = X(s) = e^{ak}\left[\frac{e^{-sk}}{s-a}\right];\ \text{ROC: } Re\{s\} > Re\{a\}$$

3(c) A Buck-Boost converter is operating at 20 kHz with Inductor L = 50 μH. The output capacitor C is sufficiently large and source voltage V_d = 15 V. The output is to be regulated at 10 V and the converter is supplying a load of 10 W. Find

(i) The duty ratio D, and

(ii) Maximum value of Inductor current. **20**

Sol:



frequency (f) = 20 kHz

Inductance (L) = 50 μH

Supply voltage (V_d) = 15 V

Output voltage (V_0) = 10 V

Load power (P_0) = 10 W

Load current $= \frac{P_0}{V_0} = \frac{10}{10} = 1\text{ A}$

Load resistance (R) $= \frac{V_0}{I_0} = \frac{10}{1} = 10\ \Omega$

Inductor current varies with slope of $\frac{V_s}{L}$ during ON time of switch and $\frac{V_0}{L}$ during OFF time of switch.

The slope of current increasing is $\frac{15}{L}$ and slope of decrement of current is $\frac{10}{L}$. So, current takes 1.5 times of ON time to decrease and come to steady state.

$T_{ON} + T_{OFF} = T$

$T_{ON} + 1.5\ T_{ON} = T$

$2.5\ T_{ON} = T$

$2.5\ DT = T$

$D = 0.4$

Any duty cycle more than 0.4 is not suitable for this operation.

Critical value of duty cycle can be obtained as follows.

$$\frac{2L}{RT} = (1 - D_{cr})^2$$

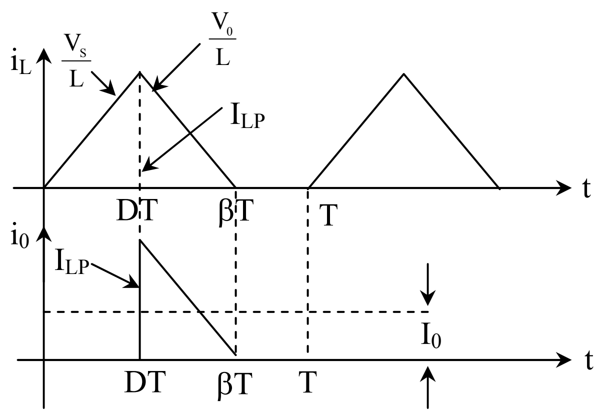
$$\frac{2 \times 50 \times 10^{-6}}{10 \times 50 \times 10^{-6}} = (1 - D_{cr})^2$$

$D_{cr} = 0.5527$

Any duty cycle more than 0.5527 makes inductor current to be continuous.

As per earlier conclusion duty cycle cannot be more than 0.4. Hence conduction is discontinuous.

Discontinuous Conduction:



As the slope of ascending is 1.5 times than descending, it takes 1.5 times of turn on time (DT) to get current to zero.

$DT = T_{ON}$ $\quad (\beta - D)T = T_{OFF}$

$(\beta - D)T = 1.5\ DT$

$\beta = 2.5\ D$

Peak value of inductor current $(I_{LP}) = \frac{V_s}{L} DT$

Average value of output current $(I_0) = \frac{\frac{1}{2} I_{LP} (\beta - D)T}{T}$

$$I_0 = \frac{1}{2} I_{LP} (\beta - D)$$

$$I_0 = \frac{1}{2} \frac{V_s}{L} DT (\beta - D)$$

$$1 = \frac{1}{2} \times \frac{15}{50\times10^{-6}} \times D \times 50 \times 10^{-6} \times (2.5D - D)$$

$$1 = \frac{15}{2} \times 1.5\ D^2$$

$$\Rightarrow D = 0.2981$$

Duty ratio D = 0.2981

Maximum value of inductor current

$$(I_{LP}) = \frac{V_s}{L} DT$$

$$= \frac{15}{50\times10^{-6}} \times 0.2981 \times 50 \times 10^{-6}$$

$$= 4.472\ A$$

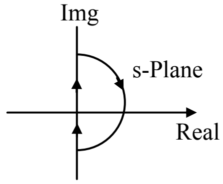
4(a) The open loop transfer function of a unity feedback system is given by

$$G(s)H(s) = \frac{K}{(s+20)(s^2-2s+1)}$$

Use Nyquist stability criteria to find the range of K for closed loop stability. **20**

Sol: $G(s)H(s) = \frac{k}{(s+20)(s^2-2s+1)} = \frac{k}{(s+20)(s-1)^2}$

Nyquist contour is shown below



Mapping of +ve imaginary axis of the Nyquist Contour.

Substitute $s = j\omega \quad 0 \le \omega \le \infty$

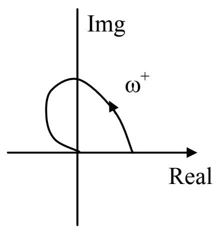
$$G(j\omega)H(j\omega) = \frac{k}{(j\omega+20)(j\omega-1)^2}$$

$$M = |G(j\omega)H(j\omega)| = \frac{k}{\sqrt{(\omega^2+400)(\omega^2+1)^2}}$$

$$\phi = \angle G(j\omega)H(j\omega) = -\left[\tan^{-1}\frac{\omega}{20} + 2\left[180^\circ - \tan^{-1}\omega\right]\right]$$

$$\phi = 2\tan^{-1}\omega - \tan^{-1}\frac{\omega}{20}$$

ω	Magnitude (M)	Phase angle (ϕ)
0	$\frac{k}{20}$	0°
⋮		
∞	0	90°

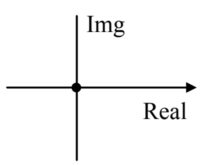


Mapping of radius 'R' semicircle of the Nyquist contour:

Substitute: $s = \underset{R\to\infty}{Lt}\ Re^{j\theta}$ θ is from 90° to 0° to –90°

$$G(Re^{j\theta})H(Re^{j\theta}) = \frac{k}{(Re^{j\theta}+20)(Re^{j\theta}-1)^2} \cong 0$$

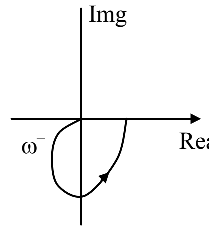
This section merges with the origin.



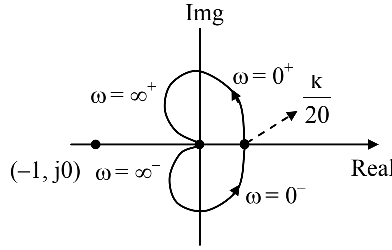
Mapping of the negative imaginary axis of Nyquist contour

Substitute $s = j\omega$ $\quad -\infty \le \omega \le 0$

This section becomes the mirror image of the positive imaginary axis and is drawn such that the Nyquist plot is symmetrical with respect to real axis.



Combining all the above three sections the Nyquist plot is shown below.



N = 0 from the plot

P = 2 (from the given G(s) H(s))

N= P– Z

Z = P – N = 2 – 0

Z = 2

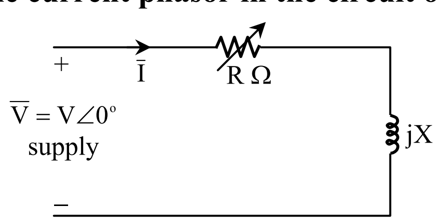
Closed loop system is unstable with two right half of s-plane poles.

For all values of k system is unstable.

4(b) Draw and elaborate (with appropriate mathematical justification) the graphical locus of induction motor (voltage, current and power) for a complete range of slip from approximate equivalent circuit model. Justify its circular nature for naming it as circle diagram of induction motor.

Also, state and explain with the help of the circle diagram, how to obtain rotor/stator copper losses, torque and slip at any arbitrary point on circle diagram. 20

Sol: 1. Locus of the tip of the current phasor in the circuit of fig. 1:



R varies from ∞ to +∞.

$\overline{V}$, X are constants

Fig. 1

$$\bar{I} = \frac{V}{R + jX} = \frac{V(R - jX)}{R^2 + X^2} = \frac{VR}{R^2 + X^2} - j\frac{VX}{R^2 + X^2} = I_x - jI_y.$$

The phasor $\bar{I}$ is shown in fig. 2.

For R = 0, $\bar{I} = -j\dfrac{V}{X}$

For R = ±∞, $\bar{I} = 0$.

What is the locus of the tip of the $\bar{I}$ phasor as R varies?

2. Proof that locus of (I_x, I_y) point is a circle:

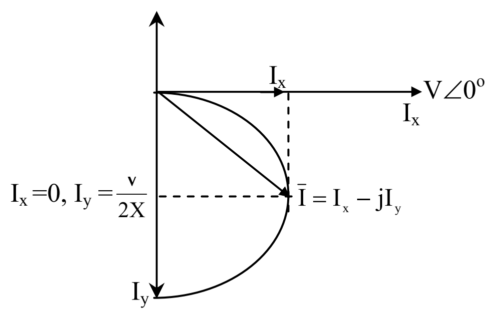


Fig. 2

We have $I_x^2 + I_y^2 = \dfrac{V^2}{R^2 + X^2}$ ……………. (1)

$\therefore\ I_x^2 + I_y^2 - \dfrac{V^2}{R^2 + X^2} = 0$ ……………. (2)

Adding $\dfrac{V^2}{4X^2}$ to both sides, we get

$I_x^2 + I_y^2 - \dfrac{V^2}{R^2 + X^2} + \dfrac{V^2}{4X^2} = \dfrac{V^2}{4X^2}$ ………… (3)

Now, $\left(I_y - \dfrac{V}{2x}\right)^2 = I_y^2 + \dfrac{V^2}{4X^2} - 2\dfrac{V}{2x}I_y$

$$= I_y^2 + \frac{V^2}{4X^2} - \frac{V}{X}\frac{VX}{R^2 + X^2}$$

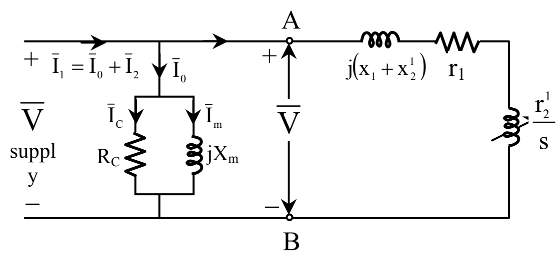
$$= I_y^2 + \frac{V^2}{4X^2} - \frac{V^2}{R^2 + X^2}$$

∴ (3) can be written as

$$I_x^2 + \left(I_y - \frac{V}{2X}\right)^2 = \left(\frac{V}{2X}\right)^2 \ \ldots\ldots\ldots\ldots..\ (4)$$

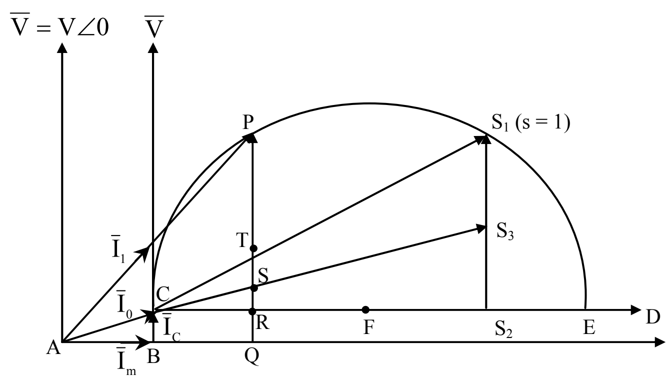
The locus of the point (I_x, I_y) (which is the tip of the $\bar{I}$ phasor of fig. 1. as R varies is a circle with center ($I_x = 0$, $I_y = \frac{V}{2X}$) and radius $\left(\frac{V}{2X}\right)$. This is the mathematic basis for the circle diagram of the 3-phase induction motor.

3. Now consider the approximate equivalent circuit/ph of a 3-phase induction motor.



The circuit to the right of the points A and B is a constant reactance variable resistance series circuit, to which a constant voltage is applied. The locus of the tip of the phasor current $\bar{I}_2$ is therefore a circle, with radius $\frac{V}{2X}$ and center $\frac{V}{2X}$, 0 where $X = (x_1 + x_2^1)$.

4. Construction of the circle diagram using fig. 3:



4.1 Draw $\bar{V} = V\angle 0^o$. (It is customary to draw $\bar{V}$ vertically).

4.2 Show $\bar{I}_c = \left(\frac{\bar{V}}{R_C}\right)$ and $\bar{I}_m = \left(-\frac{j\bar{V}}{X_m}\right)$. Show $\bar{I}_0 = \bar{I}_m + \bar{I}_C$.

4.3 From point C, show $\bar{V}$ (for convenience).

4.4 Draw the line CD lagging $\overline{V}$ by 90^o. On this line, mark the point E such that E = $\frac{V}{X}$.

Mark F on CD such that CF = $\frac{V}{2X}$

4.5 The locus of the tip of $\overline{I}_2$ is a circle with center F and radius FC. Draw the circle.

4.6 For any point P on the circle, $\overline{CP} = \overline{I}_2$ and $\overline{OP} = \overline{OC} + \overline{CP} = \overline{I}_1$.

4.7 The full circle can be completed, lower portion representing generation action.

5. Rotor/stator copper losses, torque, and slip at any point P on the circle:

5.1 At P, draw the vertical PQ

$\overline{QP}$,is the component of $\overline{I}_1$ in phase with $\overline{V}$. So to a suitable scale, QP represents the power input.

QR = BC = core losses

$\therefore$ RP = Mechanical power developed, (which includes friction losses) + stator and rotor copper losses.

5.2 From a rotor blocked test, locate the point S_1 on the circle for an applied voltage = rated voltage. At the point S_1; slip = 1.

The rotor is not rotating. There is no mechanical power.

Draw the vertical S_1 S_2 to give the stator copper losses + rotor copper loss at rotor blocked condition with applied voltage = rated value.

5.3 Locate the point S_3 on S_1S_2 such that S_1S_3 = rotor copper loss and S_3S_2 = stator copper loss.

$$\frac{S_1S_2}{S_3S_2} = \frac{r_2^1}{r_1}$$

5.4 Join CS_1 and CS_3

5.5 At the point P, PT = Mechanical power developed (output since mech losses are included in output).

TS = rotor copper losses corresponding to point of operation P,

SR = stator copper losses corresponding to P; and RQ = core losses (as given earlier)

(All these statements can be proved)

5.6 Torque and slip:

PS = PT + TS = Mech output + rotor copper loss

= Rotor power input

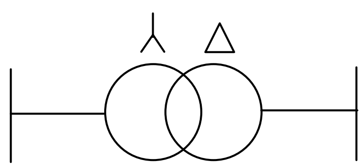
$$\text{Slip } s = \frac{\text{Rotor copper losses}}{\text{Rotor input}} = \frac{TS}{PS}$$

$$\text{Torque} = \frac{\text{Rotor input}}{\omega_S}$$

$$= \frac{PS}{\omega_S} \; (\omega_S = \text{synchronous speed, mech r/sec}).$$

4(c) Draw the wiring diagram showing currents for power and relaying circuit used for protecting a transformer of the rating 25 MVA, 220 Y/13.8 Δ kV, X = 10%. The transformer has a short-term overload capacity of 30 MVA. You are required to use CTs with common turns ratios such as 50/5A, 100/5A, 150/5A, 1000/5A, 1200/5A. If needed, auxiliary CT of adequate turns ratio may be used. **20**

Sol: The full load current in primary



$$I_{L1} = \frac{25\times10^6}{\sqrt{3}\times220\times10^3} = 65.6 \text{ A}$$

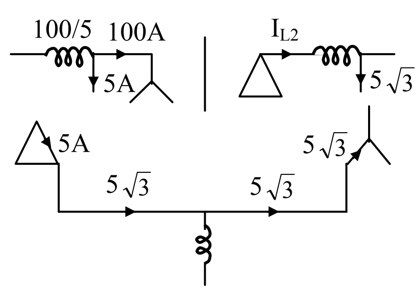
The full load current in secondary

$$I_{L2} = \frac{25\times10^6}{\sqrt{3}\times13.8\times10^3} = 1045.92 \text{ A}$$

The primary CT ratio is 100/5

Since primary of power transformer is connected so primary side CT's must be connected in delta.

Since secondary of power transformer is Δ connected, so secondary side CT's must be connected in Y



$$\sqrt{3}V_{L1}\, I_{L1} = \sqrt{3}\, V_{L2}\, I_{L2}$$

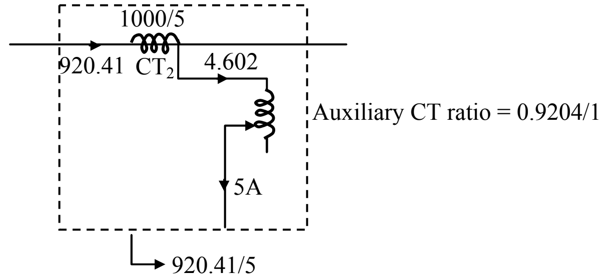
$$220 \times 100 = 13.8\, I_{L2}$$

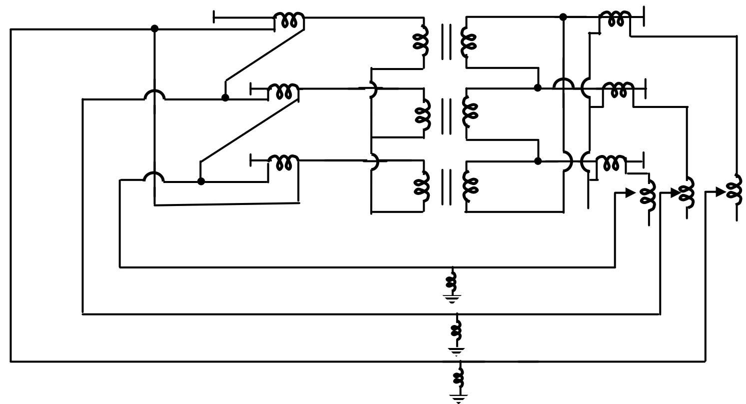
$$I_{L2} = 1594.262$$

$$\text{CT ratio on secondary side is} = \frac{1594.202}{5\sqrt{3}} = 920.41/5$$

So on secondary side CT ratio is 1000/5 but to get proper current auxiliary CT required

On secondary side of power transformer:


1000/5
920.41
CT2
4.602
Auxiliary CT ratio = 0.9204/1
5A
920.41/5



SECTION-B

5(a) The transfer function of a linear system is given by $G(s) = \frac{10}{(s+1)(s+2)}$. The sinusoidal steady state response of the system to an input is given by 1 + sin(t – 60°) + 5 sin (2t – 45°). Determine the input. **12**

Sol: Given output y(t) = 1 + sin(t – 60°) + 5 sin (2t – 45°)

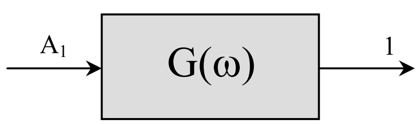
As the system is linear, there must be three different inputs to get the given output.

$x(t) = A_1 + A_2\sin(t + \phi_1) + A_3\sin(2t + \phi_2)$

$$G(s) = \frac{10}{(s+1)(s+2)}\ ; \ G(\omega) = \frac{10}{(j\omega+1)(j\omega+2)}$$

$$|G(\omega)| = \frac{10}{\sqrt{(1+\omega^2)(\omega^2+4)}}$$

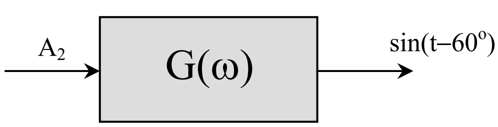
$$\angle G(\omega) = -\left[\tan^{-1}\left(\frac{\omega}{1}\right) + \tan^{-1}\left(\frac{\omega}{2}\right)\right]$$



$$|G(\omega)|_{\omega=0} = \frac{10}{\sqrt{(1+0)(0+4)}} = 5$$

$\therefore A_1 \times 5 = 1$

$\therefore A_1 = \frac{1}{5}$



$$|G(\omega)|_{\omega=1} = \frac{10}{\sqrt{(2)(5)}} = \sqrt{10}$$

$$A_2\sqrt{10} = 1 \Rightarrow A_2 = \frac{1}{\sqrt{10}}$$

$\angle G(\omega)|_{\omega=1} = -\left[\tan^{-1}\left(\frac{1}{1}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right]$

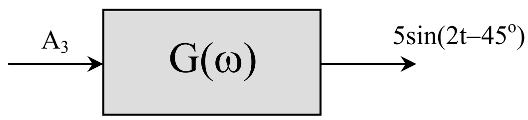
$= -\left[45^o + 26.56^o\right]$

$= -71.56^o$

$\phi_1 - 71.56 = -60^o \Rightarrow \phi_1 = -60^o + 71.56^o$

$\therefore \phi_1 = 11.56^o$

$\therefore$ Input $= \frac{1}{\sqrt{10}}\sin(t + 11.56^o)$



$|G(\omega)|_{\omega=2} = \frac{10}{\sqrt{(1+2^2)(2^2+4)}} = \frac{10}{\sqrt{5\times 8}} = \frac{10}{\sqrt{40}} = \frac{\sqrt{10}}{2}$

$A_3\frac{\sqrt{10}}{2} = 5$

$\therefore \quad A_3 = \sqrt{10}$

$\angle G(\omega)|_{\omega=2} = -\left[\tan^{-1}(2) + \tan^{-1}\left(\frac{2}{2}\right)\right]$

$= -(63.434^o + 45^o)$

$= -108.43^o$

$\phi_2 - 108.43^o = -45^o$

$\therefore \phi_2 = 63.43^o$

$\therefore$ Input is $\sqrt{10}\sin(2t + 63.43^o)$

$\therefore$ Total input of the system is

$x(t) = \frac{1}{5} + \frac{1}{\sqrt{10}}\sin(t + 11.56^o) + \sqrt{10}\sin(2t + 63.43^o)$

5(b) Draw phasor diagram of an over-excited salient-pole synchronous motor having armature resistance R_a, d-axis and q-axis reactances X_d and X_q respectively. Also prove, for lagging power factor

$$\tan(\phi-\delta)=\frac{V_t\sin\phi-I_aX_q}{V_t\cos\phi-I_aR_a}$$

Where V_t is the terminal voltage applied to motor, ϕ being the power factor angle, δ is power angle and I_a is armature current. **12**

Sol: Overexcited salient pole synchronous motor with R_a, X_d, X_q:

$\overline{V}-R_a\overline{I}-\overline{I}_q jX_q-\overline{I}_d jX_d=\overline{E}$

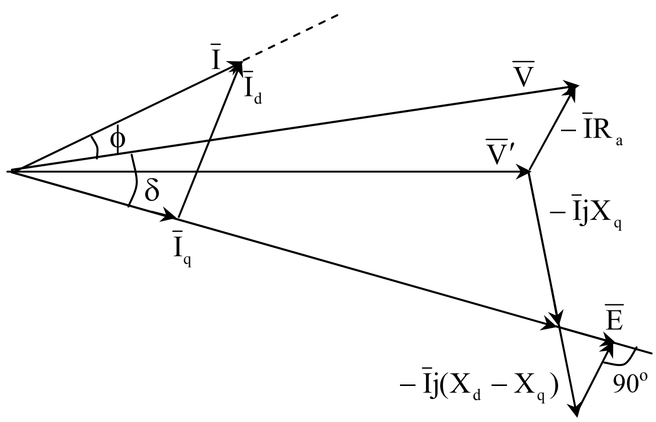
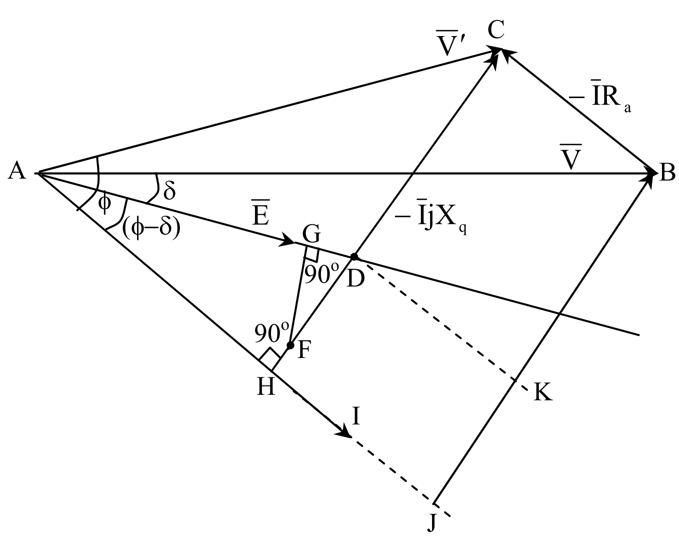


Fig. Phasor diagram with $\overline{I}$ leading $\overline{V}$ by ϕ

Phasor diagram with $\overline{I}$ lagging $\overline{V}$:



$\overline{DF} = \bar{I}[j(X_d - X_q)]$

BJ = V$\sin\phi$

CD = BK = IX_q

$\therefore$ KJ = DH = V$\sin\phi$ – IX_q

But from fig, $\tan(\phi - \delta) = \frac{DH}{AH} = \frac{V\sin\delta - IX_q}{AH}$

We must show that AH = V$\cos\phi$ – IR_a

But from fig, AJ = V$\cos\phi$.

JH = BC = IR_a (in magnitude)

$\therefore$ AH = V$\cos\phi$ – IR_a

Hence, $\tan(\phi - \delta) = \frac{DH}{AH} = \frac{V\sin\delta - IX_q}{V\cos\phi - IR_a}$

5(c) A flyback converter has the following circuit parameters:

$V_s = 24$ V

$N_1/N_2 = 3$

$L_m = 500$ μH

R = 5 Ω

C = 200 μF

f = 25 kHz

$V_0 = 10$ V

Find

(i) The average magnetizing current, and

(ii) The critical value of magnetizing inductor **12**

Sol: Given data,

$V_s = 24$ V, $\frac{N_1}{N_2} = 3$

$L_m = 500$ μH, R = 5Ω

C = 200 μF, f = 25 kHz

$V_0 = 10$ V

(i) Average magnetizing current (I_m)

$$I_0 = \frac{V_0}{R} = \frac{10}{5} = 2$$

Fly back converter operation is similar to buck boost converter

$$\frac{V_0}{V_s} = \frac{N_2}{N_1}\frac{D}{(1-D)}$$

$$\frac{D}{1-D} = \frac{N_1}{N_2}.\frac{V_0}{V_s} = (3).\frac{10}{24}$$

$$\frac{D}{1-D} = \frac{5}{4}$$

$$\Rightarrow D = \frac{5}{9} = 0.555$$

Average value of magnetizing current $= \frac{I_0}{1-D}\left(\frac{N_2}{N_1}\right)$

$$= \frac{2}{1-0.555}\left(\frac{1}{3}\right)$$

$= 1.5$ A

(ii) Critical value of inductor current can be obtained by $\frac{2L_{cr}}{RT} = (1-D)^2$

$$\Rightarrow \frac{2\times L_{cr}}{5\times 40\times 10^{-6}} = (1-0.555)^2$$

$L_{cr} = 19.75\ \mu$H

5(d) A 220 kV three-phase transmission line is 90 km long. The resistance is 0.1 Ω/km and the inductance is 1.0 mH/km. Use the short transmission line model to find

(i) Voltage at the sending end, and

(ii) Voltage regulation at the sending end. **12**

Sol: (i) (Data insufficient)

To calculate voltage @ sending end either the rating of transmission line or load current should be given.

(ii) Sending end voltage regulation = 0

When load is changed from no load to full load, receiving end voltage is changed but not sending voltage

i.e., $(V_s)_{\text{full load}} = (V_s)_{\text{no load}}$

$\therefore (V_{reg})_{\text{sending end}} = 0$

5(e) A 12-bit dual-slope ADC utilizes a 1 MHz clock and has V_{ref} = 10 V. Its analog input voltage is in the range of 0 to – 10 V. Find out the time required to convert an input signal equal to the full-scale value. Also find the integrator time constant if the peak voltage reached at the output of the integrator is 10 V. 12

Sol: $t_A = (2^{n+1} - 1)\, T_{c|k}$

$= (2^{12+1} - 1)\dfrac{1}{f_{c/k}}$

$= \dfrac{(2^{13} - 1)}{1\times 10^6}$

$= \dfrac{8191}{10^6}$

$= 8191 \times 10^{-6}$

$= 8.191\times 10^{-3}$

$= 8.191$ ms

$\cong 8.2$ ms

6(a) A 50 Hz, 4-pole turbogenerator rated 500 MVA, 22 kV has an inertia constant of 7.5 MJ/MVA. Find

(i) Rotor acceleration, if the input to the generator is suddenly raised to 400 MW for an electrical load of 350 MW,

(ii) The speed of rotor in rpm, if the rotor acceleration calculated in part (i) is constant for a period of 10 cycles, and

(iii) The change in torque angle δ in elect.degrees. 20

Sol: Given,

$f = 50$ Hz

$s = 500$ MVA

$V = 22$ kV

$H = 7.5$ MJ/MVA

(i) $P_e = 350$ MW

$P_m = 400$ MW

Swing equation in actual units and electrical degrees is

$$\frac{GH}{180^o f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{(500)\times(7.5)\times 10^6}{180^o \times (50)}.\frac{d^2\delta}{dt^2} = (400 - 350)° \times 10^6$$

$$\therefore \underset{\substack{\uparrow \\ \text{Rotor acceleration}}}{\frac{d^2\delta}{dt^2}} = 120\,\text{elec deg/sec}^2$$

(ii) $$\left(\frac{d^2\delta}{dt^2}\right)_{mech} = \frac{2}{P}\left(\frac{d^2\delta}{dt^2}\right)_{elec} = \frac{2}{4}(120)$$

$$\left(\frac{d^2\delta}{dt^2}\right)_{mech} = 60 \text{ mech deg/sec}^2$$

$$= \frac{60}{360} \text{ rotations/sec}^2$$

$$= \frac{\left(\frac{1}{6}\right)\text{rotations}}{\left(\frac{1}{60}\right)^2 \min^2}$$

$$\left(\frac{d^2\delta}{dt^2}\right)_{mech} = \frac{3600}{6} = 600 \text{ rotations/min}^2$$

$$\text{Time} = t = 10 \text{ cycles} = 10\times\frac{1}{50} = \frac{1}{5}8 = \frac{1}{5}\times\frac{1}{60}\min$$

$$t = \frac{1}{300}\min$$

Initial speed $N_0 = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500$ rpm

Final speed $N_1 = N_0 + \left(\frac{d^2\delta}{dt^2}\right)t$

$$= 1500 + 600 \times \frac{1}{300}$$

$N_1 = 1502$ rpm

(iii)change in torque angle is

$$\Delta\delta = \frac{1}{2}\alpha_e t^2$$

$$= \frac{1}{2} \times 120(\text{elec.deg/sec}^2) \times \left(\frac{1}{5}\right)^2 \text{sec}^2$$

$\Delta\,\delta = 2.4$ electrical degrees.

6(b) The full bridge inverter is used to produce a 50 Hz voltage across a series RL load using Bi-polar PWM. The dc input to the bridge is 200 V, the frequency modulation m_f is 21 and amplitude modulation m_a is 0.8. The load has resistance of R = 10 Ω and inductance L = 20 mH. Find

(i) The amplitude of fundamental voltage and current, and

(ii) Total harmonic distortion in load current. **20**

Assume harmonics (>25th order) are insignificant and normalized voltage is

$m_a = 1$		0.9	0.8	0.7	0.6	0.5
n = 1	1.0	0.9	0.8	0.7	0.6	0.5
$n = m_f$	0.6	0.71	0.82	0.92	1.01	1.15
$n = m_f \pm 2$	0.32	0.27	0.22	0.17	0.13	0.09

Sol: Full bridge inverter is operating with bipolar PWM.

DC input voltage (V_d) = 200 V

Frequency modulation (m_f) = 21

Amplitude modulation (m_a) = 0.8

Load: Resistance (R) = 10 Ω

Inductance (L) = 20 mH

Fundamental frequency (f) = 50 Hz

Inductive reactance at 50 Hz $= 2\pi \times 50 \times 20 \times 10^{-3}$

$= 6.28\ \Omega$

Impedance at fundamental frequency = $\sqrt{10^2 + 6.28^2}$

$= 11.8\ \Omega$

(i) Amplitude of fundamental voltage ($V_{01\,m}$) = m_a. V_d

$= 0.8 \times 200 = 160$ V

Amplitude of fundamental current ($I_{01\,m}$) = $\dfrac{V_{01\,m}}{Z}$

$= \dfrac{160}{11.8} = 13.56$ A

RMS value of fundamental current (I_1) = $\dfrac{13.56}{\sqrt{2}} = 9.59$ A

(ii) RMS value of 21st harmonic (V_{21}) = $0.82 \times 200 \times \dfrac{1}{\sqrt{2}}$

$= 115.9$ V

RMS value of 19th harmonic (V_{19}) = $0.22 \times 200 \times \dfrac{1}{\sqrt{2}}$

$= 31.31$ V

RMS value of 23rd harmonic (V_{23}) = $0.22 \times 200 \times \dfrac{1}{\sqrt{2}}$

$= 31.31$ V

Impedance of 21st harmonic (Z_{21}) = $\sqrt{R^2 + X_{21}^2}$

$= \sqrt{10^2 + (21 \times 6.28)^2} = 132.\ 25\ \Omega$

$Z_{19} = \sqrt{10^2 + (19 \times 6.28)^2} = 119.7\ \Omega$

$Z_{23} = \sqrt{10^2 + (23 \times 6.28)^2} = 144.78\ \Omega$

RMS value of 21st harmonic = $I_{21} = \dfrac{V_{21}}{Z_{21}}$

$$I_{21} = \frac{115.9}{132.25} = 0.8763 \text{ A}$$

$$I_{19} = \frac{V_{19}}{Z_{19}} = \frac{31.31}{119.7} = 0.2615 \text{ A}$$

$$I_{23} = \frac{V_{23}}{Z_{23}} = \frac{31.31}{144.78} = 0.216 \text{ A}$$

$$\text{RMS value of current} = \sqrt{I_1^2 + I_{19}^2 + I_{21}^2 + I_{23}^2}$$

$$= \sqrt{9.59^2 + 0.2615^2 + 0.8763^2 + 0.216^2}$$

$$I_{or} = 9.636 \text{ A}$$

$$\text{Total harmonic distortion} = \frac{\sqrt{I_{or}^2 - I_1^2}}{I_1} \times 100\% = \frac{\sqrt{9.636^2 - 9.59^2}}{9.59} \times 100\% = 9.79\%$$

6(c) Design a circuit that takes as input two 2-bit numbers, N_1 and N_2 for comparison and generates three outputs:

$N_1 = N_2$, $N_1 < N_2$ and $N_1 > N_2$. These three binary outputs are represented by F_{eq}, F_{lt} and F_{gt} respectively. Realize the output in Sum of Products (SoP) form. **20**

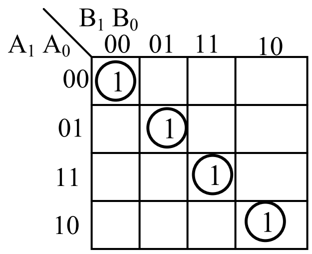
Sol: Let us consider N_1 is $A_1 A_0$ & N_2 is $B_1 B_0$

0	A_1	A_0	B_1	B_0	F_{eq}	F_{1+}	F_{gt}
0	0	0	0	0	1	0	0
1	0	0	0	1	0	1	0
2	0	0	1	0	0	1	0
3	0	0	1	1	0	1	0
4	0	1	0	0	0	0	1
5	0	1	0	1	1	0	0
6	0	1	1	0	0	1	0
7	0	1	1	1	0	1	0
8	1	0	0	0	0	0	1
9	1	0	0	1	0	0	1
10	1	0	1	0	1	0	0
11	1	0	1	1	0	1	0
12	1	1	0	0	0	0	1
13	1	1	0	1	0	0	1
14	1	1	1	0	0	0	1
15	1	1	1	1	1	0	0

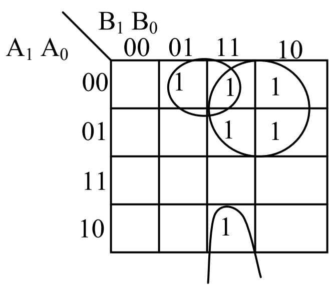
$F_{eq} = \sum m\,(0,5,10,15)$

$F_{it} = \sum m\,(1,2,3,6,7,11)$

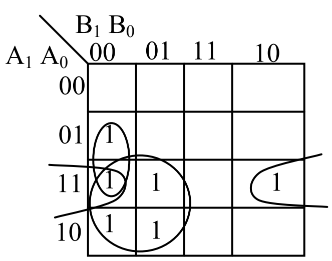
$F_{gt} = \sum m\,(4,8,9,12,13,14)$



$F_{eq}\ \ \bar{A}_1\bar{A}_0\bar{B}_1\bar{B}_0 + \bar{A}_1 A_0\bar{B}_1\bar{B}_0 + A_1 A_0 B_1 B_0 + A\bar{A}_0 B, \bar{B}_0$



$F_1 += \bar{A}_1 B_1 + \bar{A}_1\bar{A}_0 B_0 + \bar{A}_0 B_1 B_0$



$F_{gt} = A_1\bar{B}_1 = A_0\bar{B}_1\bar{B}_0 + A_1 A_0\bar{B}_0$

7(a) For a causal system specified by the transfer function $H(z) = \frac{z}{z-0.5}$

Determine the zero state response to the input

$r(k) = (0.8)^k u(k) + (2)^{k+1} u\{-(k+1)\}$. **20**

Sol: $H(z) = \frac{z}{z-0.5}$ ROC of H(z) is $|z| > 0.5$

Input $r(k) = (0.8)^k u(k) + (2)^{k+1} u\{-(k+1)\}$.

$\downarrow$ $|z| > 0.8$ $\qquad$ $\downarrow$ $|z| < 2$

Input ROC: $0.8 < |z| < 2$, ROC of H(z) is $|z| > 0.5$

$\therefore$ ROC of $Y(z) = 0.8 < |z| < 2$

$a^n u(n) \leftrightarrow \frac{1}{1-az^{-1}}$; $|z| > |a|$

$-a^n u(-n-1) \leftrightarrow \frac{1}{1-az^{-1}}$; $|z| < |a|$

Input $r(k) = (0.8)^k u(k) + (2)^{k+1} u\{-(k+1)\}$.

$\downarrow$ Z.T

$$R(z) = \frac{1}{1-0.8z^{-1}} - \frac{2}{1-2z^{-1}} ; 0.8 < |z| < 2$$

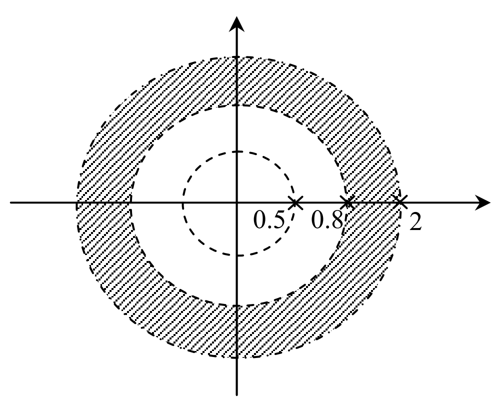
$$= \frac{1-2z^{-1} - 2 + 1.6z^{-1}}{(1-0.8z^{-1})(1-2z^{-1})}$$

$$= \frac{-1-0.4z^{-1}}{(1-0.8z^{-1})(1-2z^{-1})}$$

$Y(z) = R(z)H(z)$; ROC: $0.8 < |z| < 2$

$$Y(z) = \frac{-1-0.4z^{-1}}{(1-0.8z^{-1})(1-2z^{-1})(1-0.5z^{-1})}$$

$$= \frac{A}{1-0.5z^{-1}} + \frac{B}{1-2z^{-1}} + \frac{C}{1-0.5z^{-1}}$$



ROC of Y(z)

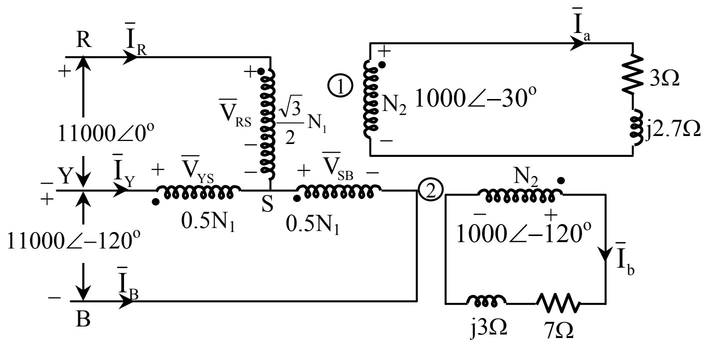
By partial fractions, $A = \frac{8}{3}$, $B = \frac{-8}{3}$; $C = -1$

Inverse Z-transform of Y(z)

$$y(n) = \frac{8}{3}(0.8)^n u(n) - \frac{8}{3}(2)^n [-u(-n-1)] - (0.5)^n u(n)$$

7(b) An unbalanced 2-ϕ, 1000 V, 50 Hz induction motor has unequal winding impedances $Z_a = 3 + j2.7$ and $Z_b = 7 + j3\ \Omega$. This motor is supplied by Scott-connected transformer combination from a 3-phase 11 kV system. Calculate phase currents I_a and I_b of the motor and line currents on 3-phase supply side. **20**

Sol:



$$\bar{V}_{RS} - \bar{V}_{YS} = 11000\angle 0^\circ$$

$$\bar{V}_{YS} = \bar{V}_{SB} = \frac{11000}{2}\angle -120^\circ$$

$$\bar{V}_{RS} = 11000\angle 0^\circ + \frac{11000}{2}\angle -120^\circ$$

$$= \frac{\sqrt{3}}{2} 11000\angle -30^\circ$$

$$\left[\frac{\sqrt{3}}{2} \times 11000\angle -30\right]\frac{N_2}{\frac{\sqrt{3}}{2}N_1} = 1000\angle -30^\circ$$

$$\frac{N_2}{N_1} = \frac{1}{11}$$

Motor phase currents:

$$\bar{I}_a = \frac{1000\angle -30^\circ}{3 + j2.7} = 247.8\angle{-72^\circ}\ \text{A}$$

$$\bar{I}_b = \frac{1000\angle -120^\circ}{7 + j3} = 131.3\angle{-143.2^\circ}\ \text{A}$$

Transformer currents::

$$\bar{I}_R = \frac{2}{\sqrt{3}}\frac{1}{11}(247.8\angle{-72°}) = 26\angle{-72°}\text{ A} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$\bar{I}_Y\left(\frac{N_1}{2}\right) - \bar{I}_B\left(\frac{N_1}{2}\right) = \bar{I}_b\, N_2$$

$$\bar{I}_Y - \bar{I}_B = \bar{I}_b\, N_2\left(\frac{2}{N_1}\right)$$

$$= \frac{2}{11}\bar{I}_b = \frac{2}{11}131.3\angle{-143.2^o}\text{ A}$$

$$\bar{I}_Y - \bar{I}_B = 23.8\angle{-143.2°} \quad \ldots\ldots\ldots\ldots(2)$$

$$\bar{I}_R + \bar{I}_Y + \bar{I}_B = 0 \quad \ldots\ldots\ldots\ldots\ldots(3)$$

From (1) & (3)

$$\bar{I}_Y + \bar{I}_B = -26\angle{-72°} = 26\angle{108°}$$

By solving equations (2) & (3)

$$\bar{I}_Y = 14.5\angle{158.8°}\text{ A}$$

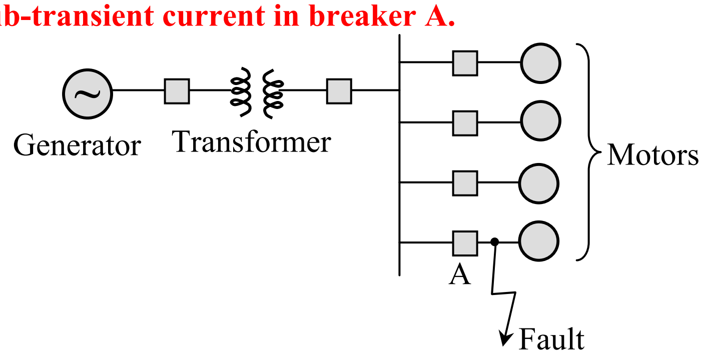
$$\bar{I}_B = 20.25\angle{74.2°}\text{ A}$$

$$\bar{I}_R = 26\angle{-72°}\text{ A}$$

7(c) A 25 MVA, 13.8 kV generator with $x_d'' = 15\%$ is connected through a 25 MVA, 13.8/6.9 kV transformer with leakage reactance of 10% to a bus which supplied four identical motors as shown in the figure. The sub-transient reactance X_d'' of each motor is 20% on base of 5 MVA, 6.9 kV. Find

(i) The sub-transient current in the fault, and

(ii) The sub-transient current in breaker A. **20**



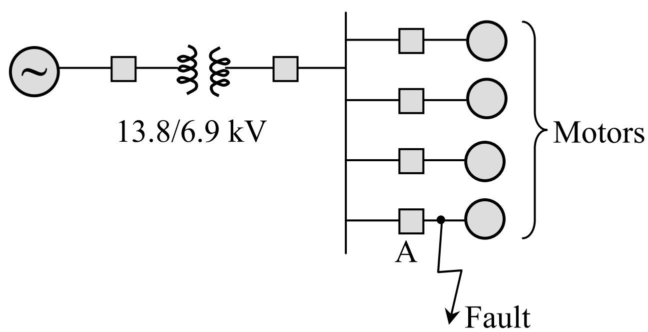
Sol: Given,

	VA-rating	V-rating	X″
Generator:	25 MVA	13.8 kV	15%
Transformer:	25 MVA	13.8 kV/6.9 kV	10%
Motor:	5 MVA	6.9 kV	20%

Network is:

$S_B = 25$ MVA $\quad$ $S_B = 25$ MVA

$V_B = 13.8$ kV $\quad$ $V_B = 6.9$ kV $\quad$ base quantities



$$\left.\begin{aligned}\text{let } S_B &= 25\,\text{MVA}\\ V_B &= 13.8\,\text{kV}\end{aligned}\right\} @\text{ generator}$$

The corresponding bases at other locations of power system shown above is represented on the circuit. For generator and transformer, base and rated quantities are same hence their pu reactances do not change. But for motor rated and base powers are not same. Hence for motor, new values of reactances should be calculated in its new values by using the formula.

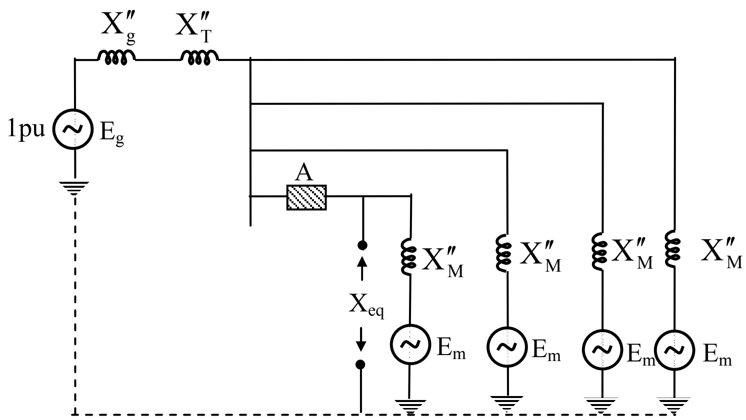
$$X_m(\text{new}) = X_m(\text{old}).\ \frac{S_{new}}{S_{old}}$$

$$= 20.\ \frac{(25)}{5}$$

$X_m(\text{new}) = 100\% = 1$ pu

Hence the reactances of motor adjusted in the new base of $S_B = 25$ MVA is 100%

Reactance diagram as seen from fault is (voltages sources are replaced by their equivalent reactance)



As seen from fault location equivalent reactance is

$$X_{eq} = (X''_n) \,||\, (X''_m) \,||\, (X''_m) \,||\, (X''_m) \,||\, (X''_T + X''_g)$$

$$= (1) \,||\, (1) \,||\, (1) \,||\, (1) \,||\, (0.15 + 0.1)$$

$X_{eq} = 0.125$

$\Rightarrow I_f$ (pu) (fault current) $= \dfrac{1}{X_{eq}} = \dfrac{1}{0.125} = 8$ pu

I_f (pu) = 8 pu

$\Rightarrow$ At motor location,

$$I_B(\text{base}) = \frac{S_B}{\sqrt{3}\,V_3} = \frac{25\times10^6}{\sqrt{3}\times6.9\times10^3} = 2.092 \text{ kA}$$

$\Rightarrow I_f$ (actual) $= I_f(\text{pu}) \times I_B$

$= 8 \times 2.092$

I_f (actual) = 16.73 kA

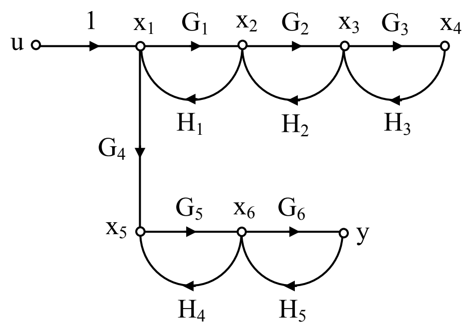
$\Rightarrow I_A = I_f - I_m = 8 - \dfrac{1}{1} = 7$ pu

$I_A = 7$ pu

$\therefore$ Current through circuit breaker A is

$I_A = 7 \times 2.092 = 14.64$ kA

8(a) Find the transfer function $\frac{Y(s)}{U(s)}$ using Mason's Gain formula. Also find $\frac{X_5(s)}{U(s)}$. **20**



Sol: Number of forward paths from U(s) to Y(s) = 1

Number of Loops = 5

Two non touching loops = 7

Three non touching loops = 2

$$\frac{Y(s)}{U(s)} = \frac{M_1\Delta_1}{\Delta}$$

Forward path gain

$M_1 = G_4 G_5 G_6$

Loops $L_1 = G_1H_1$

$L_2 = G_2H_2$

$L_3 = G_3H_3$

$L_4 = G_5H_4$

$L_5 = G_6H_5$

Two non touching loops $= L_1 L_3 = G_1 H_1 G_3 H_3$

$L_1 L_4 = G_1 H_1 G_5 H_4$

$L_1 L_5 = G_1 H_1 G_6 H_5$

$L_2 L_4 = G_2 H_2 G_5 H_4$

$L_2 L_5 = G_2 H_2 G_6 H_5$

$L_3 L_4 = G_3 H_3 G_5 H_4$

$L_3 L_5 = G_3 H_3 G_6 H_5$

The non touch loops $= L_1 L_3 L_4 = G_1 H_1 G_3 H_3 G_5 H_4$

$L_1 L_3 L_5 = G_1 H_1 G_3 H_3 G_6 H_5$

Four, five etc… non touching loops do not exist

$\therefore\ \Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1L_3 + L_1L_4 + L_1L_5 +$

$L_2 L_4 + L_2 L_5 + L_3 L_4 + L_3 L_5) - (L_1 L_3 L_4 + L_1 L_3L_5)$

$\Delta = 1 - (G_1H_1 + G_2H_2 + G_3H_3 + G_5H_4+G_6H_5) + (G_1 H_1 G_3 H_3+ G_1 H_1 G_5 H_4+ G_1 H_1 G_6 H_5 +$

$G_2 H_2 G_5 H_4+G_2 H_2 G_6 H_5+G_3 H_3 G_5 H_4+G_3 H_3 G_6 H_5)-(G_1 H_1 G_3 H_3 G_5 H_4 + G_1 H_1 G_3 H_3 G_6 H_5)$

$\Delta_1 = 1 - (L_2+ L_3)$

$\Delta_1 = 1- (G_2H_2 + G_3H_3)$

$$\frac{Y(s)}{U(s)} = \frac{M_1\Delta_1}{\Delta}$$

$$= \frac{G_4G_3G_6\left(1-(G_2H_2+G_3H_3)\right)}{\begin{aligned}&1-(G_1H_1+G_2H_2+G_3H_3+G_5H_4+G_6H_5)+(G_1\,H_1\,G_3\,H_3+\\&G_1\,H_1\,G_5\,H_4+G_1\,H_1\,G_6\,H_5+G_2\,H_2\,G_5\,H_4+\\&\quad G_2\,H_2\,G_6\,H_5+G_3\,H_3\,G_5\,H_4+G_3\,H_3\,G_6\,H_5)\\&-(G_1\,H_1\,G_3\,H_3\,G_5\,H_4+G_1\,H_1\,G_3\,H_3\,G_6\,H_5)\end{aligned}}$$

Number of forward paths form U(s) to X_5 (s) = 1

$$\frac{X_5(s)}{U(s)} = \frac{M_1\Delta_1}{\Delta}$$

Where $M_1 = G_4$

$\Delta_1 = 1 - (L_2 + L_3 + L_5) + (L_2 L_5 + L_3 L_5)$

$= 1- (G_2 H_2 + G_3 H_3 + G_6 H_5) + (G_2 H_2 G_6 H_5 + G_3 H_3 G_6 H_5)$

$$\frac{X_5(s)}{U(s)} = \frac{G_4\left(1-(G_2\,H_2+G_3\,H_3+G_6\,H_5)+(G_2\,H_2\,G_6\,H_5+G_3\,H_3\,G_6\,H_5)\right)}{\begin{aligned}&1-(G_1H_1+G_2H_2+G_3H_3+G_5H_4+G_6H_5)+(G_1\,H_1\,G_3\,H_3+\\&G_1\,H_1\,G_5\,H_4+G_1\,H_1\,G_6\,H_5+G_2\,H_2\,G_5\,H_4+\\&\quad G_2\,H_2\,G_6\,H_5+G_3\,H_3\,G_5\,H_4+G_3\,H_3\,G_6\,H_5)\\&-(G_1\,H_1\,G_3\,H_3\,G_5\,H_4+G_1\,H_1\,G_3\,H_3\,G_6\,H_5)\end{aligned}}$$

8(b) A 440 V, 50 Hz, 6-pole, Y-connected induction motor has following parameters per phase referred to the stator:

$R_s = R'_r = 0.3\ \Omega$, $X_s = X'_r = 1.0\ \Omega$ and $X_m = 40\ \Omega$

The nominal full load slip is 0.05.

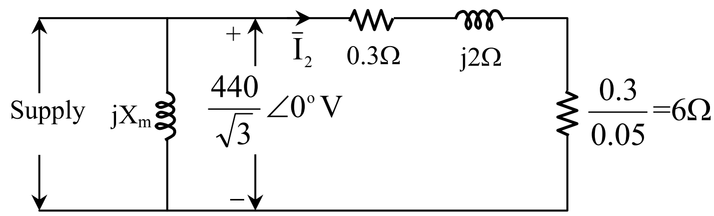
The motor is to be braked by plugging from its initial full load condition. Determine initial braking torque without braking resistor (R_B).

Also find the value of R_B so that braking current is limited to 1.5 times the full load current. What will be the corresponding braking torque as a ratio of full load torque?

Note: Assume braking resistor R_B is connected to rotor circuit. **20**

Sol: 6-pole, 50 Hz → $\omega_s = \dfrac{4\pi f}{P} = \dfrac{4\pi\times 50}{6} = \dfrac{100\pi}{3}$ mech rad/sec

From given data, approximate equivalent circuit per phase (without R_b) is shown below.



$$I_2 = \frac{440}{\sqrt{3}\sqrt{6.3^2+4}} = 38.43\ \text{A}$$

Corresponding developed torque (=load torque) $= \dfrac{3\times 38.43^2\times 6\times 3}{100\pi} = 253.9$ N-m

Now two of the supply lines are interchanged, reversing the direction of rotation of the stator mmf. (This is called plugging). The rotor now quickly comes to a stop, when the supply is to be removed.

Immediately after plugging, slip = 2 – 0.05 = 1.95.

New developed torque (which opposes rotor rotation)

$$= \frac{3\times\left(\dfrac{440}{\sqrt{3}}\right)^2(0.3)\times 3}{1.95\times 100\left[\left(0.3+\dfrac{0.3}{1.95}\right)^2+4\right]} = \frac{440^2\times 0.9}{195(4.2)} = 212.7\ \text{N-m}$$

Load torque opposes rotation, as always

Total braking torque = 212.7 + 253.9 = 466.6 N-m

Braking resistance R_b introduced into the rotor circuit at the same instant as plugging:

Before plugging, T_d = load torque = 253.9 N-m

Immediately after plugging, slip changes to 1.95, as before.

$$\text{New rotor current} = \frac{\left(\frac{440}{\sqrt{3}}\right)}{\sqrt{\left(0.3+\frac{0.3+R_b}{1.95}\right)^2+4}} = 1.5\times 38.43 = 57.65\text{ A (given)}$$

R_b can be found from the above and works out to 6.8 Ω.

New developed torque (which opposes rotation)

$$= \frac{3\times 57.65^2\times 7.1\times 3}{1.95\times 100\pi} = 346.7\text{ N-m.}$$

Adding the load torque, which also opposes rotation, total braking torque = 346.7 + 253.9 = 600 N-m.

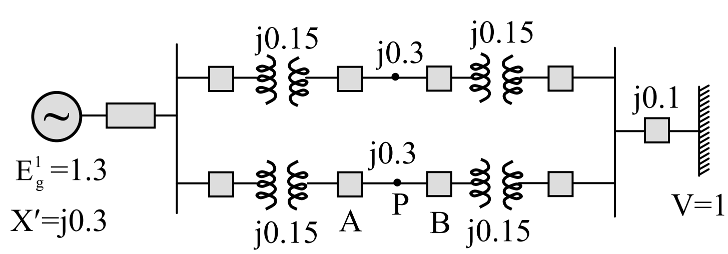
(Adding R_b both reduces the rotor current as well as increases braking torque).

8(c) A generator is connected by a double line to an infinite bus, the voltage of which is V = 1 pu as shown in the figure. Per unit values of reactances and voltages are indicated in the figure. A three-phase short circuit occurs at the point P. The circuit breakers A and B open simultaneously and remain open. The mechanical power supplied to the generator before the fault is P_m = 1 pu.

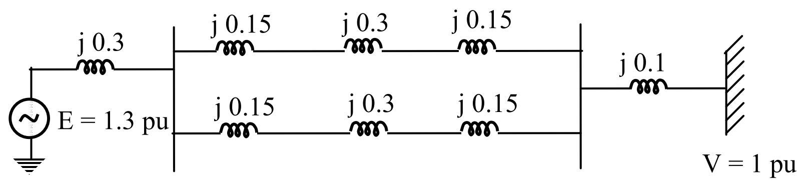
(i) Determine the electrical powers P_{e1}, P_{e2} and P_{e3} before, during and post the fault

(ii) Draw on the same graph, power angle curve for P_{e1}, P_{e2} and P_{e3}.

(iii) Calculate the angles δ_0, δ_1 and δ_{max} where δ_0 is the initial power angle, δ_1 is the post fault power angle and δ_{max} is the maximum power angle. **20**



Sol: **(i)** Before fault: (Reactance diagram)



$$P_{e_1} = \frac{EV}{X_{eq1}}$$

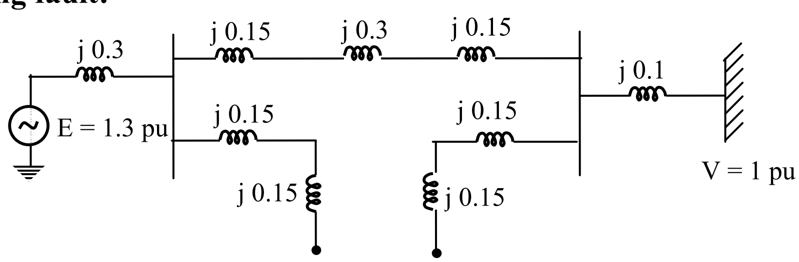
$$P_{e_1} = \frac{1.3 \times 1}{0.7}$$

$$P_{e_1} = 1.857$$

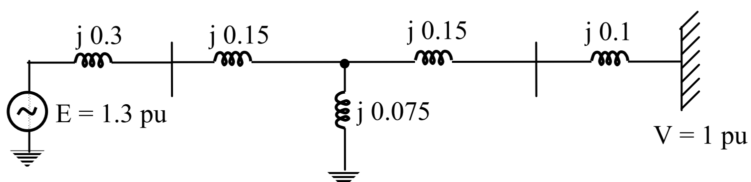
$$X_{eq1} = 0.3 + (0.15 + 0.3 + 0.15) \parallel (0.15 + 0.3 + 0.15) + 0.1$$

$$X_{eq1} = 0.7$$

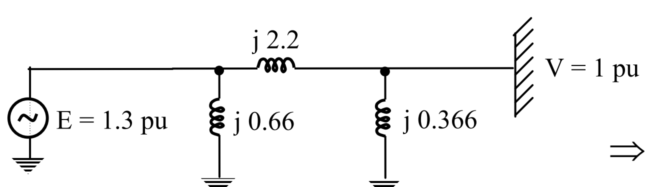
During fault:



⇓ Δ to Y



⇓ Y to Δ

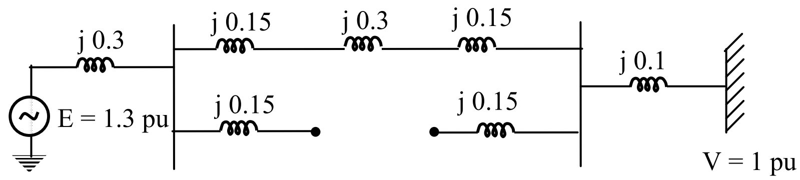


$\Rightarrow X_{eqa} = 2.2$

$$P_{e_2} = \frac{EV}{X_{eq2}} = \frac{1.3 \times 1}{2.2} = 0.591 \text{ pu}$$

$$P_{e_2} = 0.591 \text{ pu}$$

Post fault:



$$P_{e_3} = \frac{EV}{X_{eq3}}$$

$$P_{e_3} = \frac{1.3 \times 1}{1}$$

$P_{e_3} = 1.3$

$X_{eq3} = 0.3 + (0.15 + 0.3 + 0.15) + 0.1$

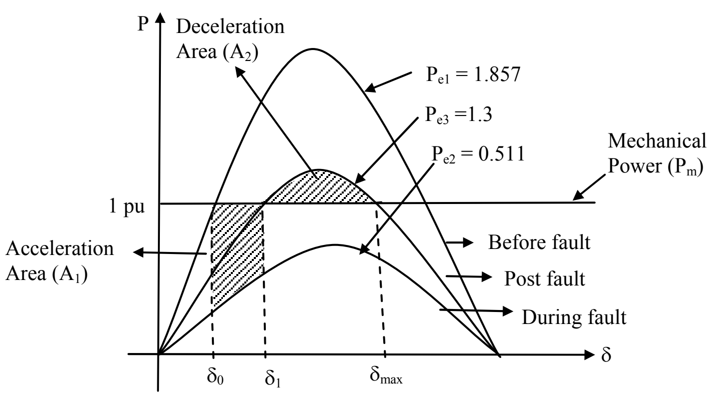
$X_{eq3} = 1$ pu

$\therefore\ P_{e_1} = 1.857$

$P_{e_2} = 0.591$

$P_{e_3} = 1.3$

(ii)



(iii) $P_{e_1}\ \sin\delta_0 = P_m$

$1.857 \sin\delta_0 = 1$

$\delta_0 = 32.58°$

$P_{e_3}\ \sin\delta_1 = P_m$

$1.3 \sin\delta_1 = 1$

$\delta_1 = 50.3°$

$\delta_{max} = 180° - \delta_1$

$\delta_{max} = 129.7°$