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COMMON HALF YEARLY EXAMINATION - 2018

Standard XII Reg.No.: 1 2 A C 4 1

MATHEMATICS

Time: 2.30 hours. Marks: 90

Instructions: 1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

Section - I

Note : i) All questions are compulsory. $20 \times 1 = 20$

ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. Rolle's theorem is applicable in the interval $-1 \leq x \leq 1$ for the function
   a) $f(x) = |x|$ b) $f(x) = x^2$ c) $f(x) = 2x^3 + 3$ d) $f(x) = x$
2. One of the foci of the rectangular hyperbola $xy = 18$ is
   a) (6,6) b) (3,3) c) (4,4) d) (5,5)
3. If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & k & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has in inverse then
   a) k is any real number b) $k = -4$
   c) $k \neq -4$ d) $k \neq 4$
4. The two lines $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z-1}{2}$ are
   a) parallel b) intersecting c) skew d) perpendicular
5. If $z_n = \cos\left(\frac{n\pi}{3}\right) + i\sin\left(\frac{n\pi}{3}\right)$ then, $z_1 z_2 z_3 \ldots\ldots z_6$ is
   a) 1 b) -1 c) i d) -i
6. A random variable X has the probability distribution

X = x	0	1	2	3
P(X = x)	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{\lambda}{10}$	$\frac{4}{10}$

   Then the mean is
   a) 1 b) 2 c) 3 d) 4
7. The radius of a cylinder is increasing at the rate of 2 cm/sec and its altitude is decreasing at the rate of 3 cm/sec. The rate of change of volume when the radius and the altitude are respectively 3 cm and 5 cm, is?
   a) 23π b) 33π c) 43π d) 53π
8. Which of the following is a contradiction?
   a) $p \vee q$ b) $p \wedge q$ c) $p \vee (\sim p)$ d) $p \wedge (\sim p)$
9. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ then $(\text{adj } A)^{-1}$ is
   a) $\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$ b) $\frac{1}{10}\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$ c) $\frac{1}{10}\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ d) $10\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$
10. The least possible perimeter of a rectangle of area 100 sq.units is
    a) 10 b) 20 c) 40 d) 60

11. If $u = y \sin x$, then, $\frac{\partial^2 u}{\partial x \partial y}$ is

a) cos x b) cos y c) sin x d) 0

12. $2+i\sqrt{3}$ is a root of the quadratic equation $x^2 + ax + b = 0$, where $a,b \in R$, then the value of $a^2 + b^2$ is

a) 65 b) 7 c) 33 d) 16

13. The volume of the solid obtained by revolving the area formed by $y = \sqrt{3+x^2}$ x=0 and x=4 is rotated about x-axis is

a) 100π b) $\frac{100}{9}\pi$ c) $\frac{100}{3}\pi$ d) $\frac{100}{3}$

14. If $|\vec{a}-\vec{b}| = |\vec{a}| = |\vec{b}| = 1$ then, the angle between $\vec{a}$ and $\vec{b}$ is

a) $\pi/3$ b) $3\pi/4$ c) $\pi/2$ d) $\pi/4$

15. The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is

a) 48 b) 24 c) 12 d) 96

16. If 2 cards are drawn from a well shuffled pack of 52 cards, then the probability that they are of the same colours without replacement is

a) $\frac{1}{2}$ b) $\frac{26}{51}$ c) $\frac{25}{51}$ d) $\frac{25}{102}$

17. If cos x is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$, then P is

a) – cot x b) cot x c) tan x d) – tan x

18. The degree of the differential equation $c = \frac{\left[1+\left(\frac{dy}{dx}\right)^3\right]^{2/3}}{\frac{d^3y}{dx^3}}$, where c is a constant, is

a) 1 b) 3 c) –2 d) 2

19. Which of the following is not a group?

a) $(Z_n, +_n)$ b) (Z, +) c) (Z, .) d) (R, +)

20. If the line $y = 2x + \lambda$ be a tangent to the hyperbola $36x^2 - 25y^2 = 3600$, then the value of λ^2 is a) -4 b) 256 c) ±16 d) ±4

Section - II

Note : Answer any 7 questions (Ques.No.30 is compulsory) 7 x 2 = 14

21. Find the rank of the matrix $\begin{bmatrix} 6 & 12 & 6 \\ 1 & 2 & 1 \\ 4 & 8 & 4 \end{bmatrix}$

22. Show that $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if and only if $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are coplanar.

23. Find the least positive integer n, such that $\left(\frac{1+i}{1-i}\right)^n = 1$

24. A reflecting telescope has a parabolic mirror for which the distance from the vertex to the focus is 9 mts. If the distance across (diameter) the top of the mirror is 160 cm, how deep is the mirror at the middle?

25. Find the critical numbers and stationary points of the function $f(\theta) = \theta + \sin\theta$ in $[0, 2\pi]$.

26. Find the intervals of concavity of the function $f(x) = 2x^3 + 5x^2 - 4x$

27. Use differential to find an approximate value of $\sqrt[3]{999}$ for one decimal place.

28. Evaluate : $\int_0^1 \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$

29. Construct the truth table for $\sim(p \rightarrow q)$

30. A pair of dice is thrown 5 times. If getting a total of 8 is considered a success, what is the probability of no success?

Section - III

Note : Answer any 7 questions (Ques.No.40 is compulsory) $7 \times 3 = 21$

31. If $\text{adj } A = \begin{pmatrix} 2 & \alpha & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{pmatrix}$ and $|A| = 20$, prove that $\alpha = 6$.

32. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a} + \vec{b} + \vec{c}| = 1$, $\vec{c} = m(\vec{a} \times \vec{b})$ where m is a scalar and $|\vec{a}| = \frac{1}{\sqrt{2}}$, $|\vec{b}| = \frac{1}{\sqrt{3}}$, $|\vec{c}| = \frac{1}{\sqrt{6}}$, show that $\vec{a}$ and $\vec{b}$ are perpendicular.

33. Solve : $x^4 + 4 = 0$

34. Find the foci of the ellipse $\frac{(x+3)^2}{6} + \frac{(y-5)^2}{4} = 1$

35. Evaluate : $\lim_{x \to \infty} \frac{\frac{1}{x^2} - 2\tan^{-1}\frac{1}{x}}{\frac{1}{x}}$

36. If $w = x + 2y + z^2$ and $x = \cos t$, $y = \sin t$, $z = t$, find $\frac{dw}{dt}$.

37. Solve : $(D^2 - 2D - 3)\, y = \sin x \cos x$

38. State and prove the reversal law on groups.

39. Alpha particles are emitted by a radio-active source at an average rate of 5 in a 20 minutes interval. Using Poisson distribution find the probability that there will be 2 emission $[e^{-5} = 0.0067]$.

40. Prove that $\int_{-\pi/2}^{\pi/2} |\sin x|\, dx = 2$

Section - IV

Note : Answer all the questions: $7 \times 5 = 35$

41. a) Solve : $x + y + 2z = 0,\ 3x + 2y + z = 0,\ 2x + y - z = 0$ by using determinant method.

(or)

b) Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ if $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$

42. a) Prove that sin(A-B) = sinA cosB - cosA sinB by using vectors.

(or)

b) Find the Cartesian equation of the plane through the points (1,2,3), (2,3,1) and perpendicular to the plane $3x - 2y + 4z - 5 = 0$.

43. a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12 mts away from the starting point. Find the angle of projetion.

(or)

b) A player running a race course observes that the sum of the distances from the two flag posts from him is always 120 meters and the distance between the flag posts is 60 meters. Find the equation of the path traced by him.

44. a) Prove that the sum of the intercepts on the co-ordinate axes of any tangent to the curve $x = a\cos^4\theta,\ y = a\sin^4\theta,\ 0 \le \theta \le \pi/2$ is equal to 'a'.

(or)

b) Show that a rectangle of largest area that can be inscribed in a circle is a square.

45. a) Find the common area enclosed by the parabolas $4y^2 = 9x$ and $3x^2 = 16y$.

(or)

b) Find c, μ and σ^2 of the normal distribution whose probability function $f(x) = ce^{-x^2+3x}$, $-\infty < x < \infty$.

46. a) Solve : $(x + y)^2 \frac{dy}{dx} = 1$

(or)

b) A cup of coffee at temperature 100°C is placed in a room whose temperature is 15°C and it cools to 60°C in 5 minutes. Find its temperature after a further interval of 5 minutes.

47. a) Show that the set {[1], [3], [5], [7]} is an abelian group under multiplication modulo 8.

(or)

b) Express the complex number $\frac{2(1+i)}{\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}}$ in $r(\cos\theta + i\sin\theta)$ form.

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