# SHRI VIDHYABHARATHI MATRIC HR.SEC.SCHOOL
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## COMMON HALFYEARLY EXAMINATION 2018

**STD: XII** **14.12.2018**

**SUBJECT: MATHEMATICS** **ANSWER KEY** **MARKS : 90**

| | SECTION – I | |
|---|---|---|
| Q.No | ANSWER KEY | MARKS |
| 1. | b) $f(x) = x^2$ | 1 |
| 2. | a)(6,6) | 1 |
| 3. | c)$k\neq -4$ | 1 |
| 4. | c)skew | 1 |
| 5. | b)-1 | 1 |
| 6. | b)2 | 1 |
| 7. | b) $33\pi$ | 1 |
| 8. | d) $p\wedge(-p)$ | 1 |
| 9. | c) $\frac{1}{10}\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$ | 1 |
| 10. | c) 40 | 1 |
| 11. | a) cosx | 1 |
| 12. | a)65 | 1 |
| 13. | c) $\frac{100}{3}\pi$ | 1 |
| 14. | a) $\frac{\pi}{3}$ | 1 |
| 15. | a)48 | 1 |
| 16. | c) $\frac{25}{51}$ | 1 |
| 17. | d)-tanx | 1 |
| 18. | b) 3 | 1 |
| 19. | c)(z,.) | 1 |
| 20. | b)256 | 1 |
| | SECTION – II | |
| 21. | $A=\begin{bmatrix}6 & 12 & 6\\ 1 & 2 & 1\\ 4 & 8 & 4\end{bmatrix}$ <br> $\sim\begin{bmatrix}6 & 12 & 6\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ $\rho(A)=1$ | 1 <br> 1 |

| Q. No. | Answer | Marks |
|---|---|---|
| 22. | $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=0$ | 1 |
| | $\Leftrightarrow 2[\vec{a}\ \vec{b}\ \vec{c}]=0$ | |
| | $\Leftrightarrow [\vec{a}\ \vec{b}\ \vec{c}]=0$ | 1 |
| 23. | $\left(\frac{1+i}{1-i}\right)^n=1$ | |
| | $(i)^n=1$ | 1 |
| | n=4 | 1 |
| 24. | $y^2=4ax$ | |
| | $y^2=3600x$ | 1 |
| | $x_1=\frac{64}{36}=\frac{16}{9}$ | 1 |
| 25. | $f(\theta)=\theta+\sin\theta$ | |
| | $f'(\theta)=1+\cos\theta$ | |
| | $\cos\theta=-1$ | |
| | critical number $\pi$ | 1 |
| | stationary point $(\pi,\pi)$ | 1 |
| 26. | $f(x)=2x^3+5x^2-4x$ | |
| | $f'(x)=6x^2+10x$ | |
| | $f''(x)=12x+10$ | |
| | $f''(x)=0$ ,x=-5/6 | 1 |

| Interval | $f''(x)=12x+10$ | Concavity |
|---|---|---|
| $\left(-\infty,-\frac{5}{6}\right)$ | − | Downward |
| $\left(-\frac{5}{6},-\infty\right)$ | + | Upward |

(Marks for the table: 1)

| Q. No. | Answer | Marks |
|---|---|---|
| 27. | Let $y=f(x)=x^{1/3}$ | |
| | $dy=f'(x)dx=\frac{1}{3}(x)^{2/3}dx$ | 1 |
| | Let $x=1000$ , $dx=\Delta x=-1$, $f(1000)$=-1/300 | |
| | $dy=-0.00333$ | |
| | $f(x+\Delta x)\approx y+dy$ | |
| | $\sqrt[3]{999}\approx$ 10-0.0033 | |
| | $\sqrt[3]{999}\approx 9.9967$ | 1 |
| 28. | $I=\int_0^1\frac{(\sin^{-1}}{\sqrt{1-x^2}}=\left[\frac{(\sin^{-1})^4}{4}\right]_0^1$ | 1 |
| | $=\frac{1}{4}\left[\frac{\pi}{2}\right]^4$ | |
| | $=\frac{\pi^4}{64}$ | 1 |

**29.**

| p | q | $p \to q$ | $\sim(p \to q)$ |
|---|---|---|---|
| T | T | T | F |
| T | F | F | T |
| F | T | T | F |
| F | F | T | F |

2

**30.**

n=5

$P=\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$p=\frac{5}{36}$ , $q=\frac{31}{36}$

$p(X=0)=5C_0\left(\frac{31}{36}\right)^5$ 1

$=\left(\frac{31}{36}\right)^5$ 1

## SECTION – III

**31.**

$|AdjA|=\begin{vmatrix} 2 & \alpha & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{vmatrix}$ 1

$|A|^2=2(-28+48)-\alpha(84-14)+4(126-126)$ 1

$400=40+60\alpha$

$360=60\alpha$

$\alpha=6$ 1

**32.**

$|\vec{a}+\vec{b}+\vec{c}|=1$

$|\vec{a}+\vec{b}+\vec{c}|^2=1$

$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}.\vec{b}+2\vec{b}.\vec{c}+2\vec{c}.\vec{a}=1$

$\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+2\vec{a}.\vec{b}+2\vec{b}m(\vec{a}\times\vec{b})+2\left[m(\vec{a}\times\vec{b}).\vec{a}\right]=1$ 1

$1+2\vec{a}.\vec{b}=1$

$2\vec{a}.\vec{b}=0$ 1

$\vec{a}\perp\vec{b}$ 1

**33.**

$x=4^{1/4}(-1)^{1/4}$

$=(2^2)^{1/4}\left[cos(2k\pi+\pi)+i\,sin(2k\pi+\pi)\right]^{1/4}$ 1

$=\sqrt{2}\left[cos(2k+1)\frac{\pi}{4}+i\,sin(2k+1)\frac{\pi}{4}\right]$ , $k=0,1,2,3.$ 1

$\therefore$ The values are $\sqrt{2}\,cis\,\frac{\pi}{4}$ , $\sqrt{2}cis\,\frac{3\pi}{4}$, $\sqrt{2}cis\,\frac{5\pi}{4}$ , $\sqrt{2}cis\,\frac{7\pi}{4}$ 1

**34.**

$a^2=6 \qquad b^2=4$

$ae=a\sqrt{\frac{a^2-b^2}{a}}=\sqrt{2}$ 1

$x+3=\pm\sqrt{2} \qquad y-5=0$ 1

$Foci(-3+\sqrt{2},5)\,\&\,(-3-\sqrt{2},5)$ 1

| | | |
|---|---|---|
| 35. | $\lim_{x\to+\infty}\frac{\frac{1}{x^2}-2\tan^{-1}\frac{1}{x}}{\frac{1}{x}}$<br>$y=\frac{1}{x}\ as\ x\to\infty, y\to 0$<br>$\lim_{x\to+\infty}\frac{\frac{1}{x^2}-2\tan^{-1}\frac{1}{x}}{\frac{1}{x}}=\lim_{y\to 0}\frac{y^2-2\tan^{-1}y}{y}\left(\frac{0}{0}form\right)$<br>$=\lim_{y\to 0}\left[\frac{2y-\frac{1}{1+y^2}}{1}\right]=0-\frac{2}{1}=-2.$ | <br><br>1<br>2 |
| 36. | $x=cost\,,\ \ y=sint,\ \ z=t.$<br>$\frac{\partial w}{\partial x}=1$ ; $\frac{\partial w}{\partial y}=2$ ; $\frac{\partial w}{\partial z}=2z$<br>$\frac{dx}{dt}=-\sin t$ ; $\frac{dy}{dt}=\cos t$ ; $\frac{dz}{dt}=1$<br>$\frac{dw}{dt}=\frac{\partial w}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial w}{\partial y}\cdot\frac{dy}{dt}+\frac{\partial w}{\partial z}\cdot\frac{dz}{dt}$<br>$\frac{dw}{dt}=1(-\sin t)+2\cos t+2z$<br>$=-\sin t+2\cos t+2t$ | <br><br>1<br><br>1<br>1 |
| 37. | $(D^2-2D-3)y=sinxcosx$<br>The characteristic equation is $p^2-2p-3=0$<br>$p=-1; p=3$<br>The complementary function is $C.F.=Ae^{-x}+Be^{3x}$<br>$P.I.=\frac{1}{D^2-2D-3}\left(\frac{1}{2}sin2x\right)$<br>$=\frac{1}{-2}\times\frac{4cos2x-7sin2x}{-65}=\frac{4cos2x-7sin2x}{130}$<br>The general solution is $y=C.F.+P.I.=Ae^{-x}+Be^{3x}+\frac{4cos2x-7sin2x}{130}$ | <br><br><br>1<br><br>1<br>1 |
| 38. | statement<br>**Proof :** (i) $(a*b)*(b^{-1}*a^{-1})=a*(b*b^{-1})*a^{-1}\ =e$<br>(ii) $(b^{-1}*a^{-1})*(a*b)=b^{-1}*(a^{-1}*a)*b$<br>$=e$<br>$\therefore$ The inverse of $a*b$ is $b^{-1}*a^{-1}$.<br>$\therefore\ (a*b)^{-1}=b^{-1}*a^{-1}$ | 1<br><br>1<br><br>1 |
| 39. | $\lambda=5$<br>(i)$P(X=2)=\frac{e^{-\lambda}\lambda^2}{2!}=\frac{e^{-5}5^2}{2!}=0.0067\times\frac{25}{2}=0.0838$ | 1<br>2 |
| 40. | $f(-x)=\|\sin x\|$ = even function<br>$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\|\sin x\|dx=2\int_{0}^{\frac{\pi}{2}}\|\sin x\|dx$<br>$=2$ | 1<br>1<br>1 |
| | | |

## SECTION – IV

**41.** (a) ) $\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 0$ 1

Since $\Delta = 0$, it has infinitely many solutions, Also atleast one 2x2 minors of $\Delta \neq 0$, the system is reduced to 2 equations. 1

$\therefore$ Assigning arbitrary value for $z$.

Let $z = k, k \in R$ and taking first and last equations. 1

$x + y = -2k$

$2x + y = k$

$\Delta_x = \begin{vmatrix} -2k & 1 \\ k & 1 \end{vmatrix} = -3k \qquad \Delta_y = \begin{vmatrix} 1 & -2k \\ 2 & k \end{vmatrix} = 5k$ 1

By Cramer's Rule $x = \frac{\Delta_x}{\Delta} = 3k,\ y = \frac{\Delta_y}{\Delta} = -5k$

**The solution set is** 1

$\{x, y, z\} = \{3k, -5k, k\}, k \in R.$

(b) $f = \tan u = \dfrac{x^3 + y^3}{x - y}$

f is a homogenous fun of degree n=2 1

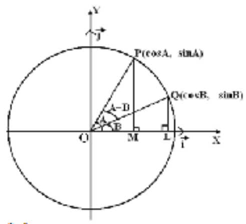
By Euler's *thm* $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = 2f$

$$x\frac{\partial}{\partial x}(\tan u) + y\frac{\partial}{\partial y}(\tan u) = 2f$$ 2

$$x\sec^2 u\frac{\partial u}{\partial x} + y\sec^2 u\frac{\partial u}{\partial y} = 2\tan u$$ 1

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$ 1

**42.** (a) Diagram 1




$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = cosA\vec{i} + sinA\vec{j}$ 1

$\overrightarrow{OQ} = \overrightarrow{OL} + \overrightarrow{LQ} = cosB\vec{i} + sinB\vec{j}$

$\overrightarrow{OQ} \times \overrightarrow{OP} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ cosB & sinB & 0 \\ cosA & sinA & 0 \end{vmatrix} = \vec{k}[\sin A cosB - cosA sinB] ---\to (1)$ 1

By definition, $\overrightarrow{OQ} \times \overrightarrow{OP} = |\overrightarrow{OQ}||\overrightarrow{OP}|\sin(A-B)\vec{k} = \sin(A-B)\vec{k} ---\to (2$ 1

From (1) and (2), **sin(A-B)= $\sin A cosB - cosA sinB$** 1

(b)

VECTOR FORM:

$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k},\ \vec{b} = 2\vec{i} + 3\vec{j} + \vec{k}$ and $\vec{v} = 3\vec{i} - 2\vec{j} + 4\vec{k}$

The equation of the plane is 1

$$\vec{r} = (1 - s)\vec{a} + s\vec{b} + t\vec{v}$$

$$\vec{r} = (1 - s)(\vec{i} + 2\vec{j} + 3\vec{k}) + s(2\vec{i} + 3\vec{j} + \vec{k}) + t(3\vec{i} - 2\vec{j} + 4\vec{k})$$ 1

CARTESIAN FORM:

$(x_1, y_1, z_1) = (1,2,3), (x_2, y_2, z_2) = (2,3,1)$ and $(l_1, m_1, n_1) = (3,-2,4)$

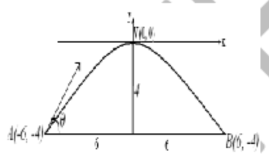
The equation of the plane is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \end{vmatrix} = 0$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 3 & -2 & 4 \end{vmatrix} = 0$$ 2

1

$$\mathbf{2y + z - 7 = 0}$$

43. (a) **Diagram** 1



Equation of the parabola is

$x^2 = -4ay$ 1

It passes through the point B(6, -4)

$(6)^2 = -4a(-4) \Rightarrow a = \frac{9}{4} \Rightarrow x^2 = -9y - - - - - - (1)$ 1

Diff. with respect to x,

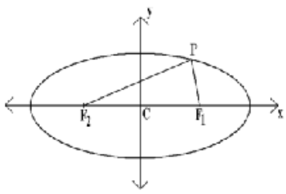
$$2x = -9\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-2x}{9}$$

$$\left(\frac{dy}{dx}\right)_{(-6,-4)} = \frac{-2}{9}(-6)$$ 1

$$Tan\theta = \frac{4}{3} \qquad \Rightarrow \theta = Tan^{-1}\frac{4}{3}$$

**The angle of projection is $\boldsymbol{\theta = Tan^{-1}\frac{4}{3}}$** 1

(b) Diagram 1



$F_1P + F_2P = 2a \Rightarrow 2a = 120 \Rightarrow a = 60$ 1

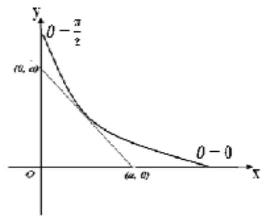
$F_1F_2 = 2ae = 60 \Rightarrow ae = 30$

$b^2 = a^2 - (ae)^2 = 3600 - 900 = 2700$ 1

**The equation of the path is**

$$\frac{x^2}{3600} + \frac{y^2}{2700} = 1$$ 2

44. (a)



$x = acos^4\theta, y = asin^4\theta$

Take any point '$\theta$' as $(acos^4\theta, \ asin^4\theta)$

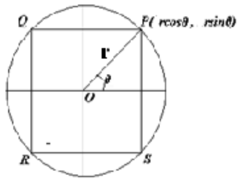
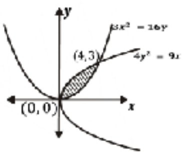
$$\frac{dx}{d\theta} = -4acos^3\theta sin\theta \ ; \quad \frac{dy}{d\theta} = 4asin^3\theta cos\theta$$

$$\frac{dy}{dx} = -\frac{sin^2\theta}{cos^2\theta}$$ 1

Slope of the tangent at '$\theta$' is $-\frac{sin^2\theta}{cos^2\theta}$

Equation of tangent at '$\theta$' is $y - asin^4\theta = -\frac{sin^2\theta}{cos^2\theta}(x - acos^4\theta)$ 2

$$ycos^2\theta - asin^4\theta cos^2\theta = -xsin^2\theta + acos^4\theta sin^2\theta$$

| | | |
|---|---|---|
| | $$xsin^2\theta + ycos^2\theta = asin^4\theta cos^2\theta + acos^4\theta sin^2\theta$$ $xsin^2\theta + ycos^2\theta = asin^2\theta cos^2\theta$ <br> $\frac{x}{acos^2\theta} + \frac{y}{asin^2\theta} = 1$ <br> ∴ Sum of intercepts = $acos^2\theta + asin^2\theta$ <br> $= a(cos^2\theta + sin^2\theta) = a$ | 1 <br> 1 |
| | (b)   <br> Let PQRS be the rectangle inscribed in a circle . <br> Let $\theta$ be the angle made by OP with the positive direction of x-axis. <br> Dimensions of the rectangle are $2rcos\theta,\ 2rsin\theta,\ 0 \le \theta \le \frac{\pi}{2}$ <br> Area of the rectangle $A = 4r^2 sin\theta cos\theta$ <br> $A(\theta) = 2r^2 sin2\theta$ <br> $A'(\theta) = 4r^2 cos2\theta$ <br> $A''(\theta) = -8r^2 sin2\theta$ <br> $A'(\theta) = 0 \implies 4r^2cos2\theta = 0 \implies 2\theta = \frac{\pi}{2} \implies \theta = \frac{\pi}{4}$ <br> When $\theta = \frac{\pi}{4}, A''\left(\frac{\pi}{4}\right) = -8r^2 < 0$ <br> When $\theta = \frac{\pi}{4}$, $A$ is maximum. <br> When $\theta = \frac{\pi}{4}$, $Length = 2x = \sqrt{2}r;\ Breadth = 2y = \sqrt{2}r$ <br> The dimensions of the rectangle are $\sqrt{2}r$ & $\sqrt{2}r$ <br> $i.e.,$ length = breadth = $\sqrt{2}r$ <br> ∴ The given rectangle is also a square | 1 <br> 1 <br> 1 <br> 1 <br> 1 |
| 45. | (a) **Diagram** <br> The point of intersection of $4y^2 = 9x$ and $3x^2 = 16y$ are $(0,0)$ and $(4,3)$. <br> Required area can be solved about x-axis. <br> Required area $= \int_0^4 (y_1 - y_2)dx$   <br> R.A.= $\int_0^4 \left[\frac{3}{2}\sqrt{x} - \frac{3}{16}x^2\right]dx$ <br> $= \left[x^{\frac{3}{2}} - \frac{x^3}{16}\right]_0^4$ <br> $= 4$ square units. | 1 <br> 2 <br> 1 <br> 1 |
| | (b) By definition, <br> $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} ---\rightarrow (1)$ <br> Given = $f(x) = ce^{-x^2+3x}$ <br> $= ce^{-(x^2-3x)}$ <br> $= ce^{-\left(x-\frac{3}{2}\right)^2+\frac{9}{4}}$ | |

$$= ce^{-\left(x-\frac{3}{2}\right)^2} e^{\frac{9}{4}}$$ **2**

$$= ce^{\frac{9}{4}} e^{-\frac{1}{2}\cdot\frac{\left(x-\frac{3}{2}\right)^2}{1/2}} \; ---\rightarrow (2)$$

Comparing (1) and (2), **2**

$$\mu = \frac{3}{2}\,,\ \sigma^2 = \frac{1}{2} \Rightarrow \sigma = \frac{1}{\sqrt{2}}$$

$$ce^{\frac{9}{4}} = \frac{1}{\sigma\sqrt{2\pi}}$$ **1**

$$c\,e^{\frac{9}{4}} = \frac{1}{\frac{1}{\sqrt{2}}\sqrt{2\pi}} \quad \Rightarrow \quad c = \frac{e^{\frac{-9}{4}}}{\sqrt{\pi}}$$

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**46.** (a)

$$(x+y)^2\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{(x+y)^2}$$

Let $x + y = z$

$$1 + \frac{dy}{dx} = \frac{dz}{dx}$$ **1**

$$\left(\frac{z^2}{z^2+1}\right)dz = dx$$

$$\left(\frac{z^2+1-1}{z^2+1}\right)dz = dx$$

$$\left(1 - \frac{1}{z^2+1}\right)dz = dx$$ **2**

$$\int\left(1 - \frac{1}{z^2+1}\right)dz = \int dx$$

$$x + y - tan^{-1}(x+y) = x + c$$ **2**

$$\boldsymbol{y - tan^{-1}(x+y) = c}$$

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(b) Let T be the temperature of the coffee at any time t.

$\frac{dT}{dt} \propto (T-15)$ since S = 15°C $\Rightarrow \frac{dT}{dt} = k(T-15) \Rightarrow T - 15 = ce^{kt}$ **1**

When $t = 0,\ T = 100 \Rightarrow 100 - 15 = ce^0 \quad \Rightarrow c = 85$

$$\therefore T - 15 = 85e^{kt}$$ **1**

When t = 5, T = 60 $\Rightarrow \quad 60 - 15 = 85e^{5k} \quad \Rightarrow 45 = 85e^{5k}$

$$\Rightarrow e^{5k} = \frac{45}{85}$$ **1**

When t = 10, $\quad T - 15 = 85e^{10k}$

$$T = 15 + 85\left(e^{5k}\right)^2$$

$$= 15 + 23.82°C = 38.82°C$$ **2**

**The required temperature after a further interval of 5 minutes is 38.82°C**

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**47.** (a) **Cayley's table:** **2**

Let $G = \{[1],[3],[5],[7]\}$

| $\cdot_8$ | [1] | [3] | [5] | [7] |
|---|---|---|---|---|
| [1] | [1] | [3] | [5] | [7] |
| [3] | [3] | [1] | [7] | [5] |
| [5] | [5] | [7] | [1] | [3] |
| [7] | [7] | [5] | [3] | [1] |

**i) Closure Axiom:**
All the entries in the Cayley's table are the members of G.
∴ Closure axiom is true.

**(ii) Associative axiom:**
Multiplication modulo 8 is always associative.
∴ Associative axiom is true.

**(iii) Identity axiom:**
The identity element is $[1] \in G$ 1
∴The identity axiom is true.

**(iv) Inverse axiom:**
The inverse of [1] is [1]
The inverse of [3] is [3] 1
The inverse of [5] is [5] $\in G$
The inverse of [7] is [7]
∴The inverse axiom is true.
$(G, \cdot_{11})$ is a group.

**(v) Commutative axiom:**
From the table, the commutative axiom is also true. 1
$(G, \cdot_{11})$ is an abelian group.

(b)

$$1+i=\sqrt{2}cis\frac{\pi}{4}$$ 1

$$\frac{2(1+i)}{cis(-\frac{\pi}{6})}=\frac{2\sqrt{2}cis\frac{\pi}{4}}{cis(-\frac{\pi}{6})}$$ 2

$$=2\sqrt{2}e^{i\frac{\pi}{4}}.e^{i\frac{\pi}{6}}$$

$$=2\sqrt{2}e^{i\left(\frac{3\pi+2\pi}{12}\right)}$$

$$=2\sqrt{2}e^{i\left(\frac{5\pi}{12}\right)}$$ 1

$$=2\sqrt{2}\left[\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right]$$ 1

# Department of mathematics

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