SHRI VIDHYABHARATHI MATRIC HR.SEC.SCHOOL

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COMMON HALFYEARLY EXAMINATION 2018

STD: XI 19.12.2018

SUBJECT: BUSINESS MATHEMATICS **ANSWER KEY** **MARKS : 90**

PART – A

S.NO.		
1.	b	4
2.	b	0, 1
3.	a	$\frac{1}{4}$
4.	a	2^n
5.	c	6
6.	d	1
7.	b	$-\frac{7}{5}$
8.	b	$\sin 50^\circ$
9.	a.	$x = 1$ (Question is not completed)
10.	b	$-\frac{1}{x^2}$
11.	c	MC = AC
12.	b	6y
13.	b	8.75%
14.	a	10
15.	a	$P(A \cap B) = 0$
16.	d	$AM \geq G.M \geq H.M$
17.	c	$\frac{3}{25}$ (Question is not completed)
18.	a.	a local maximum at c.
19.	a.	$x = y$ or $x + y = n$
20.	c.	$\frac{3}{4}$.

PART – B

21. $$\begin{vmatrix} x & y & z \\ 2x+2a & 2y+2b & 2z+2c \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} x & y & z \\ 2x & 2y & 2z \\ a & b & c \end{vmatrix} + \begin{vmatrix} x & y & z \\ 2a & 2b & 2c \\ a & b & c \end{vmatrix}$$

$$= 0$$

22. $$I - B = \begin{bmatrix} 0.50 & -0.25 \\ -0.40 & 0.33 \end{bmatrix}$$

$|I-B| = 0.065 > 0$

Hawkins Simon Condition satisfied.

The given system is Viable.

23. W.K.T. ${}^nC_x = {}^nC_y \Rightarrow x + y = n$

$15C_{3r} = 15C_{r+3}$

$3r + r + 3 = 15$

$r = 3$

24. Slope of $2x - y + 3 = 0$ is $m_1 = 2$

Slope of $x + y + 2 = 0$ is $m_2 = -1$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{2+1}{1+2(-1)}\right|$$

$\tan\theta = 3$

$\theta = \tan^{-1}(3)$

25. Supply Price Relation $x^2 = 5p - 15 = 5(p-3)$

$X^2 = 4aP$ where $X = x$, $P = p - 3$

The supply curve is parabola whose vertex is $(X=0, P=0)$

The curve is a parabola whose vertex is $(0, 3)$.

26. L.H.S $= \sin 20^\circ \sin 40^\circ \sin 80^\circ$

$= \sin 20^\circ \sin(60^\circ-20^\circ)\sin(60^\circ+20^\circ)$

$= \sin 20^\circ (\sin^2 60^\circ - \sin^2 20^\circ)$

$= \sin 20^\circ \left[\frac{3-4\sin^2 20^\circ}{4}\right]$

$= \frac{\sin 60^\circ}{4}$

$= \frac{\sqrt{3}}{8}$

27. $f(x) = \frac{x^7-2^7}{x^5-2^5}$

$$\lim_{x\to 2} f(x) = \frac{\lim\limits_{x\to 2} \frac{x^7-2^7}{x-2}}{\lim\limits_{x\to 2}\frac{x^5-2^5}{x-2}} = \frac{7\times 2^6}{5\times 2^4}$$

$= \frac{28}{5}$

28. $x = \frac{25}{p^4}$, $1 \le p \le 5$

$\frac{dx}{dp} = \frac{-100}{p^5}$

$\eta_d = \frac{-p}{x}\frac{dx}{dp} = 4.$

29. $a =$ ₹. 50 $\quad i = 5\% = 0.05$

$A = \frac{a}{i} = \frac{50}{0.05}$

$A =$ ₹. 1000.

30. If $E_1, E_2, \ldots E_n$ are a set of n mutually exclusive and collectively exhaustive events with $P(E_i) \neq 0$ $(i = 1,2,3,\ldots n)$, then for any arbitrary event A which is associated with sample space $S = \bigcup_{i=1}^{n} E_i$ such that $P(A) > 0$

$$P(E_i/A) = \frac{P(E_i)\cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\cdot P(A/E_i)}, \quad i = 1,2,3,\ldots n.$$

PART-C

31. Let the probability of 3 horses to win the race be $P(A), P(B)$, and $P(C)$ resp.

$P(A) = 2P(B) \Rightarrow P(B) = \frac{P(A)}{2}$

$P(B) = 2P(C) \Rightarrow P(C) = \frac{P(A)}{4}$

W.K.T. $P(A)+P(B)+P(C) = 1$.

$P(A) + \frac{P(A)}{2} + \frac{P(A)}{4} = 1$

$P(A) = 4/7$

$P(B) = 2/7$, $P(C) = 1/7$.

32. $y = \frac{2x+1}{3x+2}$

$\frac{dy}{dx} = \frac{(3x+2)(2)-(2x+1)(3)}{(3x+2)^2}$

$= \frac{1}{(3x+2)^2}$

Elasticity $\eta = \frac{x}{y}\cdot\frac{dy}{dx}$

$= \frac{x}{(2x+1)(3x+2)}$

when $x = 1$, $\eta = \frac{1}{15}$

33. $u = \frac{x^2}{1+x^2}$ $\quad\quad v = x^2$

$\frac{du}{dx} = \frac{2x}{(1+x^2)^2}$ $\quad\quad \frac{dv}{dx} = 2x$

$\frac{du}{dv} = \frac{du/dx}{dv/dx}$

$= \frac{2x/(1+x^2)^2}{2x}$

$= \frac{1}{(1+x^2)^2}$.

34. $y = A\sin x + B\cos x$

$y_1 = A\cos x - B\sin x$

$y_2 = -A\sin x - B\cos x$

$y_2 = -y$

$y_2 + y = 0$

35. $\tan(x+y) = 42$

$x+y = \tan^{-1}(42)$

$\tan^{-1}(2) + y = \tan^{-1}(42)$

$y = \tan^{-1}(42) - \tan^{-1}(2)$

$= \tan^{-1}\left[\frac{42-2}{1+42(2)}\right]$

$y = \tan^{-1}\left(\frac{8}{17}\right)$

36. Let m_1 and m_2 be the slopes of $ax^2+2hxy+by^2=0$

$m_1+m_2 = \frac{-2h}{b}$, $m_1m_2 = a/b$

Given $m_2 = 2m_1$

$m_1 = \frac{-2h}{3b}$, $2m_1^2 = \frac{a}{b}$

$2\left(\frac{-2h}{3b}\right)^2 = \frac{a}{b}$

$8h^2 = 9ab$.

37. i) Atleast 2 ladies are included

Ladies(4)	Gents(6)	Combination
2	3	$4C_2 \times 6C_3 = 120$
3	2	$4C_3 \times 6C_2 = 60$
4	1	$4C_4 \times 6C_1 = 6$

Required no. of ways = 186

ii) Atmost 2 Ladies are included

Ladies (4)	Gents(6)	Combination
2	3	$4C_2 \times 6C_3 = 120$
1	4	$4C_1 \times 6C_4 = 60$
0	5	$4C_0 \times 6C_5 = 6$

Required no. of ways = 186.

38. $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$

$A^2 - 4A + 5I_2 = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - 4\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 - 4A + 5I_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 - 4A + 5I_2 = 0$

39. $(x+a)^n = nC_0 x^n a^0 + nC_1 x^{n-1} a^1 + \dots + nC_n x^0 a^n$.

$\left(x^2+\frac{1}{x^2}\right)^4 = 4C_0 (x^2)^4 + 4C_1 (x^2)^3 \cdot \frac{1}{x^2} + 4C_2 (x^2)^2 \left(\frac{1}{x^2}\right)^2 + 4C_3\, x^2 \left(\frac{1}{x^2}\right)^3 + 4C_4 \left(\frac{1}{x^2}\right)^4$

$\left(x^2+\frac{1}{x^2}\right)^4 = x^8 + 4x^4 + 6 + \frac{4}{x^4} + \frac{1}{x^8}$.

40. $A = \begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & 0 \\ 9 & 1 & 5 \end{bmatrix}$

$A_{ij} = \begin{bmatrix} 15 & -10 & -25 \\ 12 & -8 & -20 \\ -6 & 4 & 10 \end{bmatrix}$

$\text{adj}\,A = [A_{ij}]^T = \begin{bmatrix} 15 & 12 & -6 \\ -10 & -8 & 4 \\ -25 & -20 & 10 \end{bmatrix}$

$(\text{adj}\,A) \cdot A = \begin{bmatrix} 15 & 12 & -6 \\ -10 & -8 & 4 \\ -25 & -20 & 10 \end{bmatrix} \begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & 0 \\ 9 & 1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$(\text{adj}\,A) \cdot A = 0$.

PART-D

41. a.

$$B = \begin{bmatrix} \frac{1}{5} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{3} \end{bmatrix}$$

$$I-B = \begin{bmatrix} \frac{4}{5} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{2}{3} \end{bmatrix}$$

$$|I-B| = \frac{113}{240} > 0$$

The given system is viable.

$$adj(I-B) = \begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix}$$

$$(I-B)^{-1} = \frac{240}{113}\begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix}$$

$$X = (I-B)^{-1} D$$

$$= \frac{240}{113}\begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{5} \end{bmatrix}\begin{bmatrix} 23 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 34.16 \\ 17.31 \end{bmatrix}$$

Output for 1 is 34.16

Output for 2 is 17.31

41. b

$$\text{Cosec}A - \text{Cosec}B = \sec B - \sec A$$

$$\frac{1}{\sin A} - \frac{1}{\sin B} = \frac{1}{\cos B} - \frac{1}{\cos A}$$

$$\frac{\sin B - \sin A}{\sin A \sin B} = \frac{\cos A - \cos B}{\cos A \cos B}$$

$$\frac{\cancel{2}\cos\left(\frac{B+A}{2}\right)\sin\left(\frac{B-A}{2}\right)}{\sin A \sin B} = \frac{-\cancel{2}\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{\cos A \cos B}$$

$$\frac{-\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{-\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)} = \frac{\sin A \sin B}{\cos A \cos B}$$

$$\cot\left(\frac{A+B}{2}\right) = \tan A \tan B$$

42. a.

Let the Equation of the circle be

$$x^2+y^2+2gx+2fy+c=0.$$

Circle passes $(0,1)$

$$2f+c=1 \quad \text{—①}$$

Circle passes $(4,3)$

$$8g+6f+c=-25 \quad \text{—②}$$

Circle passes $(1,-1)$

$$2g-2f+c=-2 \quad \text{—③}$$

Solve ② and ③, $14f-3c=-17$ —④

Solve ① and ④, $\boxed{f=-1}$

$\boxed{c=1}$, $\boxed{g=\frac{-5}{2}}$

Required Equation of circle is

$$x^2+y^2-5x-2y+1=0.$$

42. b

$$f(x) = 3x^5-25x^3+60x+1$$

$$f'(x) = 15(x^4-5x^2+4)$$

$$f'(x) = 0$$

$$x = \pm 2, \quad x = \pm 1$$

$$-2, \pm 1 \in [-2,1], \; 2 \notin [-2,1]$$

$$f(-2) = -15$$

$$f(-1) = -37$$

$$f(1) = 39$$

Absolute maximum is 39

Absolute minimum is -37.

43.a.

$u(x,y) = (x^2+y^2)^{-1/2}$

$u(tx,ty) = t^{-1}(x^2+y^2)^{-1/2}$

u is a homogeneous function of degree −1.

By Euler's Theorem,

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -u.$

Verification :-

$\frac{\partial u}{\partial x} = \frac{-x}{(x^2+y^2)^{-3/2}}$

$x\frac{\partial u}{\partial x} = \frac{-x^2}{(x^2+y^2)^{-3/2}}$

$\frac{\partial u}{\partial y} = \frac{-y}{(x^2+y^2)^{-3/2}}$

$y\frac{\partial u}{\partial y} = \frac{-y^2}{(x^2+y^2)^{-3/2}}$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -u.$

43.b.

No. of Shares = 400

FV = ₹.10

MV = 12.50

r = 12%.

i) No. of shares = $\frac{\text{Investment}}{\text{M.V}}$

Investment = 400 × 12.50

= ₹.5000.

ii) Annual dividend = No. of Shares × FV × Rate %.

= $400 \times 10 \times \frac{12}{100}$

= ₹. 480.

iii) Rate of Interest received by him on his money = $\frac{\text{Income}}{\text{Investment}} \times 100$

= $\frac{480}{5000} \times 100 = 9.6\%$.

44.a.

Marks	f	Mid x	fx	log x	f log x	f/x
0-10	5	5	25	0.6990	3.495	1.0000
10-20	10	15	150	1.1761	11.761	0.6667
20-30	25	25	625	1.3979	34.9475	1.00
30-40	30	35	1050	1.5441	46.323	0.8571
40-50	20	45	900	1.6532	33.064	0.4444
50-60	10	55	550	1.7404	17.404	0.1818
	Σf =100		Σfx =3300		$\Sigma f\log x$ =146.9945	$\Sigma f/x$ = 4.15

i) Arithmetic Mean = $\frac{\Sigma fx}{N} = 33$

ii) GM = Antilog$\left(\frac{\Sigma f\log x}{N}\right)$ = Antilog(1.4700)

= 29.51

iii) HM = $\frac{N}{\Sigma(f/x)} = 24.10$

AM > G.M > HM

44.b.

X	f	Cf	$\lvert D\rvert = \lvert X-55\rvert$	$f\lvert D\rvert$
15	12	12	40	480
25	11	23	30	330
35	10	33	20	200
45	15	48	10	150
55	22	70	0	0
65	13	83	10	130
75	18	101	20	360
85	19	120	30	570
		N=120		$\Sigma f\lvert D\rvert$=2220

Median = size of $\left(\frac{N+1}{2}\right)^{th}$ value = size of 60.5^{th} value

= 55

Mean Deviation about Median = $\frac{\Sigma f\lvert D\rvert}{N} = \frac{2220}{120}$

M.D about Median = 18.5

Coeff. of M.D about Median = $\frac{\text{M.D about Median}}{\text{Median}}$

= 0.33.

45. a.

$E_1 \rightarrow$ x speaks truth

$E_2 \rightarrow$ x tells lie

$A \rightarrow$ x reports a six.

$P(E_1) = \frac{4}{5}$ $P(E_2) = \frac{1}{5}$

$P(A/E_1) = 1/6$, $P(A/E_2) = \frac{5}{6}$

By Baye's theorem,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{4}{9}$$

45. b.

$p = 400 - 2x - 3x^2$

$R = px = 400x - 2x^2 - 3x^3$

$MR = \frac{dR}{dx} = 400 - 4x - 9x^2$ —(1)

$\frac{dp}{dx} = -2 - 6x$, $\frac{dx}{dp} = \frac{-1}{2(1+3x)}$

$$\eta_d = \frac{-p}{x} \frac{dx}{dp}$$

$$\eta_d = \frac{400 - 2x - 3x^2}{2x(1+3x)}$$

$$1 - \frac{1}{\eta_d} = \frac{400 - 4x - 9x^2}{400 - 2x - 3x^2}$$

$$p\left(1 - \frac{1}{\eta_d}\right) = 400 - 4x - 9x^2$$ —(2)

From (1) and (2)

$$MR = p\left(1 - \frac{1}{\eta_d}\right).$$

46. a.

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$L[f'(0)] = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{h \to 0} \frac{f(-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{|-h|}{-h}$$

$$= -1$$

$$R[f'(0)] = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{|h|}{h}$$

$$= 1$$

$L[f'(0)] \neq R[f'(0)]$

$\therefore f(x)$ is not differentiable at $x = 0$.

46. b.

$$y = \frac{x^2 + x + 1}{x^2 - x + 1}$$

$$\frac{dy}{dx} = \frac{(x^2 - x + 1)\frac{d}{dx}(x^2 + x + 1) - (x^2 + x + 1)\frac{d}{dx}(x^2 - x + 1)}{(x^2 - x + 1)^2}$$

$$= \frac{-2x^2 + 2}{(x^2 - x + 1)^2}$$

$$= \frac{2(1 - x^2)}{(x^2 - x + 1)^2}$$

47. a.

$4y = x^2 + 6x + 21$

$(x+3)^2 = 4(y-3)$

$X^2 = 4Y$

where $X = x+3$, $Y = y-3$

$\boxed{a=1}$

Axis $X=0$	$x=-3$
Vertex $V(0,0)$	$V(-3,3)$
Focus $F(0,1)$	$F(-3,4)$
Equation of Directrix $Y=-1$	$y=2$
L.R $=4$	4

47. b.

$n=11, \quad x=x \quad a=1/x$

Middle terms are $= t_{\frac{n+1}{2}}, t_{\frac{n+3}{2}}$

$= t_6, t_7$

$t_{r+1} = nC_r \, x^{n-r} a^r$

$t_6 = 11C_5 \, x^{11-5} \left(\frac{1}{x}\right)^5$

$= 11C_5 \, x$

$t_7 = 11C_6 \, x^{11-6} \left(\frac{1}{x}\right)^6 = \frac{11C_6}{x}$

$= \frac{11C_5}{x}$ $\{\because nC_r = nC_{n-r}\}$

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