WAY TO SUCCESS

Leads to Success

Common Half yearly Examination -2018

XI – Mathematics – Answer Key

Part – I

1	2	6	1	11	4	16	4
2	4	7	2	12	3	17	4
3	4	8	4	13	2	18	2
4	1	9	3	14	4	19	2
5	4	10	3	15	1	20	1

Part – II

21. $n(A) = 3$

$n(B) = 2$

$A = \{x, y, z\}$

$B = \{1, 2\}$

22. $2|x+1| - 6 \le 7$

$2|x+1| \le 13$

$-\frac{15}{2} \le x \le \frac{11}{2}$

$x \in \left[\frac{-15}{2}, \frac{11}{2}\right]$

24. 6 distinct letters SIMPLE

∴ The number of permutation of the 6 letters all taken at a time $= 6P_6 = 6! = 720$

25. $\frac{n(n-1)}{2!} = -\frac{1}{9}$, $n = \frac{2}{3}$

$\frac{n(n-1)(n-2)}{2!} = \frac{4}{81}$

$(1+x)^{2/3} = 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 + \ldots$

23. LHS $= \tan(360° - 45°)[-\cot(360° + 45°)] + \cot(360° + 135°)[-\tan(360° + 225°)]$

$= [-\tan 45°][-\cot 45°] + [-\tan 45°][-\tan 45°]$

$= (-1)(-1) + (-1)(-1)$

$= 2$

26.

27. A is skew symmetric

$A^T = -A$

$$\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -3 \\ -2 & x^3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & -x^3 \\ -2 & 3 & 0 \end{pmatrix}$$

$x^3 = 3$

$x = \left(\frac{1}{3}\right)^{1/3} = \sqrt[3]{3}$

28. Let $y = 2^x = e^{x \log 2}$

$u = (\log 2)\, x$

$y = e^u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$= e^u \times \log 2$

$= \log 2 \; e^{x \log 2}$

$= (\log 2)\, 2^x$

29. Put $+5x = y \Rightarrow x = +\frac{y}{5}$

$$\lim_{x \to 0} \frac{e^{5x}-1}{x} = \lim_{y \to 0} \frac{e^y - 1}{y/5} = 5 \lim_{y \to 0} \frac{e^y - 1}{y}$$

$= 5(1)$

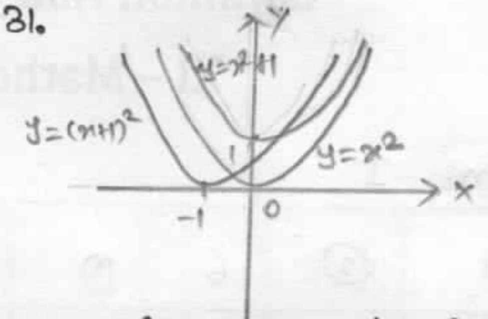
$= 5$

30. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$= \frac{41}{7}$

Part - III

31.



$f(x) = x^2+1$ causes the graph of the function $f(x) = x^2$ shifts to upward one unit.

$f(x) = (x+1)^2$ causes the graph of the function $f(x) = x^2$ shifts to left for one unit.

32. $$\frac{x}{(x+3)(x-4)} = \frac{A}{(x+3)} + \frac{B}{(x-4)}$$

$A = \frac{3}{7},\ B = \frac{4}{7}$

$$\frac{x}{(x+3)(x-4)} = \frac{3}{7(x+3)} + \frac{4}{7(x-4)}$$

33. $8^{1+\cos x+\cos^2 x+\cdots} = 8^2$

$1+\cos x+\cos^2 x+\cdots = 2$

$\frac{1}{1-\cos x} = 2$

$1 = 2(1-\cos x)$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

34. $$\frac{{}^nP_r}{{}^nC_r} = r!$$

$$r! = \frac{11880}{495} = 24 = 4!$$

${}^nC_4 = 495$

$n = 12$

35. Let $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$

$a = k^x,\ b = k^y,\ c = k^z$

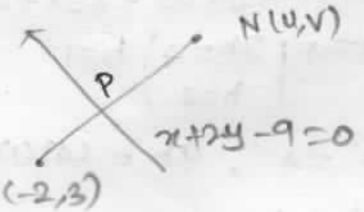
$b^2 = ac$ (∵ a, b, c are in G.P)

$k^{2y} = k^{x+z}$

$2y = x+z$

∴ x, y, z are in A.P

36.



$P\left(\frac{-2+u}{2}, \frac{3+v}{2}\right)$

P lies on $x+2y-9=0$

$u+2v-14=0 \rightarrow (1)$

$\begin{pmatrix}\text{slope of} \\ AN\end{pmatrix}\begin{pmatrix}\text{slope of} \\ x+2y-9=0\end{pmatrix} = -1$

$v-3 = 2(u+2)$

$2u - v + 7 = 0 \rightarrow (2)$

solve (1),(2)

$u = 0,\ v = 7.$

∴ Image of the point A(-2,3) is N(0,7)

37. $LHS = \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$= 0$

38. $\vec{OG} = \frac{\vec{OA}+\vec{OB}+\vec{OC}}{3}$

$\vec{GA}+\vec{GB}+\vec{GC}$

$= \vec{OA}-\vec{OG}+\vec{OB}-\vec{OG}+\vec{OC}-\vec{OG}$

$= \vec{OA}+\vec{OB}+\vec{OC} - 3\left(\frac{\vec{OA}+\vec{OB}+\vec{OC}}{3}\right)$

$= \vec{0}$

39. $\frac{3^x-1}{\sqrt{1+x}-1} = \frac{3^x-1}{\sqrt{1+x}-1} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$

$= \frac{(3^x-1)(\sqrt{1+x}+1)}{x}$

$\lim_{x\to 0} \frac{3^x-1}{\sqrt{1+x}-1} = \lim_{x\to 0}\frac{3^x-1}{x} \lim_{x\to 0}\sqrt{1+x}+1$

$= (\log 3)(2)$

$= 2\log 3$

$= \log 9$

40. $f'(x) = 4x+3$

$f'(0) = 3$

$f'(-1) = -1$

$f'(0) + 3f'(-1) = 3-3 = 0$

Part-IV

41. (a) one-one: $f(x) = f(y)$

$2x-3 = 2y-3$

$x = y$

$f(x) = f(y)$

f is one-one

onto: $y \in R$, Let $x = \frac{y+3}{2}$

$f(x) = 2\left(\frac{y+3}{2}\right) - 3 = y$

f is onto

Inverse: Let $y = 2x-3$

$x = \frac{y+3}{2}$

$f^{-1}(y) = \frac{y+3}{2}$

Replacing y as x

$f^{-1}(x) = \frac{x+3}{2}$

41. (b) $\log_{5-x}(x^2-6x+65)=2$

$\log_a b = x$

$\Rightarrow a^x = b$

$(5-x)^2 = x^2-6x+65$

$x=-10$

42. (a) $f(4)=10$

$f(-4)=-7$

$f(0)=-2$

$f(-7)=-13$

42. (b) $\sin 2A+2\sin 2B+2\sin 2C$

$= 2\sin(A+B)\cos(A-B)+\sin 2C$

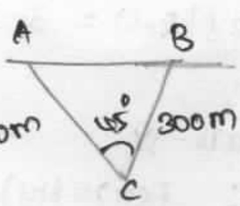
$= 2\sin C\cos(A-B)+2\sin C\cos C$

$= 2\sin C[\cos(A-B)-\cos(A+B)]$

$= 2\sin C[2\sin A\sin B]$

$= 4\sin A\sin B\sin C$

43. (a)



ΔABC, By cosine formula

$AB^2 = AC^2+BC^2-2AC.BC.\cos\frac{\pi}{4}$

$AB = 210.95m$

43. (b) $P(n) = 2+4+6\ldots+2n = n^2+n$

Basic step: $P(1) = 1+1 = 2$

$\therefore P(1)$ is true

Induction step: Assume that $P(k)$ is true for some +ve integer k

$P(k) = 2+4+6+\ldots+2k = k^2+k$

To Prove: $P(k+1)$ is also true

$[2+4+6+\ldots+2k]+2(k+1) = k^2+k+2(k+1)$

$= (k+1)^2+(k+1)$, $P(k+1)$ is true

44. (a) $\sqrt[3]{x^3+7} = x+\frac{7}{3}\times\frac{1}{x^2}-\frac{49}{9}\times\frac{1}{x^5}+\ldots$

$\sqrt[3]{x^3+4} = x+\frac{4}{3}\times\frac{1}{x^2}-\frac{16}{9}\times\frac{1}{x^5}+\ldots$

$\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4}$ $\ldots$

$=\left(x+\frac{7}{3}\times\frac{1}{x^2}\right)-\left(x+\frac{4}{3}\times\frac{1}{x^2}\right)=\frac{1}{x^2}$

44. (b) compare with

$ax^2+2hxy+by^2+2gx+2fy+c=0$

$a=4$, $2h=4$, $h=2$, $b=1$

$h^2=4$, $ab=(4)(1)=4$

$h^2=ab$

$\therefore$ Given lines are Pair of Parallel lines

Distance $=\sqrt{5}$

45. (a) $\Delta = \begin{vmatrix} -2a & a+b & a+c \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$

puting $b=-a$

$\Delta=0$, $(a+b)$ is a factor of Δ.

$|||^{ly}$ $(b+c)$, $(c+a)$ are factors of Δ.

Δ is third degree homogeneous Polynomial

$\Delta = k(a+b)(b+c)(c+a)$, k is constant

$\begin{vmatrix} -2a & a+b & a+c \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = k(a+b)(b+c)(c+a)$

Put $a=0, b=1, c=2 \Rightarrow k=4$

$\therefore \Delta = 4(a+b)(b+c)(c+a)$

45. (b) $5\hat{i}+6\hat{j}+7\hat{k} = s(7\hat{i}-8\hat{j}+9\hat{k}) + t(3\hat{i}+20\hat{j}+5\hat{k})$

Equating components

$7s+3t=5 \longrightarrow ①$

$-8s+20t=6 \longrightarrow ②$

$9s+5t=7 \longrightarrow ③$

solve ①, ② $s=1/2,\ t=1/2$

sub in ③ $9(\frac{1}{2})+5(\frac{1}{2})=7$

∴ Thus one vector is a linear combination of other two vectors

∴ Hence the given vectors are coplanar.

46. (a) $A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$,

$A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$

$k=2$

46. (b) $\vec{a}\times\vec{b} = 2\hat{i}-2\hat{j}+\hat{k}$

$|\vec{a}\times\vec{b}| = 3$

$|\vec{c}-\vec{a}|^2 = (2\sqrt{2})^2$

$|c|^2+|\vec{a}|^2-2\vec{c}\cdot\vec{a} = 8$

$|\vec{c}|=1$

$|(a\times b)\times c| = |a\times b||c|\sin 30^\circ$

$= 3(1)(\frac{1}{2})$

$= \frac{3}{2}$

47. (a) For any point $c \in (-1,1)$

$\lim_{x\to c} f(x) = \lim_{x\to c}\sqrt{1-x^2} = f(c)$

$\lim_{x\to 1^+} f(x) = 0 = f(1)$

$\lim_{x\to 1^-} f(x) = 0 = f(-1)$ ∴ Thus f is continuous on $[-1,1]$

47. (b) $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$

$y\sqrt{1-x^2} = \sin^{-1}x$

Diff. w.r.to 'x'

$-xy+(1-x^2)y_1 = 1$

Again D. w.r.to 'x'

$-[xy_1+y(1)]+(1-x^2)y_2 + y_1(-2x) = 0$

$-xy_1-y+(1-x^2)y_2-2xy_1=0$

$(1-x^2)y_2-3xy_1-y=0$

—x—

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