**SHRI VIDHYABHARATHI MATRIC HR.SEC.SCHOOL**
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**COMMON HALFYEARLY EXAMINATION 2018**

**STD: XII** **19.12.2018**

**SUBJECT: BUSINESS MATHEMATICS** **ANSWER KEY** **MARKS : 90**

## SECTION I

| | |
|---|---|
| 1. | b. $\frac{1}{k}I$ |
| 2. | a. 1 |
| 3. | b. 2 |
| 4. | c. $a^2/2$ |
| 5. | b. $3/2$ |
| 6. | d. 3 |
| 7. | a. 1000 |
| 8. | a. Rs. 50 |
| 9. | b. $64/5$ |
| 10. | b. $e^{3x}+k$ |
| 11. | a. 1, 1 |
| 12. | c. $xe^{\int P\,dy} = \int Qe^{\int P\,dy}\,dy + c$ |
| 13. | a. $3x^2+3x+1$ |
| 14. | c. 3 |
| 15. | c. 3 |
| 16. | b. $\mu \pm \sigma$ |
| 17. | c. principle of Statistical Regularity |
| 18. | c. Sampling is done to estimate the population parameter |
| 19. | a. Independent variable |
| 20. | c. four Components. |

## SECTION - II

21) $I-B = \begin{bmatrix} \frac{2}{5} & \frac{-9}{10} \\ \frac{-1}{5} & \frac{1}{5} \end{bmatrix}$ The main diagonal elements are positive.

$|I-B| = \frac{-5}{50} < 0$

The given system is not viable.

22) Let $(x, y)$ be another focus.

Centre = Mid point of foci.

$(2,3) = \left(\frac{x+3}{2}, \frac{y+3}{2}\right)$

$\frac{x+3}{2} = 2$ $\qquad \frac{y+3}{2} = 3$

$x = 1$ $\qquad y = 3$

$\therefore$ other focus is $(1, 3)$

23) At equilibrium $q_d = q_s$

$4 - 0.05p = 0.8 + 0.11p$

$0.16p = 3.2$

$p = 20$

At $p = 20$, $q = 4 - 0.05(20) = 4-1$

$q = 3$

At equilibrium $q = 3$ and $p = 20$

24) $P = 3K^2L^2 - 2L^4 - K^4$

$\frac{\partial P}{\partial L} = 6K^2L - 8L^3$ | $\frac{\partial P}{\partial K} = 6KL^2 - 4K^3$

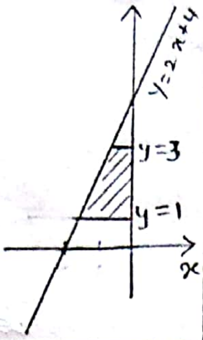
$L\frac{\partial P}{\partial L} = 6K^2L^2 - 8L^4$ | $K\frac{\partial P}{\partial K} = 6K^2L^2 - 4K^4$

$L\frac{\partial P}{\partial L} + K\frac{\partial P}{\partial K} = 4P$.

(25) $A = \int_1^3 (-x)\,dy$

$= \int_1^3 -\left(\frac{y-4}{2}\right) dy$

$= 2$ sq. units




(26) $\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$

$\tan^{-1} y = \tan^{-1} x + c$

$\tan^{-1}\left(\frac{x-y}{1+xy}\right) = c$

$x - y = c(1+xy)$

(27) $\Delta^3 [f(x_0)] = 0$

$(E-1)^3 y_0 = 0$

$(E^3 - 3E^2 + 3E - 1) y_0 = 0$

$y_3 - 3y_2 + 3y_1 - y_0 = 0$

$257 - 3y_2 + 3(226) - 200 = 0$

$y_2 = 245$

(28) $P(X=1) = 5C_1\, pq^4 = 0.4096$ — (1)

$P(X=2) = 5C_2\, p^2q^3 = 0.2048$ — (2)

From (1) × (2),

$\frac{5C_1\, pq^4}{5C_2\, p^2q^3} = \frac{0.4096}{0.2048}$

$q = 4p$

we know that, $p + q = 1$

$p = 1/5$

(29) $n = 50$, Sample mean $\bar{x} = 67.9$

95% Confidence limits

$\bar{x} \pm (Z_c)\{S.E(\bar{x})\}$.

$\Rightarrow 67.9 \pm 1.64$

The 95% Confidence intervals for estimating $\mu$ is given by $(66.2,\ 69.54)$

(30) $r(x,y) = \frac{N\Sigma xy - \Sigma x \Sigma y}{\sqrt{N\Sigma x^2 - (\Sigma x)^2}\ \sqrt{N\Sigma y^2 - (\Sigma y)^2}}$

$= \frac{11(2827) - 117(260)}{\sqrt{11(1313) - (117)^2}\ \sqrt{11(6580) - (260)^2}}$

$= 0.3566$

SECTION - III

(31) $\Delta = \begin{bmatrix} 6 & -7 \\ 9 & -5 \end{bmatrix} = 33 \neq 0$

$\Delta_x = \begin{vmatrix} 16 & -7 \\ 35 & -5 \end{vmatrix} = 165$

$\Delta_y = \begin{vmatrix} 6 & 16 \\ 9 & 35 \end{vmatrix} = 66$

$x = \frac{\Delta x}{\Delta} = 5, \quad y = \frac{\Delta y}{\Delta} = 2$

(32) The Combined Equation of asymptotes

$(3x - 4y + 7)(4x + 3y + 1) = 0$

The equation of hyperbola is

$(3x - 4y + 7)(4x + 3y + 1) = k$

Passes through $(0,0)$

$k = 7$

Combined equation is

$(3x - 4y + 7)(4x + 3y + 1) = 7$

$12x^2 - 12y^2 - 7xy + 31x + 17y = 0$

which is the required equation of the hyperbola.

(33) $y = 2x\left(\frac{x+4}{x+3}\right)+3$

Marginal cost is $dy/dx$

$$\frac{dy}{dx} = \frac{(x+3)(4x+8)-(2x^2+8x)(1)}{(x+3)^2}+0$$

$$= 2\left(1+\frac{3}{(x+3)^2}\right)$$

As $x$ increases, the MC decreases

(34) $P = \frac{200x}{150+x} - x$

$$p'(x) = \frac{30000}{(150+x)^2} - 1$$

$$p''(x) = \frac{-60000}{(150+x)^3}$$

$p'(x) = 0 \Rightarrow x = 23$

when $x = 23$, $P''(x) < 0$

$\therefore$ Profit is maximum.

(35) $MR = 9 - 2x + 4x^2$

$$R = \int(9-2x+4x^2)dx + k$$

$$= 9x - x^2 + \frac{4x^3}{3} + k$$

when $x = 0$, $R = 0 \Rightarrow k = 0$

$$R = 9x - x^2 + \frac{4x^3}{3}$$

$$p = \frac{R}{x} \Rightarrow p = 9 - x + \frac{4x^2}{3}$$

(36) $P = \cot x$ $\quad Q = 4x \operatorname{cosec} x$

$$e^{\int P dx} = \sin x$$

Solution is

$$y \sin x = \int \sin x \, 4x \operatorname{cosec} x \, dx$$

$$= 2x^2 + c$$

Given that $y = 0$ when $x = \pi/2 \Rightarrow c = -\frac{\pi^2}{2}$

$$\therefore y\sin x = 2x^2 - \frac{\pi^2}{2}.$$

(37) $h = 10$ $\quad x = 85$ $\quad x_5 = 100$

$$u = \frac{x - x_5}{h} = -1.5$$

| $x$ | $y$ | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ | $\nabla^4 y$ | $\nabla^5 y$ |
|---|---|---|---|---|---|---|
| 50 | 184 | 20 | | | | |
| | | | 2 | | | |
| 60 | 204 | 22 | | 0 | | |
| | | | 2 | | 0 | |
| 70 | 226 | 24 | | 0 | | 0 |
| | | | 2 | | 0 | |
| 80 | 250 | 26 | | 0 | | |
| | | | 2 | | | |
| 90 | 276 | 28 | | | | |
| 100 | 304 | | | | | |

$$y = y_5 + u\nabla y_5 + \frac{u(u+1)}{2!}\nabla^2 y_5 + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_5 + \cdots$$

$$y = 262.75$$

(38) $p(x_i) \geq 0$ for all $x$

$$\Sigma p(x_i) = P(x=1) + P(x=2) = 1$$

$p(x_i)$ is a p.d.f.

$$F(1) = P(x \leq 1) = P(x=1) = 1/3$$

$$F(2) = P(x \leq 2) = P(x=1) + P(x=2) = 1$$

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1/3 & \text{if } 1 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$

(39) $N = 1000$, $p = 0.55$ $\quad q = 0.45$

99% Confidence level,

$$0.55 \pm 2.58\sqrt{\frac{0.55(0.45)}{1000}}$$

$\Rightarrow 0.55 \pm 0.040635$

$\Rightarrow (0.509365, 0.590635)$

40)

| Commodity | $p_0$ | $p_1$ | V | $P=\frac{p_1}{p_0}\times 100$ | PV |
|---|---|---|---|---|---|
| A | 16.00 | 20.00 | 40 | 125.00 | 5000.00 |
| B | 40.00 | 60.00 | 25 | 150.00 | 3750.00 |
| C | 0.50 | 0.50 | 5 | 100.00 | 500.00 |
| D | 5.12 | 6.25 | 20 | 122.07 | 2441.40 |
| E | 2.00 | 1.50 | 10 | 75.00 | 750.00 |
| | | | 100 | | 12441.40 |

$$CLI = \frac{\Sigma PV}{\Sigma V} = 124.41.$$

Hence there is 24.4 % increase in CLI in 1996 compared to 1995.

## SECTION - IV

41) a.

$$B = \begin{bmatrix} 1/4 & 3/10 \\ 1/2 & 1/2 \end{bmatrix}$$

$$I-B = \begin{bmatrix} \frac{3}{4} & -\frac{3}{10} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$|I-B| = 0.225$ (+ve)

The given system has solution.

$$\text{adj}(I-B) = \begin{bmatrix} \frac{1}{2} & \frac{3}{10} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

$$(I-B)^{-1} = \frac{1}{0.225}\begin{bmatrix} \frac{1}{2} & \frac{3}{10} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

$$X = (I-B)^{-1}D = \frac{1}{0.225}\begin{bmatrix} \frac{1}{2} & \frac{3}{10} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}\begin{bmatrix} 20 \\ 15 \end{bmatrix}$$

$$X = \begin{pmatrix} 64.44 \\ 94.44 \end{pmatrix}$$

41. (b)

$\mu = 151$, $\sigma = 15$ $N = 500$

$X = 120 \Rightarrow Z = \frac{X-\mu}{\sigma} = -2.067$

$X = 155 \Rightarrow Z = 0.2667$

i) $P(120 < X < 155) = P(-2.067 < Z < 0.2667)$

$= P(0 < Z < 2.067) + P(0 < Z < 0.2667)$

$= 0.5829$

No. of students weight between 120 and 155 pounds $= 500 \times 0.5829$

$= 291$ students.

ii) $X = 185$, $Z = 2.2667$

$P(X > 185) = P(Z > 2.2667)$

$= P(0 < Z < \infty) - P(0 < Z < 2.2667)$

$= 0.0119.$

No. of students weight more than 185 pounds $\} = 500 \times 0.0119$

$= 6$ students.

42 a.

$9x^2 + 16y^2 + 36x - 32y - 92 = 0$

$$\frac{(x+2)^2}{16} + \frac{(y-1)^2}{9} = 1$$

$$\frac{X^2}{16} + \frac{Y^2}{9} = 1$$

$X = x+2$ $Y = y-1$

$a^2 = 16$ $b^2 = 9$

$e = \sqrt{7}/4$, $ae = \sqrt{7}$, $\frac{a}{e} = \frac{16}{\sqrt{7}}$

$L.R = \frac{9}{2}$

| | |
|---|---|
| Centre (0,0) | (-2,1) |
| Foci $(\sqrt{7},0)$ and $(-\sqrt{7},0)$ | $(\sqrt{7}-2,1)$ and $(-\sqrt{7}-2,1)$ |
| vertex $(\pm 4,0)$ | (2,1), (-6,1) |
| Directrix $X = \pm 16/\sqrt{7}$ | $x = \pm\frac{16}{\sqrt{7}} - 2$ |

42. (b)

$P = 0.6$

$p = 0.367$

$q = 0.633$, $n = 150$

$H_0$ : Null hypothesis $P = 0.6$

$H_1$ : Alternate hypothesis $P \neq 0.6$

$$Z = \frac{p - P}{\sqrt{\frac{Pq}{n}}} = -5.92$$

$|Z| = 5.92$

At 1% level $|Z| \geq 2.58$

$\Rightarrow Z$ is in Critical region

Null hypothesis is rejected.

43. (a) At Market equilibrium.

$$P_d = P_s \Rightarrow \frac{16}{x+4} = \frac{x}{2}$$

$x = -8$ (or) $x = 4$

At $x = 4$ $P = 2$

$x_0 P_0 = 8$

$$CS = \int_0^{x_0} f(x)dx - P_0 x_0$$

$$= \int_0^4 \frac{16}{x+4}dx - 8$$

$$= 16(\log 2) - 8 \text{ units}$$

$$PS = P_0 x_0 - \int_0^{x_0} g(x)\,dx$$

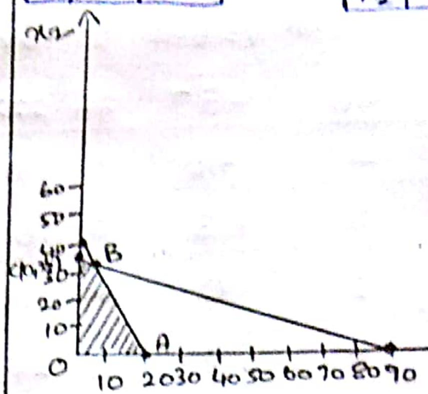
$$= 8 - \int_0^4 \frac{x}{2}dx$$

$= 4$ units.

43. (b) $2x_1 + x_2 = 40$ — (1) $\quad 2x_1 + 5x_2 = 180$ — (2)

| $x_1$ | 0 | 20 |
|---|---|---|
| $x_2$ | 40 | 0 |

| $x_1$ | 0 | 90 |
|---|---|---|
| $x_2$ | 36 | 0 |




Solve (1) and (2), $x_1 = 2.5$, $x_2 = 35$

$\therefore C(2.5, 35)$

The Coordinates of Extreme points are $O(0,0)$, $A(20,0)$ $B(2.5,35)$ $C(0,36)$.

$\therefore \max Z = 147.5$ at $x_1 = 2.5, x_2 = 35$.

44. (a) i) $R = px = 400x - \frac{x^2}{1000}$

$$\frac{dR}{dt} = \left(400 - \frac{x}{500}\right)\frac{dx}{dt}$$

when $x = 10{,}000$ and $\frac{dx}{dt} = 200$

$\frac{dR}{dt}$ = Rs. 76,000 Per week.

Revenue is increasing at a rate of Rs. 76,000 per week.

ii) $C(x) = 50x + 16000$

$$\frac{dC}{dt} = 50\frac{dx}{dt}$$

when $\frac{dx}{dt} = 200$, $\frac{dC}{dt}$ = Rs. 10,000/week

Cost is increasing at a rate of Rs. 10,000 Per week.

iii) $P = R - C \Rightarrow \frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt}$

$\frac{dP}{dt}$ = Rs. 66,000 Per week.

Profit is increasing at a Rate of Rs. 66,000/week.

44 (b) $y = ax + b$ be the line of best fit.

Let $u = x - 10$ $\quad v = y - 68$.

| $x$ | $y$ | $u = x-10$ | $v = y-68$ | $u^2$ | $uv$ |
|---|---|---|---|---|---|
| 4 | 31 | -6 | -37 | 36 | 222 |
| 9 | 58 | -1 | -10 | 1 | 10 |
| 10 | 65 | 0 | -3 | 0 | 0 |
| 12 | 68 | 2 | 0 | 4 | 0 |
| 14 | 73 | 4 | 5 | 16 | 20 |
| 22 | 91 | 12 | 23 | 144 | 276 |
| | | 11 | -22 | 201 | 528 |

$$a\Sigma u^2 + b\Sigma u = \Sigma uv$$

$$a\Sigma u + nb = \Sigma v$$

$$201a + 11b = 528 \quad \text{---(1)}$$

$$11a + 6b = -22 \quad \text{---(2)}$$

Solve (1) and (2),

$$a = 3.1428, \quad b = -9.4284$$

$$v = (3.1428)u - 9.4284$$

$$y = 3.1428x + 27.1436$$

when $x = 17$, $y = 80.5712$

45 (a) $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{\infty} x \cdot 2e^{-2x}\,dx$

By using Bernoulli formula,

$$E(x) = 2\left[x\frac{e^{-2x}}{-2} - 1\cdot\frac{e^{-2x}}{4}\right]_0^{\infty}$$

$$= \frac{1}{2}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^{\infty} x^2 2 \cdot e^{-2x}\,dx$$

$$= 2\left[x^2\frac{e^{-2x}}{-2} - \frac{(-2xe^{-2x})}{4}2 + \frac{e^{-2x}}{-8}\right]_0^{\infty}$$

$$= \frac{1}{2}$$

Variance $= E(x^2) - (E(x))^2$

$$= \frac{1}{4}.$$

45 (b) $I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}\,dx \quad \text{---(1)}$

$$I = \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x}\,dx \quad \text{---(2)}$$

$$\int_0^a f(x) = \int_0^a f(a-x)\,dx$$

(1)+(2) $\Rightarrow$

$$2I = \int_0^{\pi/2} (a+b)\,dx = (a+b)\frac{\pi}{2}$$

$$I = (a+b)\frac{\pi}{4}.$$

46. (a) $y(x-2)(x-3) - x + 7 = 0$

The curve cut the $x$-axis when $y = 0$.

$$x = 7$$

point of contact is $(7, 0)$.

$$y(x^2 - 5x + 6) - x + 7 = 0$$

$$y(2x-5) + (x^2 - 5x + 6)\frac{dy}{dx} - 1 = 0$$

At $(7,0) \Rightarrow m = \frac{dy}{dx} = \frac{1}{20}$.

Eqn. of tangent is

$$y - y_1 = m(x - x_1)$$

$$x - 20y - 7 = 0.$$

Eqn. of Normal is

$$20x + y = k$$

This equation passing through $(7,0)$

$$k = 140.$$

Equation of normal is

$$20x + y - 140 = 0.$$

46. (b)

| year | Value | 4 year moving Total | 4 year moving total Centered | Two 4 year moving Total |
|---|---|---|---|---|
| 1981 | 464 | – | – | – |
| 1982 | 515 | | | |
| 1982.5 | | 1964 | | |
| 1983 | 518 | | 3966 | 495.75 |
| | | 2002 | | |
| 1984 | 467 | | 4029 | 503.625 |
| | | 2027 | | |
| 1985 | 502 | | 4093 | 511.625 |
| | | 2066 | | |
| 1986 | 540 | | 4236 | 529.50 |
| | | 2170 | | |
| 1987 | 557 | | 4424 | 553 |
| | | 2254 | | |
| 1988 | 571 | | 4580 | 572.50 |
| | | 2326 | | |
| 1989 | 586 | | – | – |
| | | – | | |
| 1990 | 612 | | – | – |

47. (a)

put $y = vx$

$\frac{dy}{dx} = v + x$

$\frac{dx}{x} = \frac{\cos v}{\sin v} dv$

$\log x = \log \sin v + \log c$

$x = c \sin\left(\frac{y}{x}\right)$

At $(1, \pi/2) \Rightarrow c = 1$

$\therefore x = \sin(y/x)$.

47. (b)

$f(\theta) = \sin^2\theta, \quad [0, \pi]$

$f'(\theta) = \sin 2\theta$

$f''(\theta) = 2\cos 2\theta$

$f'(\theta) = 0$

$\theta = 0, \pi/2, \pi$

$0, \pi$ are end points of interval.

Critical number $\theta = \pi/2$

when $\theta = \pi/2$, $f''(\theta) < 0$

$\therefore f$ is Local maximum.

Max. Value $= 1$.

# Department of mathematics

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