CHAPTER-1
ELECTRIC CHARGES AND FIELDS

Electrostatics - study of forces, fields and potentials due to charges at rest.

Examples for static electricity are

- spark or hearing a crackle when we take off our synthetic clothes or sweater, particularly in dry weather
- Sensation of an electric shock while opening the door of a car or holding the iron bar of a bus after sliding from our seat.
- Lightning
- A comb rubbed with hair attracts small pieces of paper etc.

Electric Charge

- **Electric charge** is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.
- **The two types of charges are positive and negative (Named by Benjamin Franklin)**
- **Like charges** repels and unlike charges attracts.
- When amber rubbed with wool or silk cloth attracts light objects – discovered by Thales.
- **Electroscope** – device for charge detection
- It is a **scalar quantity** .
- SI unit of electric charge- **coulomb (C)**
- Charge of a proton is positive (1.602192×10^{-19} C)
- Charge of an electron is negative ($-1.602192 \times 10^{-19}$ C)
- Matter with **equal number of electrons and protons** are **electrically neutral.**
- Matter with excess number of electrons – negatively charged
- Matter with excess protons – positively charged.

Conductors

- Substances which allow passage of charges .
- Eg : Metals, human body etc
- The charge transferred to a conductor is distributed over the entire surface of the conductor.

Insulators

- Substances which does not allow passage of charges.
- Eg: plastic, rubber etc.
- The charge transferred to an insulator stays at the same place.

Grounding or Earthing

- The process of sharing charges with earth.
- Earthing provides a safety measure for electrical circuits and appliances.

Methods of charging a body

Rubbing (charging by friction)

- When two bodies are rubbed electrons are transferred from material with lower work function to material with higher work function.
- Work function – energy required to remove an electron from a metal surface.
- Body gains electrons- negatively charged
- Body which loses electron – positively charged.

Effect on the mass of a body due to rubbing

- Positively charged body – mass decreases
- Negatively charged body – mass increases

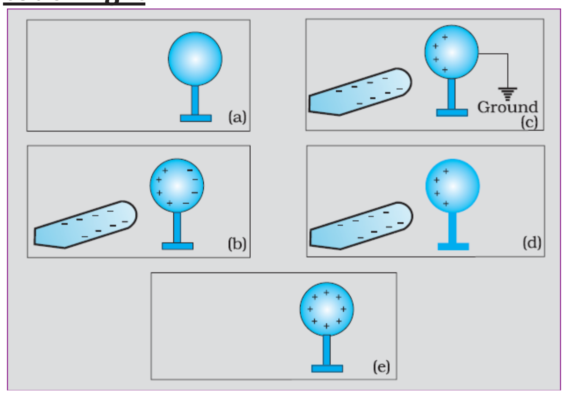
Conduction (by direct contact)

- When a charged body is brought in to contact with an uncharged conductor, charge flows from the charged body to the uncharged body.
- This is used to charge a conductor.

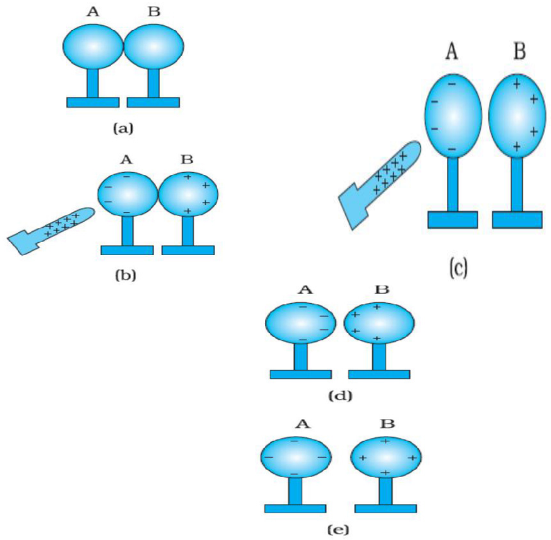
Induction – without direct contact

- When a charged body is brought near to an uncharged conductor (without touching), that end of the uncharged conductor which is near to the charged body gets oppositely charged and the farther end is charged with the same type of charge.

Charging a metal sphere positively without touching it



Charging of two spheres



Point charges

- If the sizes of charged bodies are very small as compared to the distances between them, we treat them as *point charges*.
- All the charge content of the body is assumed to be concentrated at one point in space.

Properties of electric charges

- **Charges are additive** –total charge of system is the sum of all charges.

  $Q = q_1+q_2+q_3+$ .....
- **Charges are quantized**- charge of a body in the universe is integer multiple of a basic charge (e).

  Q = ne, n- integer, e =1.6 X 10 $^{-19}$ C.
- The quantisation of charge was first suggested by the experimental laws of electrolysis discovered by Faraday.
- It was experimentally demonstrated by Millikan.
- **Charges are conserved** – the total charge of an isolated system is a constant.

Problem 1

- How many electronic charges form 1 C of charge?
- **Solution**

  q=ne, n= ?, e = 1.6×10^{-19} C,

  n= q/e = 6.25 x 10^{18}

Problem 2

- A comb drawn through person's hair causes 10^{22} electrons to leave the person's hair and stick to the comb. Calculate the charge carried by the comb.
- **Solution**

  n= 10^{22} , e = 1.6×10^{-19} C, **q = ne= 1.6x10^3 C**

  charge of comb = -1.6x10^3 C

Problem 3

- If a body gives out 10^9 electrons every second, how much time is required to get a total charge of 1C from it?
- **Solution**

  Number of electrons in 1s = 10^9

  Charge in 1s = ne = 10^9x**1.6 X 10 $^{-19}$**

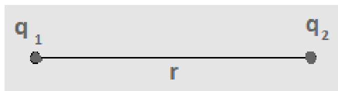
  = 1.6x10 $^{-10}$C

  Time to get 1 C charge

  = 1/(1.6x10 $^{-10}$C) = 6.25 x 10^9 s = 198.18 years

Coulomb's law

- The force of attraction or repulsion between two stationary electric charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.



- Force between two stationary charges is

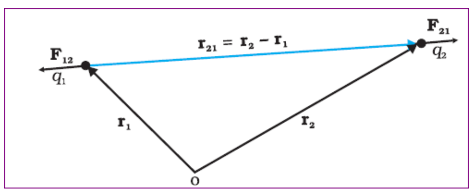
$$F=\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$$

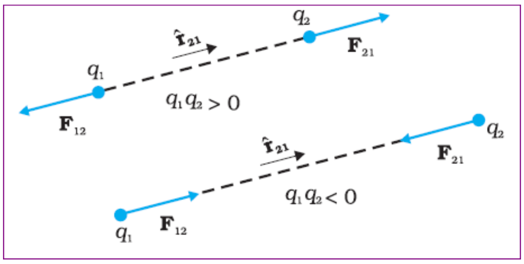
- Where ε_0 -permittivity of free space, ε_r -relative permittivity.
- Relative permittivity is given by , $\varepsilon_r=\frac{\varepsilon}{\varepsilon_0}$
- ε - Permittivity of the medium.
- Also ε_0 = 8.854x10^{-12} $C^2N^{-1}m^{-2}$
- Thus $\frac{1}{4\pi\varepsilon_0}=9\times10^9$

Definition of coulomb

- *When* $q_1 = q_2$ = 1 C, r = 1 m , F = 9 × 10^9 N
- 1 C is the charge that when placed at a distance of 1 m from another charge of the same magnitude *in vacuum* experiences an electrical force of repulsion of magnitude 9 × 10^9 N.

Coulomb's law in vector form





- Force on q_1 due to q_2 is,

$$\vec{F}_{12} = \frac{1}{4\Pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

- Force on q_2 due to q_1 is,

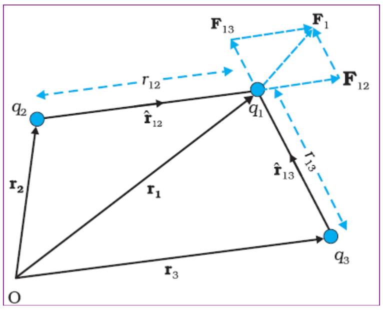
$$\vec{F}_{21} = \frac{1}{4\Pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

- Thus F_{12}= $-F_{21}$, Coulomb's law agrees with Newton's third law.

<u>Super position principle</u>

- Force on a charge due to a number of charges is the vector sum of forces due to individual charges.

$$\vec{F} = \vec{F_1} + \vec{F_2} + - - - - + \vec{F_n}$$



- The force on q_1 due to q_2 is

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

- The force on q_1 due to q_3 is

$$\mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13}$$

- Thus the total force on q_1 is

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13}$$

- For a system of n charges

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{\mathbf{r}}_{1i}$$

<u>Electric field</u>

- Region around a charge where its effect can be felt.
- Intensity of electric field at a point is the force per unit charge.

$$E = \frac{F}{q}$$

$$F = q\,E$$

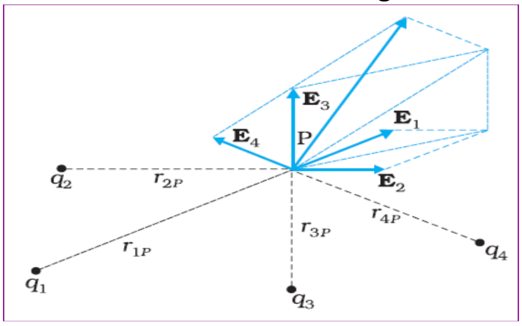
- Unit of electric field is N/C or V/m.
- It is a vector quantity.

<u>Electric field due to a point charge</u>

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

<u>Electric field due to a system of charges</u>

- Total electric field at a point due to a system of charges is the vector sum of the field due to individual charges.



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{\mathbf{r}}_{iP}$$

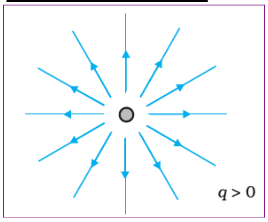
<u>Electric field lines</u>

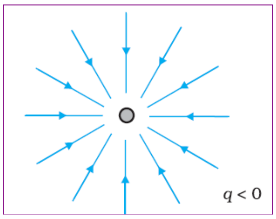
- Pictorial representation of electric field.
- Electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.

<u>Properties of field lines</u>

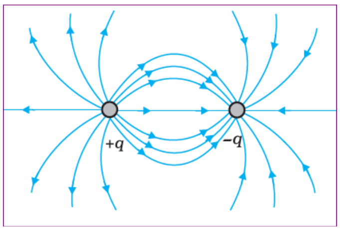
- Start from positive charge, end at negative charge. Do not form closed loops.
- Field lines are continuous in a charge free region.
- Two field lines never intersect.(Reason: two directions for electric field is not possible at a point)
- Field lines are parallel in uniform electric field.
- Tangent at any point gives direction of electric field.
- Number of field lines gives intensity of electric field.

positive charge **negative charge**

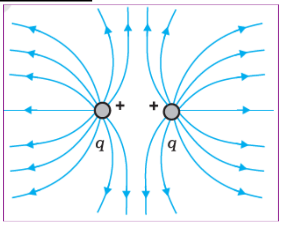




Positive and negative charge (dipole)

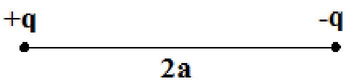


Two positive charges



Electric Dipole

- Two equal and opposite charges separated by a small distance.



- Total charge and force on a dipole is **zero.**

Dipole moment

- Product of charge and dipole length.

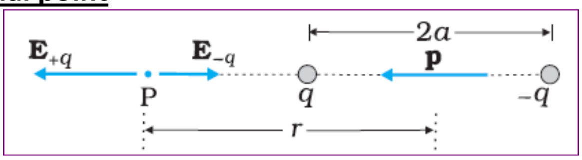
$$p = q \times 2a$$

q- charge, 2a- dipole length

- Direction is from negative to positive charge.
- SI unit- coulomb metre (C m)

Electric field due to a dipole

Axial point



- The field at the point P due to positive charge is

$$\mathbf{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\,\hat{\mathbf{p}}$$

- The field due to negative charge is

$$\mathbf{E}_{-q} = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}\,\hat{\mathbf{p}}$$

- Thus the total electric field at P is

$$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{\mathbf{p}}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r+a)^2 \times (r-a)^2}\right]\hat{p}$$

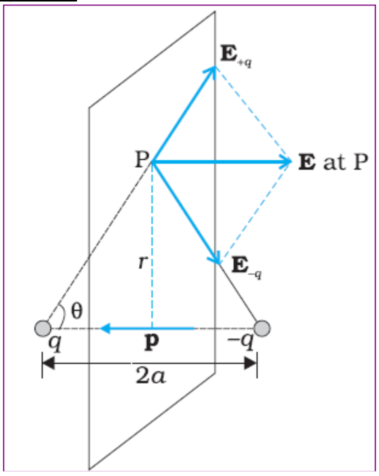
- Simplifying

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right]\hat{p}$$

- For r >> a, we get $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{4qa}{r^3}\right]\hat{p}$
- Using p= q x2a

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2p}{r^3}\right]\hat{p}$$

Equatorial point



- The magnitudes of the electric fields due to the two charges +*q* and –*q* are equal and given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2} \qquad E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2}$$

- The components normal to the dipole axis cancel away.
- The components along the dipole axis add up.
- Thus total electric field is

$$\mathbf{E} = -\left(E_{+q} + E_{-q}\right)\cos\theta\ \hat{\mathbf{p}}$$

- Substituting $\cos\theta = \frac{a}{(r^2+a^2)^{\frac{1}{2}}}$ and

simplifying we get

$$\vec{E} = \frac{-q \times 2a}{4\pi\varepsilon_0 \left(r^2 + a^2\right)^{\frac{3}{2}}} \hat{p}$$

- For r >> a, we get $\vec{E} = \frac{-q \times 2a}{4\pi\varepsilon_0 r^3} \hat{p}$
- Using p= q x2a

$$\vec{E} = \frac{-p}{4\pi\varepsilon_0 r^3} \hat{p}$$

Relation connecting axial field and equatorial field of dipole

- We have axial field

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \left[\frac{2p}{r^3}\right] \hat{p}$$

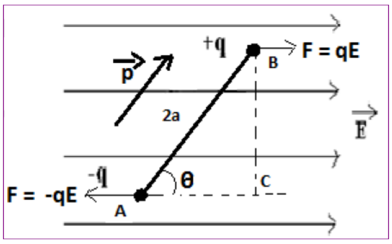
- Equatorial field

$$\vec{E} = \frac{-p}{4\pi\varepsilon_0 r^3} \hat{p}$$

- Thus

$$\boxed{\mathbf{E}_{\text{axial}} = 2 \times \mathbf{E}_{\text{equatorial}}}$$

Torque on a dipole in a uniform electric field

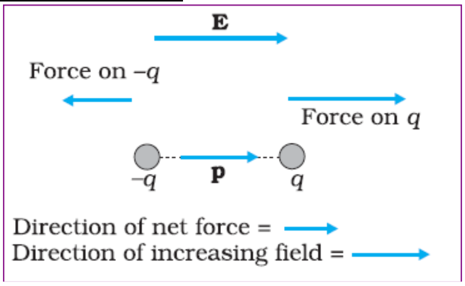


- Torque = force X perpendicular distance
  $\tau = qE \times 2a\sin\theta$, $\tau = pE\sin\theta$
  Or $\tau = p \times E$
- Torque is zero when p and E are in the same direction.
- Torque is maximum (= pE) , when p and E are perpendicular.
- The dipole rotates in a uniform electric field.
- As the total force is zero , there is no translational motion.

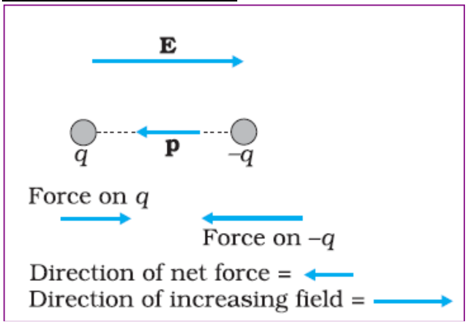
Torque on a dipole in a non uniform electric field

- In non uniform field there is a torque and net force on the dipole.
- Thus the dipole has rotational and translational motion.

- **E parallel to p**



- **E antiparallel to p**



How comb attracts tiny particles when charged?

- Comb acquires charge through rubbing.
- The charged comb induces dipole moment in the direction of the field.
- As the electric field due to the comb is not uniform, there acts a net force and paper moves.

Physical significance of electric dipole

Non Polar molecules

- The molecules in which positive centre of charge and negative centre of charge lie at the same place.
- Dipole moment is zero for a non polar molecule in the absence of an external field.
- They develop a dipole moment when an electric field is applied.
- Eg:CO_2, CH_4, etc.

Polar molecules

- The molecules in which the centres of negative charges and of positive charges do not coincide.
- Eg: water

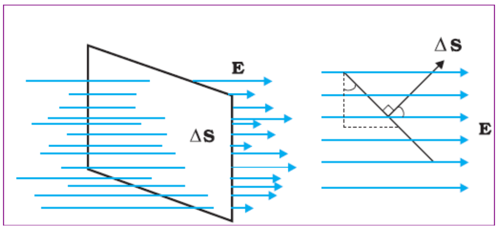
Electric flux

- Number of field lines passing normal through a surface.

$$\phi = EA\cos\theta$$

- Or

$$\boxed{\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{S} = E\,\Delta S\ \cos\theta}$$



- Unit – **Nm^2/ C**
- **It is a scalar quantity**

Charge density

Linear charge density (λ)

- It is the charge per unit length.

$$\lambda = \frac{Q}{l}$$

- SI unit is C/m.

Surface charge density (σ)

- It is the charge per unit area.

$$\sigma = \frac{Q}{A}$$

- SI unit is C/m^2.

Volume charge density (ρ)

- It is the charge per unit volume.

$$\rho = \frac{Q}{V}$$

- SI unit is C/m^3.

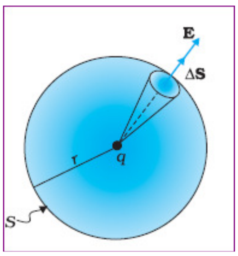
Gauss's Theorem

- Total electric flux over a closed surface is

$$\phi = \frac{q}{\varepsilon_0}$$

- Where q - total charge enclosed
- The closed surface – Gaussian surface.

Proof



- The flux through area element ΔS is

$$\boxed{\Delta\phi = \mathbf{E} \cdot \Delta\mathbf{S} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \cdot \Delta\mathbf{S}} \quad \boxed{\Delta\phi = \frac{q}{4\pi\varepsilon_0 r^2}\Delta S}$$

- The total flux through the sphere is

$$\phi = \sum_{all\ \Delta S} \frac{q}{4\pi\varepsilon_0 r^2}\Delta S$$

$$\phi = \frac{q}{4\pi\varepsilon_o r^2}\sum_{all\ \Delta S}\Delta S = \frac{q}{4\pi\varepsilon_0 r^2}S$$

- Where the total surface area S = 4πr^2.
- Thus

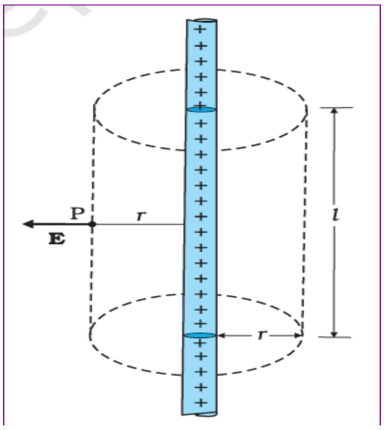
$$\phi = \frac{q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

Features of Gauss's law

- Gauss's law is true for any closed surface irrespective of the size and shape.
- The charge includes sum of all charges enclosed by the surface.
- Gauss's law is useful to calculate electric field when the system has some symmetry.
- Gauss's law is based on the inverse square dependence on distance contained in the Coulomb's law.

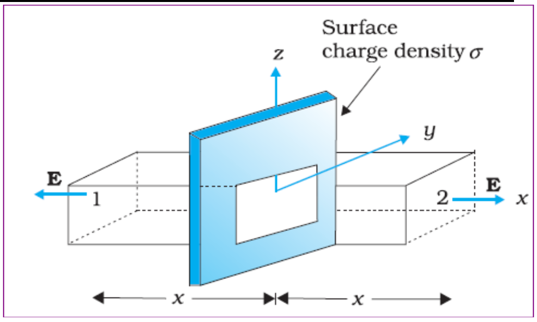
Applications of Gauss's law

Electric field due to a straight charged wire



- Total flux through the Gaussian surface is
$$\phi = E \times 2\pi r l$$
- Total charge enclosed is
$q = \lambda \times l$,   λ- charge per unit length
- Using Gauss's law
$$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$
- Thus $$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$
- In vector form $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$
- Where $\hat{n}$ - radial unit vector

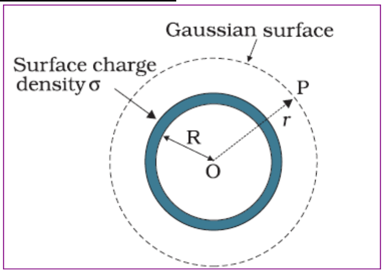
Electric field due to a plane sheet of charge



- Total flux enclosed by the Gaussian surface is $\phi = E \times (2A)$, A- area of cross section.
- Total charge enclosed is $q = \sigma A$, σ – surface charge density.
- Using Gauss's law $E \times (2A) = \frac{\sigma A}{\varepsilon_0}$
- Thus $E = \frac{\sigma}{2\varepsilon_0}$
- In vector form $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$
-

Electric field due to a charged spherical shell

Points outside the shell

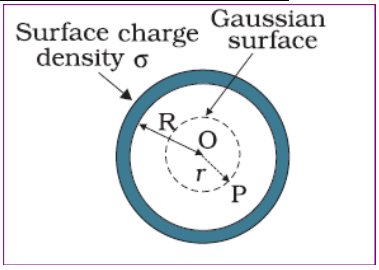


- Total flux enclosed by the Gaussian surface is $\phi = E \times (4\pi r^2)$, r- radius of Gaussian surface.
- Total charge enclosed is $q = \sigma \times (4\pi R^2)$, R –radius of shell
- Using Gauss's law $E \times (4\pi r^2) = \frac{\sigma \times 4\pi R^2}{\varepsilon_0}$
- Thus $E = \frac{\sigma R^2}{\varepsilon_0 r^2}$
- Or $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$
- In vector form $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

Points on the shell

- On the surface r=R, therefore $E = \frac{\sigma}{\varepsilon_0}$

Points inside the shell



- Total charge enclosed =0
  $E \times 4\pi r^2 = 0$
- Thus E= 0 inside the shell.
- Vanishing of electric field (E=0) inside a charged conductor is called **electrostatic shielding**

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