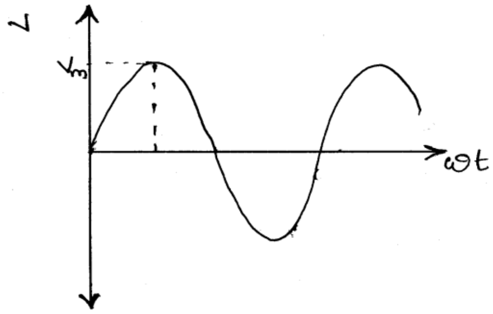
CHAPTER 7
ALTERNATING CURRENT

1. Write the equations for the instantaneous values of voltages and currents.
Ans:



A.C voltages and currents are represented by

$$v = v_m \sin \omega t \text{ and}$$
$$i = i_m \sin \omega t$$

v → instantaneous value of voltage
v_m → Peak value of voltage
ω → Angular frequency
ωt → Phase angle $\left[\omega = \frac{\theta}{t} \Rightarrow \theta = \omega t\right]$
i → instantaneous value of current
i_m → Peak value of current

2. Define r.m.s. value of a.c. Give the relation between the rms value and the peak value.
Ans: **r.m.s**. value of a.c. is defined as the d.c. equivalent which produces the same amount of heat energy in same time as that of an a.c.
Relation between r.m.s. value and peak value is $I_{rms} = \frac{i_m}{\sqrt{2}}$

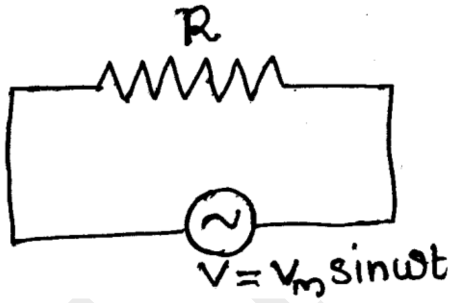
$$v_{rms} = \frac{v_m}{\sqrt{2}}$$

3[Q]. If the instantaneous current from an ac source is 10sin 314t ampere**, what will be the effective current in the circuit?**

4[P]. (a) The peak voltage of an ac supply is 300V. What is the rms voltage?

(b) The rms value of current in an ac circuit is 10A.What is the peak current?

5. Derive an expression for the current when an A.C. voltage applied to a resistor. What is the average power consumed in a complete cycle?
Ans:



Consider an a.c. voltage
$v = v_m \sin \omega t$ applied to a resistor R.

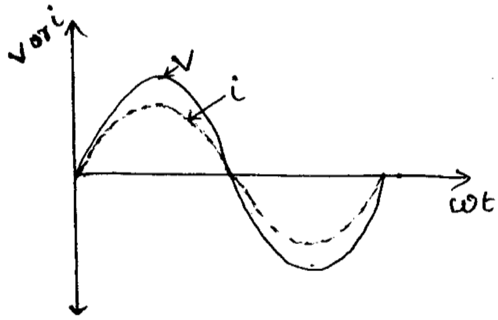
$$v = v_m \sin\omega t \;\ldots.\; (1)$$

Dividing equation (1) by R

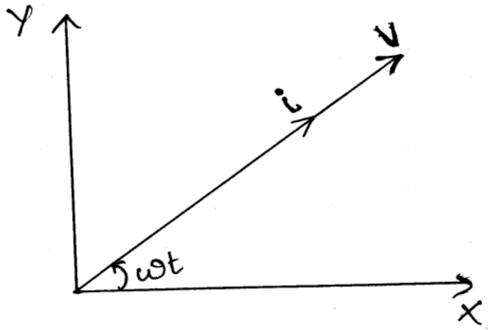
$$\frac{v}{R} = \frac{v_m}{R} \sin \omega t$$

$$\Rightarrow \boxed{i = i_m \sin\omega t} \;\ldots\ldots\ldots\; (2)$$

From equations (1) and (2), we can see that **both the voltage and current are in phase.**



<u>Phasor diagram</u>

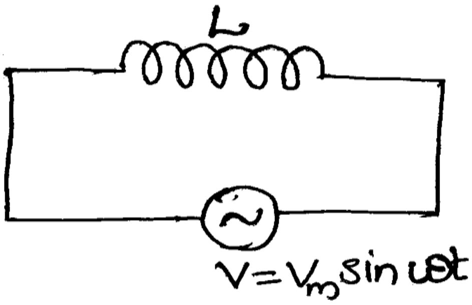


Average power consumed in a complete cycle:

$\langle P\rangle = V_{rms} I_{rms}$

6[P]. A 100-ohm resistor is connected to a 220 V, 50 Hz supply.
(a) What is the rms value of current in the circuit?
(b) What is the net power consumed over a full cycle?

7. Derive the expression for current in an A.C. circuit containing an inductor only.
Ans:



Consider an AC voltage
$v = v_m \sin \omega t$ applied to an inductor (L)

$V = v_m \sin \omega t$ ….. (1)

By kirchhoff's voltage rule

$$v - L\frac{di}{dt} = 0$$

$$v = L\frac{di}{dt}$$

$$V_m \sin \omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin \omega t \, dt$$

| $\sin(\pi/2 - \theta) = \cos\theta$
| $\sin(\theta - \frac{\pi}{2}) = -\cos\theta$

$$\therefore i = \int di$$

$$= \int \frac{V_m}{L}\sin \omega t \, dt$$

$$= \frac{V_m}{L}\cdot\frac{-\cos \omega t}{\omega}$$

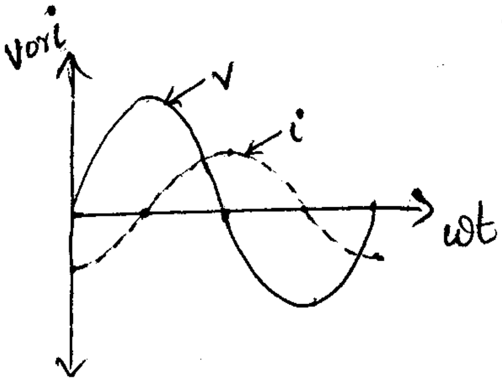
$$= \frac{V_m}{L\omega}(-\cos \omega t)$$

$= i_m(-\cos \omega t)$, but $L\omega = X_L$ is the inductive reactance

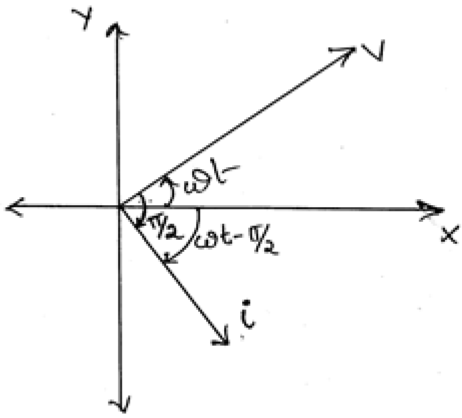
$= i_m \sin(\omega t - \frac{\pi}{2})$ $\quad\Big|\ \frac{V_m}{L\omega} = i_m$

$\boxed{i = i_m \sin(\omega t - \frac{\pi}{2})}$ ……… (2)

From equations (1) and (2) we can see that **the current lags behind the voltage by a phase angle** $\frac{\pi}{2}$**.**



Phasor Diagram



8. Define inductive reactance
Ans: Inductive reactance (X_L) is the resistance offered by the inductor towards the flow of a.c.
$\mathbf{X_L = L\omega}$
SI unit of X_L is **ohm.**

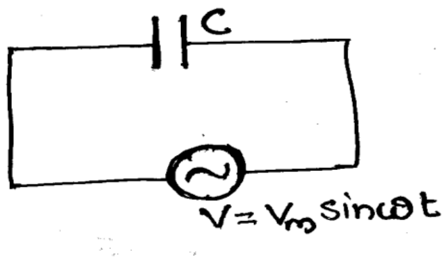
9. What is the average power consumed by inductor in a complete cycle.
Ans: Average power, $\langle P\rangle = 0$

10[P]. A 44mH **inductor is connected to** 220V, 50 Hz **ac supply,**

Determine the rms value of the current in the circuit.

11. Derive an expression for the current in an A.C circuit containing a capacitor.

Ans:



Consider an a.c. voltage

$v = v_m \sin \omega t$ applied to a circuit containing a capacitor only.

$v = v_m \sin \omega t$ …….. (1)

In a capacitor $v = \dfrac{q}{C}$

$\Rightarrow \dfrac{q}{C} = v_m \sin \omega t$

$i = \dfrac{dq}{dt} = \dfrac{d}{dt}(Cv_m \sin \omega t)$

$= Cv_m \dfrac{d}{dt}(\sin \omega t)$

$= C\, v_m \cos \omega t \times \omega$

$= (C\omega) v_m \cos \omega t$

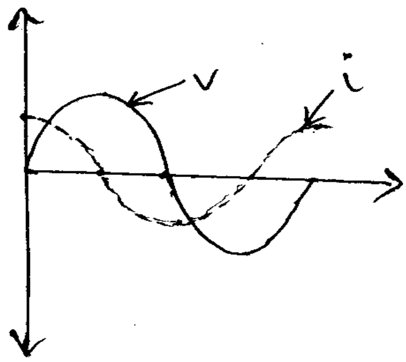
$= \dfrac{v_m \cos \omega t}{\left(\dfrac{1}{C\omega}\right)}$, but $\dfrac{1}{C\omega} = X_c$ is the capacitive reactance.

$i = \dfrac{V_m}{X_L} \cos \omega t$

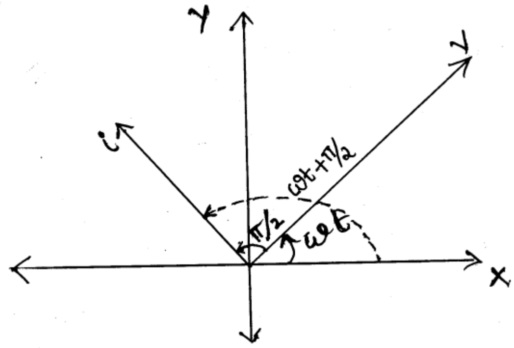
$\Rightarrow i = i_m \cos \omega t$

$\Rightarrow \boxed{i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)}$ …… (2)

From equations (1) and (2), We can see that the current leads the voltage by a phase angle $\frac{\pi}{2}$.



Phasor Diagram



12. Define Capacitative reactance (X_c)

Ans: Capacitative reactance is the is the resistance offered by the capacitor towards the flow of a.c.

$Xc = \dfrac{1}{c\omega}$

SI unit of X_c is **ohm**.

13. Capacitor blocks dc. Why?

Ans: In the case of d.c., $\boldsymbol{\omega = 0}$

So $X_c = \dfrac{1}{c\omega} = \dfrac{1}{c \times 0} = \dfrac{1}{0} = \infty$. Since reactance is infinity, capacitor blocks d.c.

14. What is the average power consumed by a capacitor in a complete cycle of a.c

Ans: $\langle P \rangle = 0$

15[P]. A 60μF **capacitor is connected to a** 110V, 60Hz **ac supply. Determine the rms value of current in the circuit**

16. What is meant by watt less current (idle current)?

Ans: In an a.c. circuit containing an inductor or capacitor only, the phase difference between voltage and current is $\frac{\pi}{2}$.

Therefore, the average power

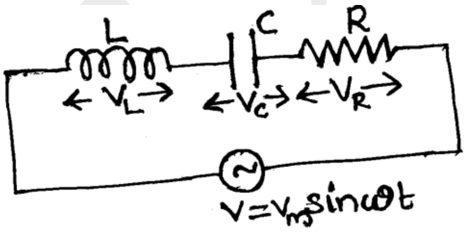
$$\langle P \rangle = V_{rms} I_{rms} \cos\phi$$
$$= V_{rms} I_{rms} \cos\frac{\pi}{2}$$
$$= V_{rms} I_{rms} \times 0 = 0$$

Since the **average power is zero**, the current in such circuits is called wattles current.

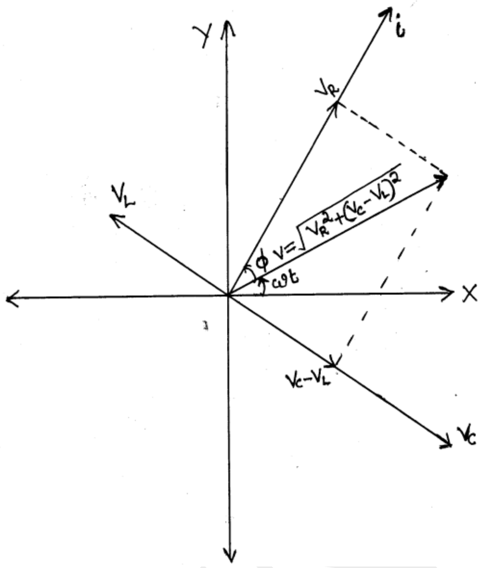
17[Q]. Show graphically the variation of Ohmic resistance, inductive reactance and capacitive reactance with frequency of AC.

LCR CIRCUIT

18. For a series LCR circuit
(i) **Draw the phasor diagram**
(ii) Give the expression for current
(iii) Derive an expression for impedance
(iv) Draw the impedance triangle
(v) Give the expression for the phase angle
Ans:



(i) Phasor Diagram



Consider an a.c. voltage $v = v_m \sin\omega t$ applied to a series circuit containing an **inductor** (L), **capacitor** (C) and a **resistor** (R).

(ii) Expression for current

From the phasor diagram, we can see that current **i** leads the resultant voltage by a phase angle ϕ.

Therefore $\mathbf{i = i_m \sin(\omega t + \phi)}$

If we assume $V_L > V_C$, we will obtain $\mathbf{i = i_m \sin(\omega t + \phi)}$

Combining the above expressions,

$$\mathbf{i = i_m \sin(\omega t \pm \phi)}$$

(iii) Impedance

Impedance (**Z**) means the total resistance offered by L, C and R towards AC.

From the phasor diagram we have,

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

At the maximum value of **v** and **i**

$V_R = i_m R$, $V_C = i_m X_C$, $V_L = i_m X_L$

Therefore,

$$V_m = \sqrt{(i_m R)^2 + (i_m X_C - i_m X_L)^2}$$

$$\Rightarrow V_m = \sqrt{i_m^2 R^2 + i_m^2 (X_C - X_L)^2}$$

$$\Rightarrow V_m = i_m \sqrt{R^2 + (X_C - X_L)^2}$$

$$\Rightarrow \frac{V_m}{i_m} = \sqrt{R^2 + (X_C - X_L)^2}; \text{ but } \frac{V_m}{i_m} = Z$$

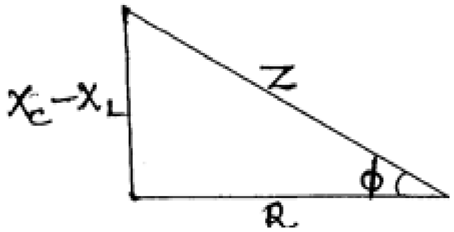
$$\Rightarrow \boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$$, is called the **impedance** of LCR circuit.

Where, $X_C = \frac{1}{C\omega}$, is the capacitative reactance,

$X_L = L\omega$, is the inductive reactance and R is the ohmic resistance

(iv) Impedance triangle

It is a right angled triangle, whose base represents the **ohmic resistance**, altitude represents the **net reactance** $(X_C - X_L)$ and the hypotenuse represents the **impedance (Z)** of LCR circuit.



(v) Phase Angle (ϕ)

From the impedance triangle,

$$\tan\phi = \frac{X_C - X_L}{R}$$

$$\Rightarrow \boldsymbol{\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)}$$

If $X_C > X_L$, ϕ is positive, ie, the current leads the voltage.

If $X_C < X_L$ then ϕ is negative ie, the current lags behind the voltage.

19. Explain the resonance in a series LCR circuit.

Ans: For a particular frequency of a.c. voltage (ω_0) X_L becomes equal to X_C, then impedance

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

becomes minimum (Z=R). Hence maximum current flows through the circuit. This phenomenon in which the current through an **LCR** circuit, becomes maximum at a particular frequency is called **resonance**. The frequency at which resonance occurs is called resonance frequency.

20. Derive an expression for the resonance frequency.

Ans: Resonance condition is

$$X_L = X_C$$

$$\Rightarrow L\omega_0 = \frac{1}{C\omega_0}$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$,where ω_0 is the resonance angular frequency

But $\omega_0 = 2\pi f_0$

Therefore, $2\pi f_0 = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \mathbf{f_0} = \frac{1}{2\pi\sqrt{LC}}$$, this is the expression for resonance frequency (f_0)

21. What is the value of impedance at resonance?

Ans: We have $Z = \sqrt{R^2 + (X_C - X_L)^2}$

At resonance $X_C = X_L$

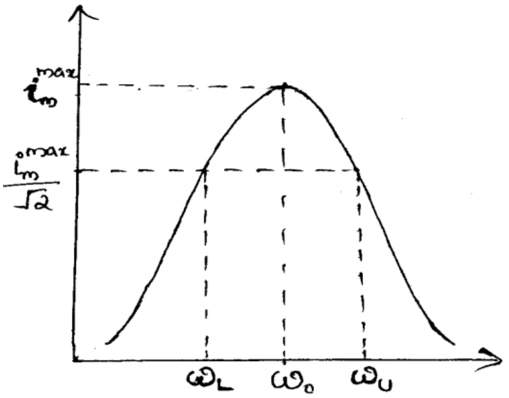
Therefore $Z = \sqrt{R^2 + 0} = R$

Thus ***at resonance the entire applied voltage appears across R***.

22. Define (i) Band width (ii) Sharpness (iii) Quality factor

Ans: <u>(i)Band width (β)</u>

Band width is defined as "*the difference between the frequencies on either side of resonance frequency for which the value of current is $\frac{1}{\sqrt{2}}$ times the peak value at resonance*".



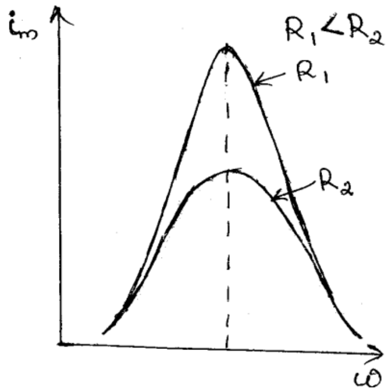
Band width, $\boldsymbol{\beta = \omega_U - \omega_L}$

ω_L is the Lower cut off frequency and ω_U is the Upper cut off frequency

Note :- The resonance curve is sharp if the band width is small.

(ii) **Sharpness of resonance**

The peak value of current at resonance depends only on the ohmic resistance (R) of the circuit. If the value of R is small the peak value of current is high at resonance frequency.



Sharpness of LCR circuit is defined as " ***the ratio of resonant frequency to the band width****"*

$$\boxed{S = \frac{\omega_0}{\omega_U - \omega_L}}$$

(iii) Quality factor (Q)

Q- factor is defined as ***the ratio of inductive reactance or capacitative reactance to the resistance at resonance.***

$$Q = \frac{L\omega}{R} \text{ or } Q = \frac{\left(\frac{1}{C\omega}\right)}{R} = \frac{1}{C\omega R}$$

Note:-

Quality factor and sharpness are numerically equal.

23. Derive an expression for the average power consumed by a series LCR circuit in a complete cycle of ac.

Ans: In an LCR circuit the instantaneous value of voltage is

$v = v_m \sin\omega t$ and that of current is

$I = i_m \sin(\omega t + \phi)$

The average power consumed in a complete cycle

$$\langle P \rangle = \frac{\int_0^T P\,dt}{T}$$

$$= \frac{1}{T}\int_0^T vi\,dt$$

$$= \frac{1}{T}\int_0^T v_m \sin\omega t . i_m \sin(\omega t + \phi)dt$$

$$= \frac{v_m i_m}{T}\int_0^T \frac{1}{2}[\cos(\omega t - (\omega t + \phi)) - \cos(\omega t + \omega t + \phi)]dt$$

$$= \frac{v_m i_m}{2T}\int_0^T [\cos(-\phi) - \cos(2\omega t + \phi)]dt$$

$$= \frac{v_m i_m}{2T}\left[\int_0^T \cos\phi\,dt - \int_0^T \cos(2\omega t + \phi)]dt\right]$$

$$= \frac{v_m i_m}{2T}\left[\int_0^T \cos\phi dt - 0\right]$$

$$= \frac{v_m i_m}{2T}\cos\phi \int_0^T dt$$

$$= \frac{v_m i_m}{2T}\cos\phi [t]_0^T$$

$$= \frac{v_m i_m}{2T}\cos\phi [T - 0]$$

$$= \frac{v_m i_m}{2}\cos\phi$$

$$= \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi$$

$$= V_{rms} I_{rms} \cos\phi$$

$$\langle P \rangle = V_{rms} I_{rms} \cos\phi$$

The term $V_{rms}I_{rms}$ is called apparent power and the term $\cos\phi$ is called power factor.

From the impedance triangle we have

$$\cos\phi = \frac{R}{Z}$$

In the case of resistor,

$\phi = 0$

$\langle P \rangle = V_{rms}I_{rms}\cos 0$

$= V_{rms}I_{rms} \times 1$

$= V_{rms}\, I_{rms}$

In the case of Inductor

$\phi = \frac{\pi}{2}$

$\langle P \rangle = V_{rms}I_{rms}\cos\phi$

$= V_{rms}I_{rms}\cos\frac{\pi}{2}$

$= V_{rms}I_{rms} \times 0 = 0$

In the case of capacitor

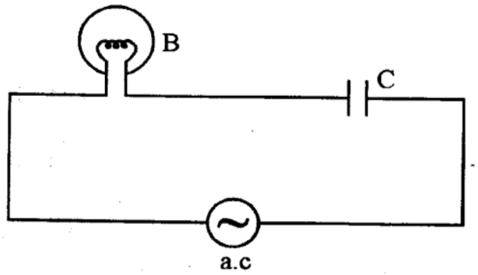
$\phi = \frac{\pi}{2}$

$\langle P \rangle = V_{rms}I_{rms}\cos\frac{\pi}{2}$

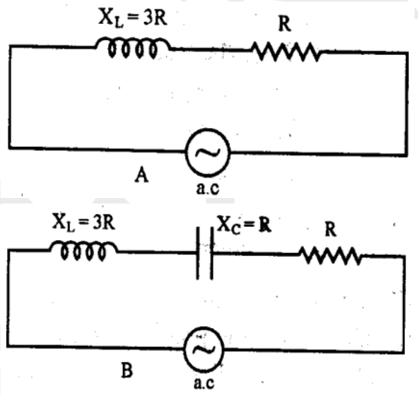
$= V_{rms}I_{rms} \times 0 = 0$

24. Draw the variation of impedance of an LCR circuit with frequency of AC.

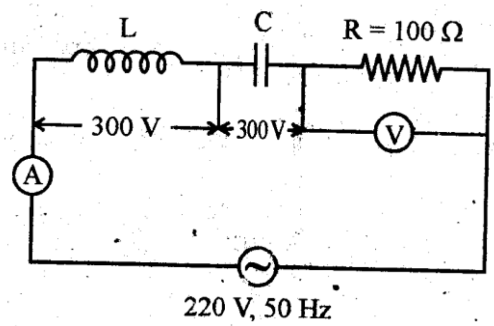
25. An electric bulb B and a parallel plate capacitor C are connected in series as shown in figure. The bulb glows with some brightness. How will the glow of the bulb affected on introducing a dielectric slab between the plates of the capacitor? Give reasons in support of your answer.



26. Given below are two electric circuits A and B. What is the ratio of power factor of the circuit B to that of A?



27. Find the voltmeter and ammeter readings in the given circuit.



28. Obtain the resonant frequency of a series LCR circuit with L=2.0H, C=32μF and R=10Ω. What is the Q-value of this circuit?

29. A series LCR circuit with R=20Ω, L=1.5H and C= 35 μF is connected to a variable-frequency 200V supply.

When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

30. A coil of self-inductance 0.50H and resistance 100**Ω** is connected to a 240 V, 50 Hz ac supply.

(a) What is the maximum current in the circuit?

(b) What is the time lag between voltage maximum and current maximum?

31. A 100 μF capacitor in series with a 40**Ω** resistance is connected to a 110V, 60Hz supply.

(a) What is the maximum current in the circuit?

(b) What is the time lag between voltage maximum and current maximum?

32. Explain how oscillations are produced in an LC circuit.

Ans: An oscillator is an arrangement that can produce continuous alternating voltage using a d.c. source. Consider a fully charged capacitor connected across an inductor through a switch.

By Kirchhoff's loop rule

$$\frac{q}{c} - L\frac{di}{dt} = 0$$

But i = $\frac{-dq}{dt}$ (-ve sign show that when **i** increases, q decreases).

$$\therefore \frac{q}{c} + L\frac{d^2q}{dt^2} = 0$$

$$\Rightarrow L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

Dividing by L,

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

This equation has the form of

$\frac{d^2x}{dt^2} + \omega_0^{\,2}x = 0$ for a simple harmonic oscillator. The charge, therefore oscillates with a natural frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ and varies sinusoidally with time as

$q = q_m\cos(\omega_0 t + \phi)$

When the key is closed, the charge flows from the capacitor and the current in the circuit increases. Thus the charge on the capacitor decreases and it energy, $U_E = \frac{q^2}{2C}$ decreases. At the same time, the current through the inductor increases and a magnetic field is set up in it. Hence energy in the inductor, $U_B = \frac{1}{2} Li^2$ increases. The current in the circuit increases gradually and becomes maximum when the capacitor is fully discharged.

Now the current flows in the opposite direction and the capacitor is charged with opposite polarities. This process continues till the capacitor is fully charged. Then the whole process repeats. Thus, the energy in the system oscillates between the capacitor and inductor.

LC oscillations are damped due to resistance of the components. The effect of this resistance brings damping effect on charge and current in the circuit and the oscillations finally die away.

The total energy of the system will not remain constant. Some energy is radiated away in the form of electromagnetic waves.

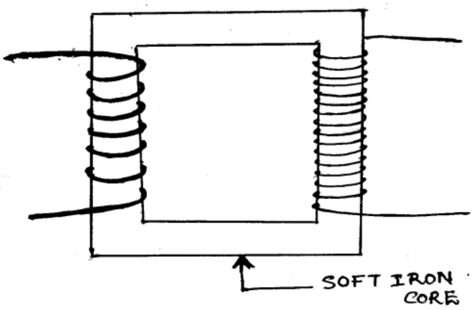
33. A 30μF capacitor is connected to a 27mH inductor. What is the angular frequency of free oscillations of the circuit?

34. What is the use of transformers?
Ans: Transformer is a device used to increase or decrease A.C. voltage.

35. What is the principle of a transformer?
Ans: Transformer works on the principle of mutual induction.
When a.c. flows through the primary coil, a changing magnetic field is produced around it. The secondary coil is placed in this changing magnetic field and hence an e.m.f. is induced across the secondary coil.

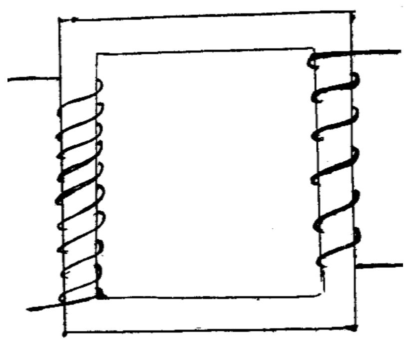
36. Derive the transformer equation.
Ans:

step up transformer



$N_s > N_p$, $V_s > V_p$ and $i_s < i_p$

Step down transformer



$N_s < N_p$, $V_s < V_p$ and $i_s > i_p$

Let N_p and N_s be the number of turns in the primary and secondary of the transformer. The voltage induced in the secondary, when **AC** flows through the primary is given by

$$V_s = N_s \frac{d\phi}{dt} \quad \text{.......... (1)}$$

At the same time, due to self-induction, the back e.m.f. produced in the primary is

$$V_p = N_p \frac{d\phi}{dt} \quad \text{........ (2)}$$

Dividing equation (1) by (2)

$$\frac{Vs}{Vp} = \frac{Ns}{Np}$$

37. Define the efficiency of a transformer.

Ans: Efficiency, $\eta = \dfrac{\text{output power}}{\text{input power}}$

38. What is an ideal transformer? Derive an expression connecting the currents and voltages in the primary and secondary coils of an ideal transformer.
Ans: Ideal transformer is a transformer having efficiency η=1.
⇨ Input power = output power
Ie, $V_p i_p = V_s i_s$

$$\Rightarrow \frac{i_p}{i_s} = \frac{V_s}{V_p}$$

Therefore we have

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s}$$

39. What are the different power losses in a transformer?

Ans: The different power losses in a transformer are:

1. **Copper Loss**

As the current flows through the primary and secondary copper wires, electric energy is wasted in the form of heat.

2. Eddy current Loss (Iron Loss)

The eddy currents produced in the soft iron core of the transformer produce heating. Thus electric energy is wasted in the form of heat.

3. Magnetic flux leakage

The entire magnetic flux produced by the primary coil may not be available to the secondary coil. Thus some energy is wasted.

4. Hysteresis Loss

Since the soft iron core is subjected to continuous cycles of magnetization, the core gets heated due to hysteresis. Thus some energy is wasted.