Chapter Ten

WAVE OPTICS

Introduction

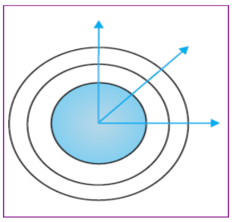
- Wave optics deals with the wave behavior of light.
- Christian Huygens proposed wave theory of light.
- According to wave theory, a luminous body is a source of disturbance and the disturbance propagated in the form of waves and energy is distributed equally in all directions.
- Interference, diffraction, polarization, etc can be explained by wave optics.
- Thomas Young confirmed the wave nature of light through double-slit experiment.
- Maxwell proposed electromagnetic theory of light.
- Transverse nature of light is established by polarization.

Wavefront

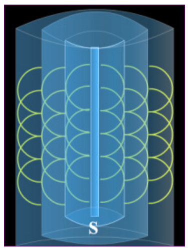
- A wavefront is locus of all points in a medium which are at the same phase of vibration.
- The speed with which the wavefront moves outwards from the source is called the speed of the wave.
- The energy of the wave travels in a direction perpendicular to the wavefront.

Types of wavefront

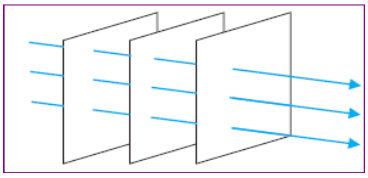
- **Spherical wavefront –** wavefront from a point source



- **Cylindrical wavefront**- wavefront from a linear source.

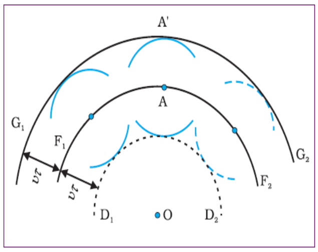


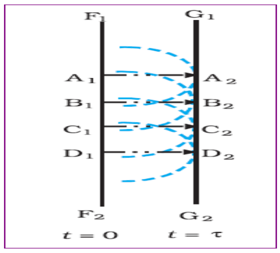
- **Plane wavefront : - w**avefront at large distances from a point source.



Huygen's Principle

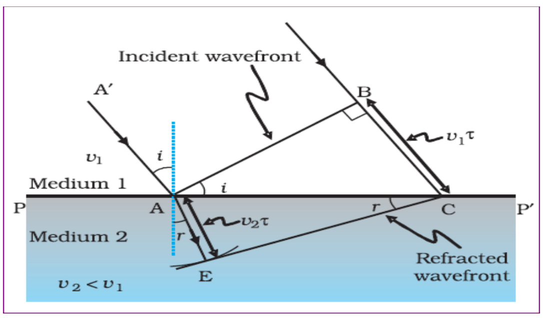
- According to Huygens principle, *each point of the wavefront is the source of a secondary disturbance and the wavelets (secondary wavelets) emanating from these points spread out in all directions with the speed of the wave.*
- *By drawing a common tangent to all these spheres, we obtain the new position of the wavefront at a later time.*





- Huygens argued that the amplitude of the secondary wavelets is maximum in the forward direction and zero in the backward direction

Refraction of a plane wave



- Let τ be the time taken by the wavefront to travel the distance BC.
- Thus $BC = v_1 \tau$
- From the triangle ABC we get

$$\sin i = \frac{BC}{AC} = \frac{v_1 \tau}{AC}$$

- Also from triangle AEC

$$\sin r = \frac{AE}{AC} = \frac{v_2 \tau}{AC}$$

- Thus

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

- If *c represents the speed of light in* vacuum, then,

$$n_1 = \frac{c}{v_1} \qquad n_2 = \frac{c}{v_2}$$

- Therefore

$$n_1 \sin i = n_2 \sin r$$

- This is the ***Snell's law of refraction***.
- If λ_1 and λ_2 denote the wavelengths of light in medium 1 and medium 2, respectively, then

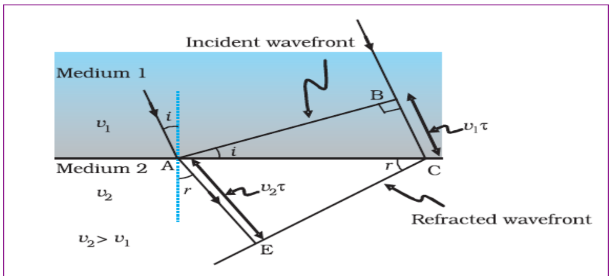
$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{v_1}{v_2}$$

- That is

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

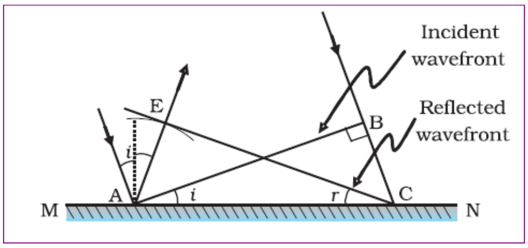
- This implies that **when a wave gets refracted into a denser medium, the wavelength and the speed of propagation decrease but the frequency ν (=v/λ) remains the same.**

Refraction at a rarer medium



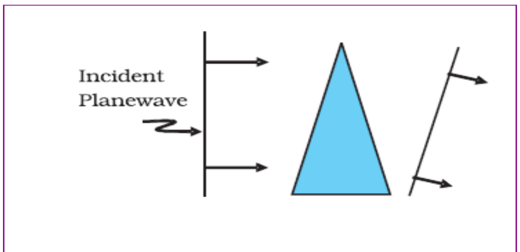
- The angle of refraction will be greater than angle of incidence.
- Thus, if *i = i_c then sin r = 1 and r = 90°.*
- Therefore $\sin i_c = \frac{n_2}{n_1}$
- The angle i_c is known as the **critical angle** and for all angles of incidence greater than the critical angle the wave will undergo **total internal reflection.**

Reflection of a plane wave by a plane surface



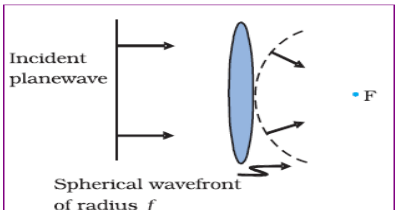
- If *v represents the speed of the wave in the medium and if* τ represents the time taken by the wavefront to advance from the point B to *C then* $BC = v\tau$
- Also $AE = BC = v\tau$
- The triangles EAC and BAC are congruent
- Therefore the angles *i* and *r would* be equal. This is the *law of reflection*.

A plane wave passing through a thin prism.



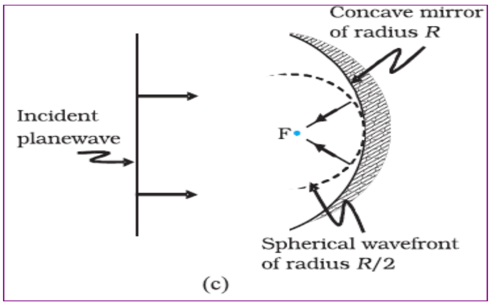
(a)

A plane wave incident on a thin convex lens



(b)

A plane wave is incident on a concave mirror



The Doppler effect

- The apparent change in frequency of light seen by an observer ,whenever there is a relative motion between source and observer is called Doppler Effect.
- When **the source is moving towards the observer** with a velocity v, then the apparent frequency of light

$$\nu' = \nu(1 + \frac{v}{c})$$

- Where *ν*- actual frequency, v – velocity
- Therefore the fractional change in frequency

$$\frac{\Delta\nu}{c} = \frac{v}{c}$$

- If the source is moving away from the observer, the apparent frequency

$$\nu' = \nu(1 - \frac{v}{c})$$

- Hence the fractional change in frequency is

$$\frac{\Delta\nu}{c} = -\frac{v}{c}$$

Red shift

- When the source moves away from the observer, there is an apparent decrease in the frequency of light. This is called **red shift**.

Blue shift

- When the source moves towards the observer, there is an apparent increase in the frequency of observed light. This is called **blue shift**.

INTERFERENCE OF LIGHT

Superposition Principle

- When more than one wave is passed through the same medium at the same instant, then the resultant displacement is the vector sum of displacements due to individual waves.
- Intensity of a wave is proportional to the square of its amplitude.

$$I \propto a^2$$

Coherent Sources of light

The coherent sources which emits light waves of :

- Same wavelength or frequency
- Nearly equal amplitude
- Are in phase or having a constant phase difference
- Eg: light from a double slit

Interference

- The modification in the distribution of light energy when waves from more than one coherent sources superpose each other.

Constructive interference

- When crests of two waves or two troughs meet together the amplitude of the resultant wave becomes maximum. This is called c**onstructive interference**.
- The intensity of the wave is given by

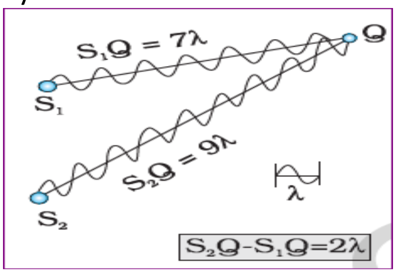
$$I_{max} \propto (a_1 + a_2)^2$$

Condition for constructive interference

- For constructive interference, the path difference of the waves reaching at point is the integral multiple of the wavelength.

$$S_1P \sim S_2P = n\lambda \quad (n = 0, 1, 2, 3, ...)$$

- The resultant intensity of constructive interference is $4I_0$., where I_0 is the intensity of the wave before interference.



Destructive interference

- When crest of one wave meet with trough of the other the amplitude of the resultant wave becomes minimum. This is called **destructive interference**.
- The intensity of the wave is given by

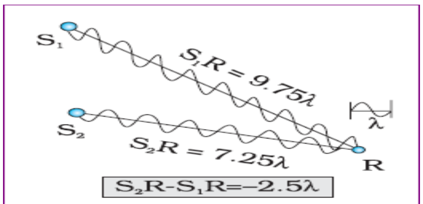
$$I_{min} \propto (a_1 - a_2)^2$$

Condition for Destructive Interference

- The path difference is given by

$$S_1P \sim S_2P = (n+\frac{1}{2})\lambda \quad (n = 0, 1, 2, 3, \ldots)$$

- The resultant intensity of destructive interference will be zero.



(b)

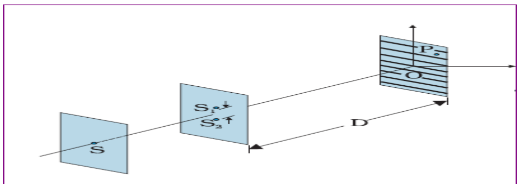
Relation between Path Difference and Phase Difference

- Path difference of λ corresponds to a phase difference of 2π.
- If Δx is the path difference, then the phase difference

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

YOUNG'S DOUBLE- SLIT EXPERIMENT

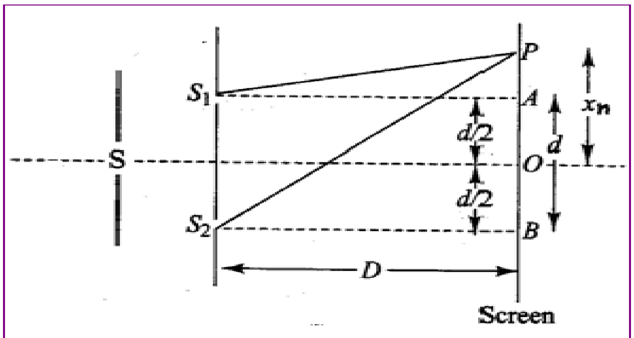
- Thomas Young designed an double slit arrangement to study interference.
- Young derived two coherent sources of light using a double slit.



(a)

- When light from two coherent sources S_1 and S_2 superimpose, alternate dark and bright bands are formed on the screen.

Expression for band width (Fringe width)



- The dark and bright bands appear on the screen are called ***fringes.***
- The distance between two consecutive bright fringes or two consecutive dark fringes is called the **fringe width**.
- From the triangle S_1AP

$$S_1P^2 = S_1A^2 + AP^2$$

- That is

$$S_1P^2 = D^2 + [x_n - d/2]^2 \ldots\ldots\ldots\ldots(1)$$

- From triangle S_2BP

$$S_2P^2 = S_2B^2 + BP^2$$

- That is

$$S_2P^2 = D^2 + [x_n + d/2]^2 \ldots\ldots\ldots\ldots(2)$$

- Subtracting equation 1 from 2, we get

$$S_2P^2 - S_1P^2 = \left[x_n + \frac{d}{2}\right]^2 - \left[x_n - \frac{d}{2}\right]^2$$

$$= \left[x_n^2 + 2x_n\frac{d}{2} + d^2/4\right] - \left[x_n^2 + 2x_n\frac{d}{2} - d^2/4\right]$$

- Thus

$$S_2P^2 - S_1P^2 = 2dx_n$$

- Or

$$[S_2P - S_1P]\,[S_2P + S_1P] = 2dx_n \ldots\ldots\ldots(3)$$

- If the point P is near to O, then

$$S_2P \approx S_1P \approx D$$

- Therefore

$$[S_2P - S_1P]\,2D = 2dx_n$$

- Or

$$S_2P - S_1P = \frac{dx_n}{D}$$

- Thus

$$\textbf{Path Difference} = \frac{x_n d}{D}$$

- For the point P to be bright, the path difference =nλ, thus

$$\frac{x_n d}{D} = n\lambda$$

- Therefore the distance to n^{th} band is,

$$x_n = \frac{n\lambda D}{d}; \; n = 0, \pm 1, \pm 2, ..$$

- Thus distance to $(n+1)^{th}$ band is

$$x_{n+1} = \frac{(n+1)\lambda D}{d}$$

- The band width is given by

$$\beta = x_{(n+1)} - x_n$$

- Thus

$$\beta = \frac{\lambda D}{d}$$

- This is the combined width of a dark band and a bright band.
- The dark and bright bands are equally spaced.
- If P is dark, then

$$x_n = (n + ½)\frac{\lambda D}{d}; \; n = 0, \pm 1, \pm 2$$

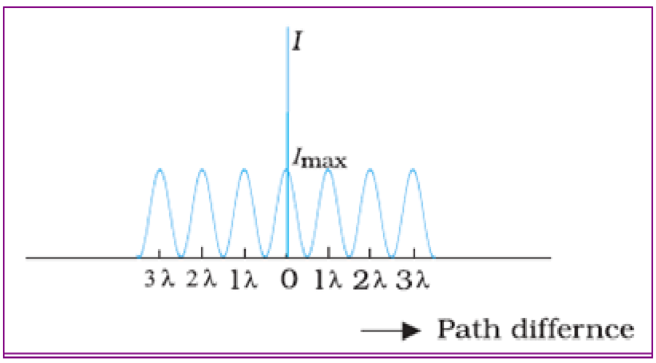
Fringe width can be increased by

- Increasing the wavelength of light (λ)
- Increasing the distance between the sources and screen (D)
- By decreasing the distance between the two coherent sources (d).

Conditions for getting sustainable interference pattern

- The two sources must be coherent
- The coherent sources must be narrow and very close to each other.
- The screen must be at large distance from the sources.

The intensity distribution of light on the screen in Young's Double Slit Experiment



Some observations

- If one of the slits is covered with black paper – **no interference pattern.**
- If the source is moved towards the slits, the **fringe width do not change** but **intensity increases**.
- If white light is used, then a **white band at the centre** and **colored bands on either side** are formed.
- If the system is immersed in a medium of refractive index n, then the new fringe width β', is given by

$$\beta' = \frac{\beta}{n}$$

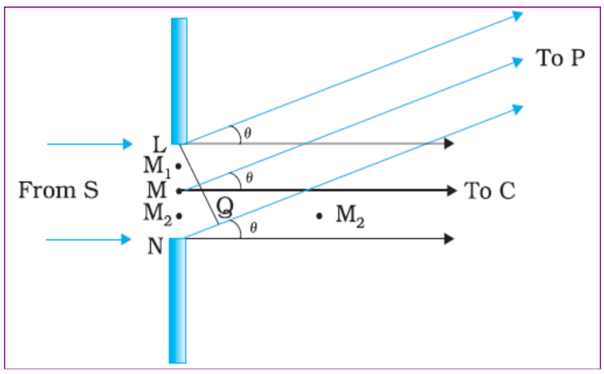
- The color of thin films of soap solution, or oil or petrol spread over water is due to interference.

DIFFRACTION

- It is the bending of the light at the sharp corners of obstacles.
- Diffraction of light occurs when the size of obstacle is comparable to the wavelength of light.
- All types of waves namely , light waves, sound waves, matter waves, waves on water etc shows diffraction.
- The **finite resolution** of our eye or of optical instruments such as telescopes or microscopes **is limited** due to the phenomenon of diffraction.

The single slit Diffraction

- When the double slit in Young's experiment is replaced by a single narrow slit (illuminated by a monochromatic source), a broad pattern with a central bright region is seen.
- On both sides, there are alternate dark and bright regions, the intensity becoming weaker away from the centre.



- The path difference NP – LP between the two edges of the slit is

$$NP - LP = NQ = a \sin \theta \approx a\theta$$

- To find the intensity at any point P on the screen we divide the slit into much smaller parts, and add their contributions at P with the proper phase differences.

Central maximum

- At 'C', the path difference between the rays coming from LM and MN is zero. Hence constructive interference takes place. This point is called **central maximum or principal maximum**.
- Since the light rays from different points of the slit interfere constructively, the point C is maximum bright.

Positions of secondary minima

- The secondary minima occurs at

$$\theta = \pm \frac{n\lambda}{a}$$

- Where n= 1, 2, 3,……

Positions of secondary maxima

- The secondary minima occur at

$$\theta \approx (n+1/2)\ \lambda/a$$

- Where n= 1, 2, 3,……

The conditions for minima and maxima of diffraction at a single slit experiment

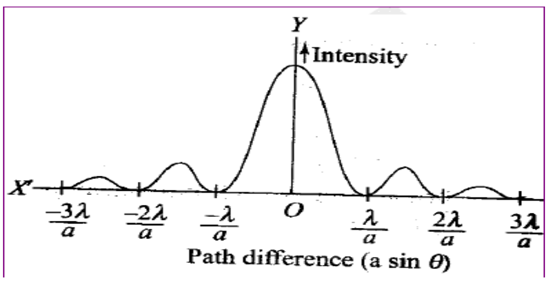
- **For minima**

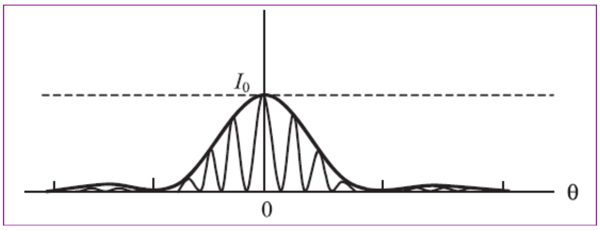
$$a \sin \theta = n\lambda, \quad n = 1, 2, 3\ldots$$

- **For maxima**

$$a \sin \theta = (2n+1)\frac{\lambda}{2} \quad n=1, 2, 3\ldots.$$

Intensity distribution of diffraction pattern





Some observations

- Diffraction is more when the slit width is decreased.
- When the wavelength of the source is increased the angular deviation also increases.
- Only few band are observable in diffraction.

Comparison between interference and diffraction

Interference	Diffraction
It is the superposition of secondary waves from two different wave fronts.	It is the superposition of secondary waves from different parts of the same wave front.
Fringes may or may not be of equal width.	Fringes are never of equal width.
All bright fringes have same intensity.	Intensity of bright fringes decreases as we move from the central bright fringe.
The regions of minimum intensity are perfectly dark.	The regions of minimum intensity are not perfectly dark.

Energy conservation in interferncce and diffraction

- Energy is conserved in both interference and diffraction
- The total energy is redistributed in interference and diffraction

Resolving power of optical instruments

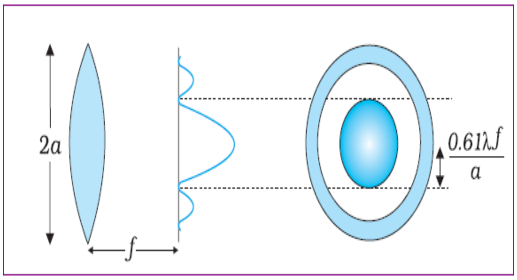
- The ability to resolve two neighboring objects which are very close to each other is the resolving power.
- Human eye can resolve two objects if they subtend an angle of one minute at the eye
- The resolving power is measured as the reciprocal of the angle subtended by the object.

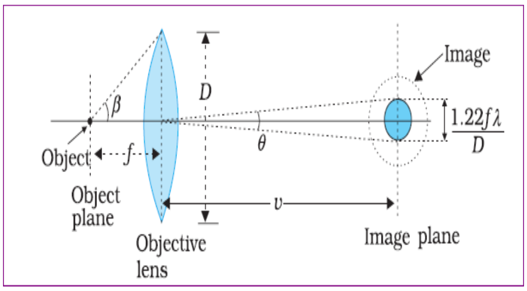
- The resolving power of optical instruments is limited by diffraction.

Resolving power of a Telescope

For a telescope with objective lens of diameter '2a', when alight of wavelength λ falls on it,

$$\text{Resolving power} = \frac{2a}{1.22\,\lambda}$$





- This implies that the telescope will have better resolving power if a is large.
- It is for this reason that for better resolution, a telescope must have a large diameter objective.

Resolving power of a microscope

For a **microscope** with objective lens of diameter 'D' and making angle 2β by the diameter of the objective lens at the focus of the microscope, then

$$\text{Resolving power} = \frac{2\sin\beta}{1.22\,\lambda}$$

If 'n' is the refractive index of the medium between the object and objective then,

$$\text{Resolving power} = \frac{2n\sin\beta}{1.22\lambda}$$

- The product $n\sin\beta$ *is called* ***the numerical aperture***
- The resolving power can be increased by choosing a medium of higher refractive index. Such an arrangement is called an ***'oil immersion objective'***.

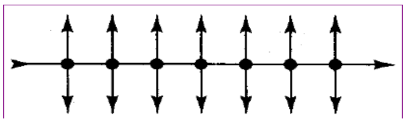
POLARISATION

- The phenomenon by is called polarization.
- When ordinary light passes through certain crystals like tourmaline crystal, the vibrations of electric field vector are restricted. This phenomenon is called **polarization.**
- Polarization shows that **light is a transverse wave.**
- Sound waves cannot polarize**.**

Unpolarised light

- The ordinary light which contains the vibrations of electric field vector in every plane perpendicular to the direction of propagation is called unpolarised light.

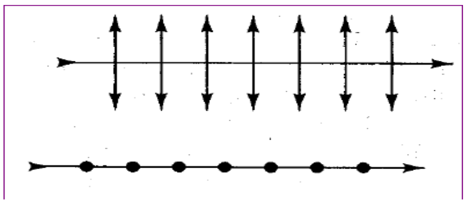
Representation of unpolarised light



Plane polarized light

- The polarized light in which the electric field vibrations of light are confined to a single plane are called plane polarised light.

Representation of plane polarised light



Plane of vibration

- It is the plane in which the vibrations of the polarized light take place.

Plane of polarization

- It is the plane perpendicular to the plane of vibration of the plane polarized light.

Polarizer

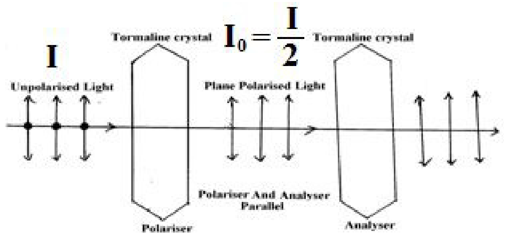
- The crystal which produces polarized light is called a polarizer.

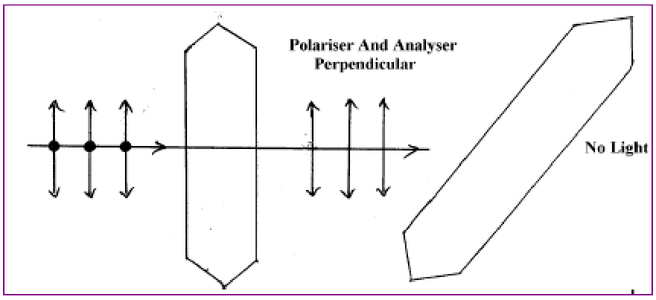
Analyzer

- The crystal which is used to check whether the light is polarized or not is called analyzer or detector.

An experiment to study polarization of light

- When unpolarized light passes through polarizer the light coming out of it is plane polarized.





- If the polarizer and analyser are parallel the intensity of light coming through the analyser will be maximum.
- If the analyser is rotated through 90^0 the intensity of light coming out of it becomes zero.

Polaroids

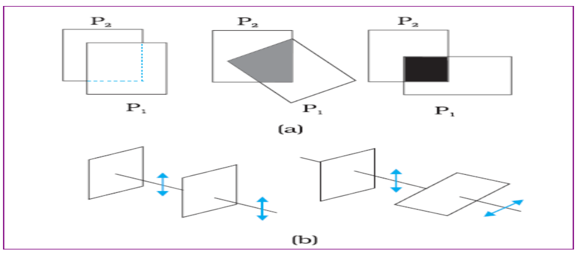
- Polaroid is an artificially made polarising material that produce intense beam of polarised light by selective absorption.
- Polaroids are in sunglasses, windowpanes, photographic cameras, 3D movie cameras etc.

Malus' law

- Malus's law states that when a beam of plane polarised light is incident on the analyser, then the intensity of the emergent light is directly proportional to square of the cosine of the angle between the polariser and analyser.

$$I = I_0 \cos^2\theta$$

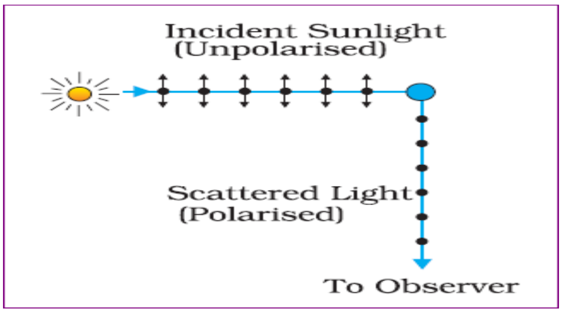
- Where θ is the angle between the axes of polarizer and analyzer.



Methods of polarization

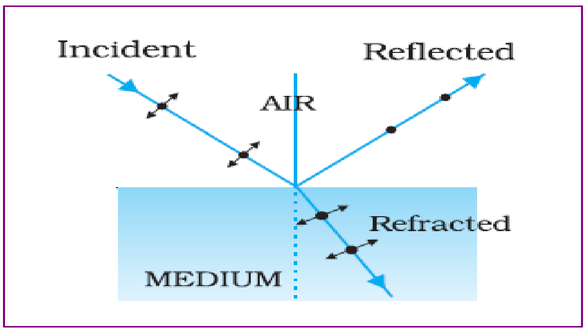
- **Polarization by scattering**
- **Polarisation by reflection**

Polarization by scattering



- When sunlight is incident on the gas molecules in the atmosphere, it gets scattered.
- The scattered light seen in a direction perpendicular to the direction of incidence is found to be plane polarised. This phenomenon is called polarisation by scattering.
- When this polarised light is viewed through a polaroid which is rotated, then the intensity changes with rotation.
- The scattering of light by molecules was intensively investigated by C.V. Raman and his collaborators .Raman was awarded the Nobel Prize for Physics in for this work.

Polarisation by reflection



- When ordinary light falls on a surface separating two transparent media, a part of the light is reflected and the other part is transmitted (refracted).
- When reflected wave is perpendicular to the refracted wave, the reflected wave is a totally polarised wave.

Brewster's angle (polarizing angle)

- The angle of incidence at which the reflected ray is totally polarized is called ***Brewster's angle*** *and is denoted by* i_B.

Brewster's law

- Brewster's law states that the tangent of the Brewster's angle is equal to the refractive index of the medium.

$$\tan i_B = \mu$$

Proof

- From Snell's law

$$\mu = \frac{\sin i_B}{\sin r} = \frac{\sin i_B}{\sin(\pi/2 - i_B)}$$

$$= \frac{\sin i_B}{\cos i_B} = \tan i_B$$

Distinguishing a polarized light and unpolarized light

- When we observe unpolarised light (ordinary light) through a Nicol prism (tourmaline crystal), the intensity of the light coming out of the prism does not change if the crystal is rotated.
- But when we observe polarized light through a Nicol prism, the intensity of the light coming out of the prism changes if the crystal is rotated.

PROBLEMS

1 What speed should a galaxy move with respect to us so that the sodium line at 589.0 nm is observed at 589.6 nm?

Sol

Since $\nu\lambda = c$, $\frac{\Delta\nu}{\nu} = -\frac{\Delta\lambda}{\lambda}$ (for small changes in ν and λ). For

$\Delta\lambda = 589.6 - 589.0 = +0.6$ nm

we get $\frac{\Delta\nu}{\nu} = \frac{v_{radial}}{c}$

$\frac{\Delta\nu}{\nu} = -\frac{\Delta\lambda}{\lambda} = -\frac{v_{radial}}{c}$

or, $v_{radial} \cong +c\left(\frac{0.6}{589.0}\right) = +3.06 \times 10^5\ ms^{-1}$

$= 306$ km/s

Therefore, the galaxy is moving away from us.

2 Two slits are made one millimetre apart and the screen is placed one metre away. What is the fringe separation when blue-green light of wavelength 500 nm is used?

Sol

$$\text{Fringe spacing} = \frac{D\lambda}{d} = \frac{1 \times 5 \times 10^{-7}}{1 \times 10^{-3}}\ m$$

$$= 5 \times 10^{-4}\ m = 0.5\ mm$$

3 Green light of wavelength 5100 Å from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, find the slit separation.

Sol

Wavelength of light $= \lambda = 5100$ Å $= 5100 \times 10^{-10}$ m

Width of 10 fringes $= x = 2$ cm $= 0.02$ m

Fringe width $= \beta = \frac{x}{N} = \frac{0.02}{10} = 0.002$ m

Distance between the slits and the screen $= D = 200$ cm $= 2.00$ m

Slit separation $= d = ?$

$B = \frac{\lambda D}{d}$, $d = \frac{\lambda D}{\beta}$

$d = \frac{5100 \times 10^{-19} \times 2.00}{0.002} = 5.1 \times 10^{-4}$ m.

4 Two coherent sources of monochromatic light of wavelength 6000 Å produce an interference pattern on a screen kept at a distance if 1m from them. The distance between two consecutive bright fringes on the screen is 0.5 mm. Find the distance between the two coherent sources (E.Q]

Sol

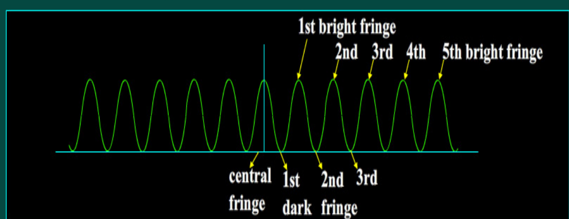
$\lambda = 6000$ Å $= 6000 \times 10^{-10}$ m, $D = 1$m, $\beta = 0.5$ mm $= 0.5 \times 10^{-3}$ m

$d = \frac{\lambda D}{\beta} = \frac{6000 \times 10^{-10} \times 1}{0.5 \times 10^{-3}} = 1.2 \times 10^{-3}$ m.

5 In a Young's double slit experiment the slits are 1mm apart and the screen is kept at a distance of 1.5 m from the slits. It is found that the 5th bright fringe is at a distance of 3.0 mm from the second dark fringe. Find the wavelength of the light used

Sol

$d = 1mm = 1.0 \times 10^{-3}$m, $D = 1.5$ m



The distance between 5th bright fringe and the 2nd dark ring

$= 5\beta - 1.5\beta = 3.5\beta = 3.0 \times 10^{-3}$ m. $\quad \beta = \frac{3.0 \times 10^{-3}}{3.5} = 0.857 \times 10^{-3}$ m.

$\lambda = \frac{\beta d}{D} = \frac{0.857 \times 10^{-3} \times 1.0 \times 10^{-3}}{1.5} = 0.571 \times 10^{-6}$ m $= 571.0 \times 10^{-9}$ m

6 Find the angular width of the central bright maximum in the Fraunhofer pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wave length $6000\,\mathring{A}$

Sol

Width of the slit = $a = 12 \times 10^{-5}$ cm $= 12 \times 10^{-7}$m

Wavelength of light = $\lambda = 6000\,\mathring{A} = 6000 \times 10^{-10}$m

Angular width of the central maximum = 2θ =?

$$\text{Sin}\,\theta = \frac{\lambda}{a} = \frac{6000 \times 10^{-10}}{12 \times 10^{-7}} = 0.5 \qquad \theta = 30^0,\ 2\theta = 60^0$$

7 When the angle of incidence on a certain material is 60^0, the reflected light is completely polarized. Find the refractive index for the material and also the angle of refraction.

Sol

Polarising angle = $i = 60^0$, Angle of refraction = r = ?

Refractive index of the material = n = tan i = tan 60 =1.732

$$r = 90 - i = 90 - 60 = 30^0$$

8 The critical angle of incidence of water for total internal reflection is 48^0 for a certain wavelength at room temperature. What is the polarising angle and the angle of refraction for light incident on the surface of water at an angle that gives maximum polarisation of the reflected light?

Sol

Critical angle = $C = 48^0$, $n = \frac{1}{\sin C} = \frac{1}{\sin 48} = 1.345$

For maximum polarisation, the angle of incidence must be the polarising angle i

$\tan i = n$, $i = \tan^{-1}(1.345) = 53.37^0$

Angle of refraction, = $r = 90 - i = 90 - 53.37 = 36.63^0$

9 What is the Brewster angle for air to glass transmission? Refractive index of glass = 1.5

Sol

$\tan i = n$

$i = \tan^{-1}$

$n = \tan^{-1}(1.5) = 56.3^0$

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