Chapter Thirteen

NUCLEI

Atomic number (Z)

- It is the number of protons in the nucleus.
- It is denoted by Z.

Mass number (A)

- It is the total number of nucleons
- Total no. of nucleons = no. of protons + number of neutrons
- Mass number is denoted by A.

Neutron number (N)

- It is the total number of neutrons.
- Denoted by N and N= A-Z.

Representation of nuclei

- An atom is represented as $_ZX^A$.
- A- mass number, Z- atomic number

Atomic mass

- Accurate measurement of atomic masses is carried out with a **mass spectrometer**.
- **Atomic mass unit (u)**, is used for expressing atomic masses.
- It is defined as 1/12th of the mass of the carbon (^{12}C) atom.

$$1\,\text{u} = \frac{\text{mass of one } ^{12}\text{C atom}}{12} = \frac{1.992647 \times 10^{-26}\ \text{kg}}{12} = 1.660539 \times 10^{-27}\ \text{kg}$$

Composition of nucleus

- Nucleus contains protons and neutrons
- The mass of a proton is

$$m_p = 1.00727\,\text{u} = 1.67262 \times 10^{-27}\ \text{kg}$$

- James Chadwick-discovered neutrons
- Mass of a neutron is

$$m_n = 1.00866\ \text{u} = 1.6749 \times 10^{-27}\ \text{kg}$$

- A free neutron is unstable.
- It decays into a proton, an electron and a antineutrino (another elementary particle), and has a mean life of about 1000s.
- It is stable inside the nucleus

Isotopes

- Atomic species with same atomic number but different mass number are called ***isotopes.***
- Hydrogen has three isotopes having masses 1.0078 u (protium), 2.0141 u (deuterium), and 3.0160 u (tritium).
- Tritium nuclei, being unstable, do not occur naturally and are produced artificially in laboratories

Isobars

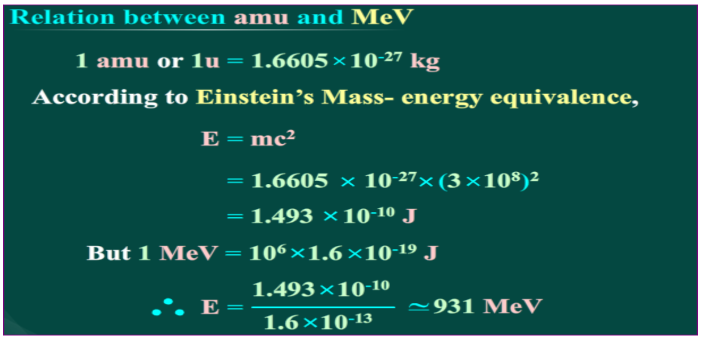
- All nuclides with same mass number *A and different atomic number are called isobars.* Eg:

$$^3_1\text{H and } ^3_2\text{He}$$

Isotones

- Nuclides with same neutron number *N but different atomic number Z* are called *isotones*

$$^{198}_{80}\text{Hg} \quad ^{197}_{79}\text{Au}$$



Relation between amu and MeV

1 amu or 1u = 1.6605×10^{-27} kg

According to Einstein's Mass- energy equivalence,

$$E = mc^2$$

$$= 1.6605 \times 10^{-27} \times (3 \times 10^8)^2$$

$$= 1.493 \times 10^{-10}\ \text{J}$$

But 1 MeV = $10^6 \times 1.6 \times 10^{-19}$ J

$$\therefore E = \frac{1.493 \times 10^{-10}}{1.6 \times 10^{-13}} \simeq 931\ \text{MeV}$$

SIZE OF THE NUCLEUS

- The radius of a nucleus with mass number A is given by

$$R = R_0 A^{1/3}$$

- Where

$$\boxed{R_0 = 1.2 \times 10^{-15}\,\mathrm{m}}$$

- *Thus the density of nucleus is* a constant, independent of *A, for all nuclei.*

- The density of nuclear matter is

$$\boxed{2.3 \times 10^{17}\,\mathrm{kg\,m^{-3}}}$$

Mass – Energy

- Einstein showed that mass is another form of energy and one can convert mass-energy into other forms of energy, say kinetic energy and vice-versa.

$$\boxed{E = mc^2}$$

- In a reaction the conservation law of energy states that the initial energy and the final energy are equal provided the energy associated with mass is also included.

Mass Defect

- The difference in mass of a nucleus and its constituents, ΔM, *is called the mass defect, and is given by*

$$\boxed{\Delta M = [Zm_p + (A - Z)m_n] - M}$$

Mass of 8 neutrons = 8 × 1.00866 u

Mass of 8 protons = 8 × 1.00727 u

Mass of 8 electrons = 8 × 0.00055 u

Therefore the expected mass of $^{16}_{8}\mathrm{O}$ nucleus = 8 × 2.01593 u = 16.12744 u.

- The atomic mass of $^{16}{}_{8}\mathrm{O}$ found from mass spectroscopy experiments is seen to be 15.99493 u.

- Substracting the mass of 8 electrons (8 × 0.00055 u) from this, we get the experimental mass of O nucleus to be 15.99053 u.

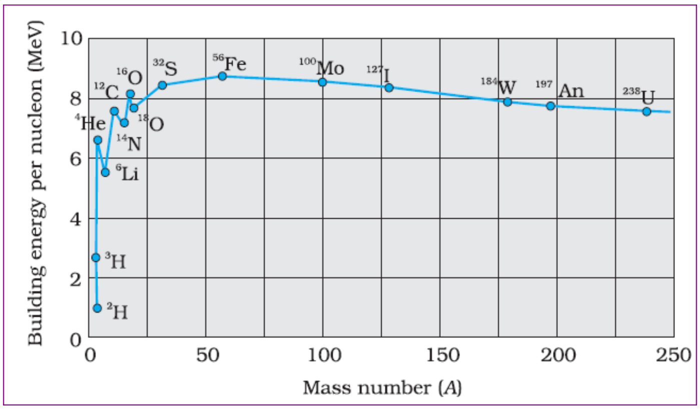
Nuclear Binding Energy

- **It is the energy equivalent of mass defect.**

$$\boxed{E_b = \Delta M c^2}$$

- If a certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass, an energy E_b *will be released* in the process.
- The ratio of the binding energy E_b *of a* nucleus to the number of the nucleons, A, in that nucleus is called ***binding energy per nucleon***

Plot of the binding energy per nucleon E_{bn} versus the mass number A



Features of the Graph

- The binding energy per nucleon, E_{bn}, *is practically* constant, i.e. practically independent of the atomic number for nuclei of middle mass number (30 < A < 170).

- The curve has a maximum of about 8.75 MeV for *A = 56* and has a value of 7.6 MeV for *A = 238*

- E_{bn} *is lower for both light* nuclei (*A<30) and heavy* nuclei (*A>170).*

Conclusions:

- A very heavy nucleus, say *A = 240, has lower binding energy per* nucleon compared to that of a nucleus with

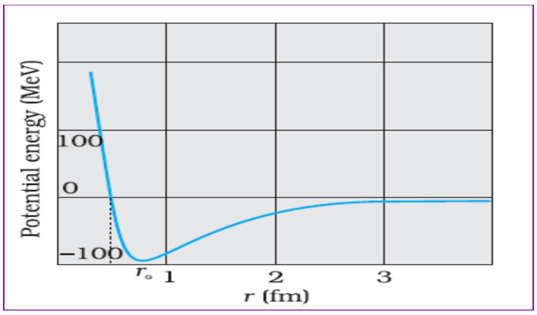
A = 120. Thus if a nucleus *A = 240 breaks into two A = 120 nuclei, nucleons get more* tightly bound.

- Thus energy would be released when a heavy nucleus is broken into light nucleus- the process- **nuclear *fission***
- **Similarly when two light nuclei (*A≤ 10) are joined together to form a heavy nucleus , energy is released- nuclear Fusion***

NUCLEAR FORCE

- Force that binds the nucleons together.
- Strongest force in nature.
- Short range force.
- Does not depend on charge.
- The property that a given nucleon influences only nucleons close to it is also referred to as **saturation property** of the nuclear force.
- The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres
- Acts through the exchange of π-mesons

Plot of the potential energy between two nucleons as a function of distance



- The potential energy is a minimum at a distance *r0 of about* 0.8 fm.
- This means that the force is attractive for distances larger than 0.8 fm and repulsive if they are separated by distances less than 0.8 fm.

RADIOACTIVITY

- **H. Becquerel** discovered radioactivity in 1896.
- Radioactivity is a nuclear phenomenon in which an unstable nucleus undergoes a decay.This is referred to as ***radioactive decay***.
- ***Three types of radioactive decay* occur in nature** :
- **α-decay** in which a helium nucleus (He) is emitted;
- **β-decay** in which electrons or positrons (particles with the same mass as electrons, but with a charge exactly opposite to that of electron) are emitted;
- **γ-decay** in which high energy (hundreds of keV or more) photons are emitted.

Law of radioactive decay

- ***This law states that the number of nuclei undergoing the decay per unit time is proportional to the total number of nuclei in the sample.***
- If a sample contains N undecayed nuclei and let dN nuclei disintegrate in dt second, thus the rate of disintegration

$$\frac{dN}{dt}\alpha - N$$

- The negative sign shows that the number of nuclei decreases with time.
- Thus

$$\boxed{\frac{dN}{dt} = -\lambda N}$$

- Where **λ** is called the **radioactive *decay constant or disintegration constant*.**

$$\boxed{\text{or, } \frac{dN}{N} = -\lambda dt}$$

- Now, integrating both sides of the above equation, we get

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt$$

$$\text{or, } \ln N - \ln N_0 = -\lambda (t - t_0)$$

- Here N_0 is the number of radioactive nuclei in the sample at some arbitrary time t_0 and N is the number of radioactive nuclei at any subsequent time t.

- Setting $t_0 = 0$

$$\ln \frac{N}{N_0} = -\lambda t$$

- Thus **$N = N_0 e^{-\lambda t}$**

Decay Rate

- It gives the number of nuclei decaying per unit time

$$R = -\frac{dN}{dt}$$

$$R = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$$

$$\text{or, } R = R_0 e^{-\lambda t}$$

- Here R_0 *is the radioactive decay* rate at time *$t = 0$, and R is the rate at any subsequent* time *t.*

- Thus

$$R = \lambda N$$

- The **total decay rate R** *of a sample of one or more radionuclide's* is called ***the activity*** *of that sample.*

- *The **SI unit for activity is** **becquerel**,* named after the discoverer of radioactivity.

- **1 becquerel = 1Bq = 1 decay per second**

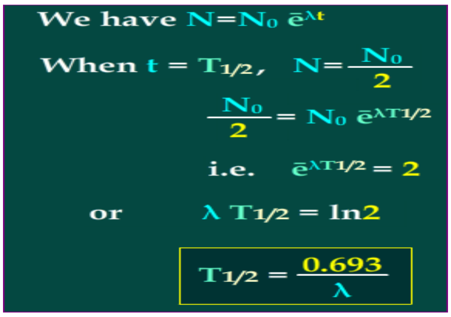
- An older unit, the **curie**, is still in common use.

1 curie = 1 Ci = 3.7×10^{10} Bq (decays per second)

Half life period ($T_{1/2}$)

- **It is the time in which the number of undecayed nuclei falls into half of its original number.**
- Thus it is the time at which both N and R have been reduced to one-half their initial values.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$



We have $N = N_0 \bar{e}^{\lambda t}$

When $t = T_{1/2}$, $N = \frac{N_0}{2}$

$\frac{N_0}{2} = N_0 \bar{e}^{\lambda T_{1/2}}$

i.e. $\bar{e}^{\lambda T_{1/2}} = 2$

or $\lambda T_{1/2} = \ln 2$

$\boxed{T_{1/2} = \frac{0.693}{\lambda}}$

Mean life (τ)

- It is the average life of all the nuclei in a radioactive sample.

- Mean life = total life time of all nuclei / total number of nuclei present initially

$$\tau = \frac{1}{\lambda}$$

- The number of nuclei which decay in the time interval *t to t + Δt* is

$$R(t)\Delta t = (\lambda N_0 e^{-\lambda t} \Delta t)$$

- Each of them has lived for time *t*. Thus the total life of all these nuclei would be

$$t \lambda N_0 e^{-\lambda t} \Delta t$$

- Therefore mean life is given by

$$\tau = \frac{\lambda N_0 \int_0^\infty t e^{-\lambda t} dt}{N_0} = \lambda \int_0^\infty t e^{-\lambda t} dt$$

One can show by performing this integral that

$\tau = 1/\lambda$

We summarise these results with the following:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2 \qquad (1$$

Alpha decay

- When a nucleus undergoes *alpha-decay, it transforms to a different* nucleus by emitting an alpha-particle (a helium nucleus)

$$\boxed{{}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}}$$

$$\boxed{{}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}}$$

- The difference between the initial mass energy and the final mass energy of the decay products is called the ***<u>Q value of the process or the disintegration energy</u>***.

$$\boxed{Q = (m_X - m_Y - m_{He})\, c^2}$$

- This energy is shared by the daughter nucleus and the alpha particle,in the form of kinetic energy
- Alpha-decay obeys the radioactive law
- Alpha particles are positively charged particles
- Can be deflected by electric and magnetic fields.
- Can affect photographic plates.

Beta decay

- **A nucleus that decays spontaneously by emitting an electron or a positron is said to undergo *beta decay.***

- **<u>In beta-minus decay</u>,** a neutron transforms into a proton within the nucleus according to

$$\boxed{n \longrightarrow p + e^- + \bar{\nu}}$$

  - Where ν is the antineutrino

- In beta minus (β^-) decay, an electron is emitted by the nucleus.
- **Eg:**

$$\boxed{{}^{32}_{15}\mathrm{P} \rightarrow {}^{32}_{16}\mathrm{S} + e^- + \bar{\nu} \qquad (T_{1/2} = 14.3\ \mathrm{d})}$$

- When β^- particles are emitted, **<u>the atomic number increases by one.</u>**
- **<u>In beta-plus decay</u>,** a proton transforms into neutron (inside the nucleus)

$$\boxed{p \longrightarrow n + e^+ + \nu}$$

- Where ν is the neutrino
- In beta plus ($\beta+$) decay, a positron is emitted by the nucleus,
- **Eg:**

$$\boxed{{}^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + e^+ + \nu \quad (T_{1/2} = 2.6\ \mathrm{y})}$$

- When β^+ particles are emitted the **<u>atomic number decreases by one.</u>**

Neutrinos and Antineutrinos

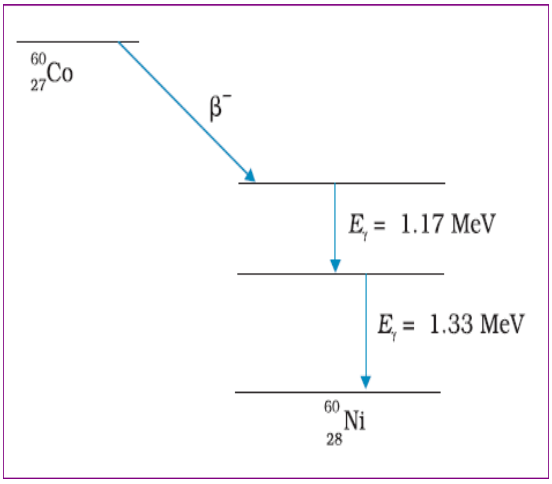
- The particles which are emitted from the nucleus along with the electron or positron during the decay process.
- Neutrinos interact only very weakly with matter; they can even penetrate the earth without being absorbed.

Gamma decay

- There are energy levels in a nucleus, just like there are energy levels in atoms.

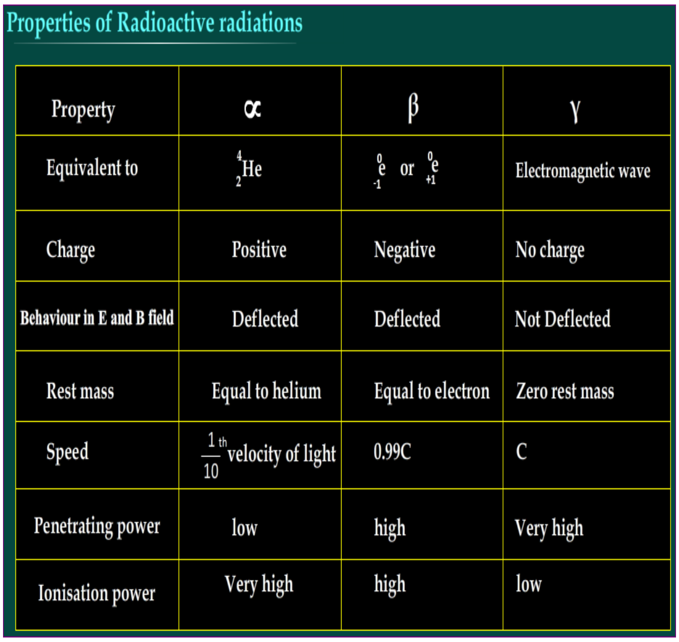
- When a nucleus is in an excited state, it can make a transition to a lower energy state by the emission of electromagnetic radiation.

- As the energy differences between levels in a nucleus are of the order of MeV, the photons emitted by the nuclei have MeV energies and are called gamma rays.



- Most radionuclides after an alpha decay or a beta decay leave the daughter nucleus in an excited state.

- The daughter nucleus reaches the ground state by a single transition or sometimes by successive transitions by emitting one or more gamma rays.



Properties of Radioactive radiations

Property	α	β	γ
Equivalent to	^4_2He	$^0_{-1}e$ or $^0_{+1}e$	Electromagnetic wave
Charge	Positive	Negative	No charge
Behaviour in E and B field	Deflected	Deflected	Not Deflected
Rest mass	Equal to helium	Equal to electron	Zero rest mass
Speed	$\frac{1}{10}$th velocity of light	0.99C	C
Penetrating power	low	high	Very high
Ionisation power	Very high	high	low

NUCLEAR ENERGY

- In conventional energy sources like coal or petroleum, energy is released through chemical reactions.

- One kilogram of coal on burning gives 10^7 J of energy, whereas 1 kg of uranium, which undergoes fission, will generate on fission 10^{14} J of energy.

Nuclear Fission

- Enrico Fermi found that when neutrons bombard various elements, new radioactive elements are produced.
- Eg:

$$^1_0n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3\,^1_0n$$

$$^1_0n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{133}_{51}Sb + ^{99}_{41}Nb + 4\,^1_0n$$

Still another example is

$$^1_0n + ^{235}_{92}U \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^1_0n$$

- The fragment nuclei produced in fission are highly neutron-rich and unstable.
- They are radioactive and emit beta particles in succession until each reaches a stable end product.
- The **<u>energy released (the *Q value*) in the *fission reaction of nuclei like* uranium</u>** is of the order of **<u>200 MeV</u>** per fissioning nucleus.
- The disintegration energy in fission events first appears as the kinetic energy of the fragments and neutrons.
- Eventually it is transferred to the surrounding matter appearing as heat.
- The source of energy in **<u>nuclear reactors, which produce electricity, is nuclear fission</u>**.
- The enormous energy released in an **<u>atom bomb comes from uncontrolled nuclear</u>** fission.

Nuclear reactor

- Neutrons liberated in fission of a uranium nucleus were so energetic that they would escape instead of triggering another fission reaction.
- Slow neutrons have a much higher intrinsic probability of inducing fission in U (235) than fast neutrons.
- The **average energy of a neutron produced in fission of U (235) is 2 MeV**.
- In reactors, light nuclei called ***moderators*** *are provided along with the fissionable nuclei for* ***slowing*** **down fast neutrons**.
- The moderators commonly used are **water, heavy water (D2O) and graphite**.
- The Apsara reactor at the Bhabha Atomic Research Centre (BARC), Mumbai, uses water as moderator.
- The other Indian reactors, which are used for power production, use heavy water as moderator.

Multiplication factor

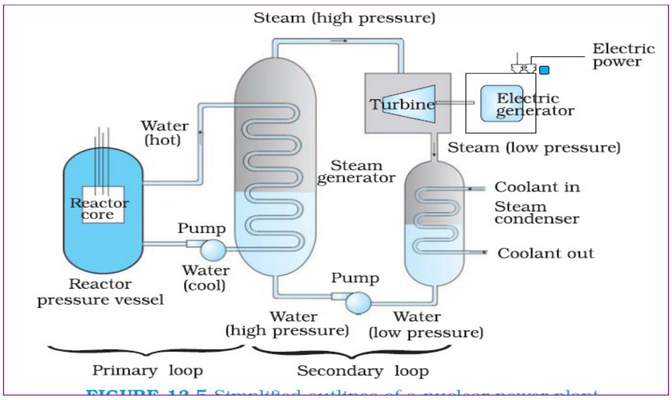
- It is the ratio of number of fission produced by a given generation of neutrons to the number of fission of the preceding generation.
- It is the measure of the growth rate of the neutrons in the reactor.
- For *K = 1, the operation of the reactor* is said to be *critical, which is what we wish it to be for steady power* operation.
- If *K becomes greater than one, the reaction rate and the reactor* power increases exponentially.
- Unless the factor *K is brought down very* close to unity, the reactor will become supercritical and can even explode.
- The explosion of the Chernobyl reactor in Ukraine in 1986 is a sad reminder that accidents in a nuclear reactor can be catastrophic.
- The reaction rate is controlled through **control-rods** made out of neutron-absorbing material such as **cadmium.**
- In addition to control rods, reactors are provided with ***safety rods*** which, when required, can be inserted into the reactor and K can be reduced rapidly to less than unity.
- The abundant U(238) isotope, which does not fission, on capturing a neutron leads to the formation of plutonium.

$$^{238}_{92}\mathrm{U} + \mathrm{n} \rightarrow {}^{239}_{92}\mathrm{U} \rightarrow {}^{239}_{93}\mathrm{Np} + e^- + \bar{\nu}$$

$$^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + e^- + \bar{\nu}$$

- Plutonium is highly radioactive and can also undergo fission under bombardment by slow neutrons

Pressurized-water reactor



- In such a reactor, water is used both as the moderator and as the heat transfer medium

- In the *primary-loop, water is circulated through the reactor vessel and transfers* energy at high temperature and pressure (at about 600 K and 150 atm) to the steam generator, which is part of the *secondary-loop.*
- *In the steam* generator, evaporation provides high-pressure steam to operate the turbine that drives the electric generator.
- The low-pressure steam from the turbine is cooled and condensed to water and forced back into the steam generator.
- A kilogram of U(235) on complete fission generates about 3 × 10^4 MW.
- in nuclear reactions highly radioactive elements are continuously produced.
- Therefore, an unavoidable feature of reactor operation is the accumulation of radioactive waste, including both fission products and heavy *transuranic elements such as plutonium and* americium.

Nuclear fusion

- Energy can be released if two light nuclei combine to form a single larger nucleus, a process called *nuclear fusion.*

$$ {}_1^1\text{H} + {}_1^1\text{H} \rightarrow {}_1^2\text{H} + e^+ + \nu + 0.42\ \text{MeV} $$

$$ {}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + n + 3.27\ \text{MeV} $$

$$ {}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_1^3\text{H} + {}_1^1\text{H} + 4.03\ \text{MeV} $$

- The fusion reaction in the sun is a multi-step process in which hydrogen is burned into helium, hydrogen being the ‘fuel’ and helium the ‘ashes’.
- The ***proton-proton (p, p) cycle by which this occurs is represented*** by the following sets of reactions:.

$$ {}_1^1\text{H} + {}_1^1\text{H} \rightarrow {}_1^2\text{H} + e^+ + \nu + 0.42\ \text{MeV} \qquad \text{(i)} $$

$$ e^+ + e^- \rightarrow \gamma + \gamma + 1.02\ \text{MeV} \qquad \text{(ii)} $$

$$ {}_1^2\text{H} + {}_1^1\text{H} \rightarrow {}_2^3\text{He} + \gamma + 5.49\ \text{MeV} \qquad \text{(iii)} $$

$$ {}_2^3\text{H} + {}_2^3\text{H} \rightarrow {}_2^4\text{He} + {}_1^1\text{H} + {}_1^1\text{H} + 12.86\ \text{MeV} \qquad \text{(iv)} $$

- The combined reaction is

$$ 4{}_1^1\text{H} + 2e^- \rightarrow {}_2^4\text{He} + 2\nu + 6\gamma + 26.7\ \text{MeV} $$

$$ \text{or}\ \ (4{}_1^1\text{H} + 4e^-) \rightarrow ({}_2^4\text{He} + 2e^-) + 2\nu + 6\gamma + 26.7\ \text{MeV} $$

- In sun it has been going on for about 5 × 10^9 y, and calculations show that there is enough hydrogen to keep the sun going for about the same time into the future.
- In about 5 billion years, however, the sun’s core, which by that time will be largely helium, will begin to cool and the sun will start to collapse under its own gravity.
- This will raise the core temperature and cause the outer envelope to expand, turning the sun into what is called a ***red giant***.
- If the core temperature increases to 10^8 K again, energy can be produced through fusion once more – this time by burning helium to make carbon.

Controlled thermonuclear fusion

- The first thermonuclear reaction on earth occurred at Eniwetok Atoll on November 1, 1952, when USA exploded a fusion device, generating energy equivalent to 10 million tons of TNT (one ton of TNT on explosion releases 2.6 × 10’22 MeV of energy).
- A sustained and controllable source of fusion power is considerably more difficult to achieve.

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