HOT QUESTIONS – After study the note, do the following questions yourself. Solution will be given in the next session.

1. If $f(x)=x+7$ and $g(x)=x-7, x\in R$, find $(fog)(7)$.
2. Let $*$ be a binary operation, defined by $a*b=3a+4b-2$, find $4*5$.
3. Show that the relation R defined by $(a,b)R\,(c,d)\Rightarrow a+d=b+c$ on the set $N\times N$ is an equivalence relation.
4. Show that the relation R defined by $R=\{(a,b): a-b \, is \ divisible \ by \ 3;\ a,b\in N\}$ is an equivalence relation.
5. If the binary operation $*$ on the set of integers Z, is defined by $a*b=a+3b^2$, then find the value of $2*4$.
6. If $*$ be a binary operation on N given by $a*b=HCF(a,b),\ a,b\in N$. Write the value of $22*4$.
7. If the binary operation $*$ defined on Q is defined as , $a*b=2a+b-ab$ for all $a,b\in Q$, find the value of $3*4$.
8. If *f* is an invertible function, defined as $f(x)=\frac{3x-4}{5}$, write $f^{-1}(x)$.
9. Prove that the relation R in a set $A=\{1,2,3,4,5\}$ given by $R=\{(a,b): |a-b| \ is \ even\}$, is an equivalence relation.
10. Let $f:N\to N$ defined by $f(n)=\begin{cases}\frac{n+1}{2}, & if\ n\ is\ odd\\ \frac{n}{2}, & if\ n\ is\ odd\end{cases}$ for all $n\in N$. Find whether the function *f* is bijective.
11. Show that the relation S in the set R of real numbers defined as $S=\{(a,b): a,b\in R\ and\ a\le b^2\}$ is neither reflexive, nor transitive.
12. If $f:R\to R$ be defined by $f(x)=(3-x^3)^{\frac{1}{3}}$,
13. If $f(x)=27x^3$ and $g(x)=x^{\frac{1}{3}}$, find $(gof)(x)$.
14. Show that the relation R defined by $R=\{(a,b): a-b \, is \ divisible \ by \ 3;\ a,b\in N\}$ is an equivalence relation.
15. Show that the relation S in the set R of real numbers defined as $S=\{(a,b): a,b\in R\ and\ a\le b^3\}$ is neither reflexive, nor symmetric nor transitive.
16. Show that the relation S in the set $A=\{(a,b)\in Z, |a-b| \ is\ divisible\ by\ 4\}$ is an equivalence relation. Find the set of all elements related to 1.
17. Consider $f:R\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible.
18. Let and * be a binary operation on A defined by (a,b) * (c,d) = (a+c,b+d). Show that * is commutative as well as associative. Also find its identity, if it exists.
19. Show that the relation S in the set R of real numbers, defined as $S=\{(a,b): a,b\in R \text{ and} \le b^3\}$ is neither reflective, nor symmetric not transitive.
20. Show that the relation S in the set

    $A=\{x\in z: 0\le x\le 12\}$ given by

$S=\{(a,b): a,b\in z, |a-b| \text{ is divisibe by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

21. Consider $f:R\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible with $f^{-1}(y)=\left(\frac{\sqrt{y+6}-1}{3}\right)$.

22. Let $A=N\times N$ and $*$ be a binary operation on A defined by (a,b) $*$ (c,d)=(a+c,b+d). Show that $*$ is commutative and associative. Also, find the identity element for $*$ on A, if any.

23. State the reason for the relation R in the set $\{1,2,3\}$ given by $\{(1,2),(2,1)\}$ not to be transitive.

24. Let $A=\{1,2,3\}$; $B=\{4,5,6,7\}$ and let

    $f=\{(1,4),(2,5),(3,6)\}$ be a function from A to B. State whether f is one-one or not.

25. If $R\to R$ is defined by $f(x)=3x+2$, define $f(f(x))$.

26. Consider the binary operation $*$ on the set $\{1,2,3,4,5\}$ fined by $a*b$ = min $\{a,b\}$. Write the operation table of the operation $*$.

27. Let $f:R\to R$ be defined as $f(x)=10x+7$. Find the function $g:R\to R$ such that $gof=fog=1_R$.

28. Write fog, if $f:R\to R$ and $g:R\to R$ are given by $f(x)=|x|$ and $g(x)=|5x-2|$.

29. Write fog, if $f:R\to R$ and $g:R\to R$ are given by $f(x)=8x^3$ and $g(x)=x^{1/3}$

30. A binary operation $*$ on the set $\{0,1,2,3,4,5\}$ is defined as : $a*b=\begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & \text{if } a+b\geq 6\end{cases}$

    Show that zero is the identify for this operation and each element a of the set is invertible with $6-a$, beings the inverse of a.

31. Consider $f:R_+\to[4,\infty)$ given by $f(x)=x^2+4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y)=\sqrt{y-4}$ where R_+ is the set of all non- negative real numbers.

32. Let $*$ be a binary operation on N given by $a*b$=lcm (a,b) for all $a,b\in N$. Find $5*7$.

33. The binary operation $*$ $R\times R\to R$ is defined as $a*b=2a+b$. Find $(2*3)*4$.

34. Let $A=R-\{3\}$ and $B=R-\{1\}$ consider the function $f:A\to B$ defined by $f(x)=\frac{x-2}{x-3}$ show that f is one-one and onto and hence find f^{-1}

35. If the binary operation $*$ on the set Z of integers is defined by $a*b=a+b-5$, then write the identity elements for the operation $*$ in Z.

36. Show that $f:N\to N$, given by $f(x)=\begin{cases} x+1, \text{if } x \text{ is odd} \\ x-1, \text{if } x \text{ is even}\end{cases}$, is bijective (both one-one and onto).

37. Consider the binary operation $*:R\times R\to R$ and $o:R\times R\to R$ defined as $a*b=|a-b|$ and $a\,o\,b=a$ for all $a,b\in R$. Show that $*$ is commutative but not associative, o is associative but not commutative.

38. If $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$, show that $(fof)(x)=x$ for all $x\neq\frac{2}{3}$. What is the inverse of f?