CHAPTER 3 & 4

MATRICES AND DETERMINANTS

SAY 2018

1. a) Construct a 2×2 matrix $A=[a_{ij}]$ whose elements are given by $a_{ij}=2i+j$ (1)

   b) Find A^2 (2)

2. Consider the matrix $A=\begin{bmatrix}1&0&2\\0&1&2\\0&4&9\end{bmatrix}$

   a) Find A^{-1} using elementary row operations. (3)

   b) Find the solution of the system of equations given below: (3)
   (A^{-1} obtained above may be used)
   $x+2z=2;\ y+2z=1;\ 4y+9z=3$

3. a) Show that

   $$\begin{vmatrix}1&a&bc\\1&b&ca\\1&c&ab\end{vmatrix}=(a-b)(b-c)(c-a)\quad (3)$$

   b) If $A=\begin{bmatrix}2&3\\4&-1\end{bmatrix}$, verify that

   $A(adj\,A)=|A|I$ (3)

MARCH 2018

4. Using elementary row operations, find the inverse of the matrix $\begin{bmatrix}1&2\\2&-1\end{bmatrix}$ (3)

5. a) Find x and y if

   $$x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}\quad (2)$$

   b) Express the matrix $A=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrices. (4)

6. a) Prove that $\begin{vmatrix}a&b&c\\a+2x&b+2y&c+2z\\x&y&z\end{vmatrix}=0$ (2)

   b) If $A=\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}$; $B=\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}$

   i) Prove that $B=A^{-1}$.

   ii) Using A^{-1}, solve the system of linear equations given below:
   $x-y+2z=1;\ 2y-3z=1;$
   $3x-2y+4z=2$ (4)

SAY 2017

7. a) The number of all possible 2×2 matrices with entries 0 or 1 is

   a) 8 b) 9
   c) 16 d) 25 (1)

   b) If the area of a triangle whose vertices are $(k,0),(5,0),(0,1)$ is 0 square units, then find k (2)

   c) Using elementary transformation find the inverse of the matrix $\begin{bmatrix}2&1\\1&1\end{bmatrix}$ (3)

8. a) If A is a 2×2 matrix with $|A|=5$, then $|adj\,A|$ is

a) 5 b) 25

c) $\frac{1}{5}$ d) $\frac{11}{25}$ (1)

b) Solve the system of equations using matrix method: $x+y+z=1;\ 2x+3y-z=6$ and $x-y+z=-1$ (4)

MARCH 2017

9. a) The value of k such that the matrix $\begin{pmatrix} 1 & k \\ -k & 1 \end{pmatrix}$ is symmetric is

a) 0 b) 1

c) -1 d) 2 (1)

b) If $A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that $A^2=\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$ (3)

c) If $A=\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}$ then find $|3A|$ (2)

10. a) If $A=\begin{bmatrix} a & 1 \\ 1 & 0 \end{bmatrix}$ is such that $A^2=I$ then a equals

a) 1 b) -1

c) 0 d) 2 (1)

b) Solve the system o equations:
$x-y+z=4$
$2x+y-3z=0$. Using matrix method (4)
$x+y+z=2$

SAY 2016

11. a) If the matrix A is both symmetric and skew-symmetric, then A is ……… (1)

b) If $A=\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, then show that $A^2-5A+10I=0$ (3)

c) Hence find A^{-1}. (2)

12. a) The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix}$ is ………….. (1)

b) Using matrix method, solve the system of linear equations:
$x+y+2z=4;\ 2x-y+3z=9$
$3x-y-z=2$ (4)

MARCH 2016

13. a) If $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, then $BA=$ …………… (1)

b) Write $A=\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix. (2)

c) Find the inverse of $A=\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$ (3)

14. a) The value of $\begin{vmatrix} x & x-1 \\ x+1 & x \end{vmatrix}$ is ……….. (1)

b) Using properties of determinants, show that
$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}=\left(1-x^3\right)^2.$$ (4)

SAY 2015

15. Consider a 2×2 matrix $A=[a_{ij}]$ with $a_{ij}=2i+j$.

a) Construct A (1)

b) Find $A+A'$ and $A-A'$. (1)

c) Express A as sum of a symmetric and skew-symmetric matrix. (1)

16. Consider a matrix $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$

a) Find A^2. (2)

b) Find k so that $A^2 = ka - 2I$. (1)

17. a) If $\begin{vmatrix}x & 1\\ 1 & x\end{vmatrix}=15$, then find the values of x. (1)

b) Solve the following system of equations:

$3x-2y+3z=8;\ 2x+y-z=1;$

$4x-3y+2z=4$ (4)

MARCH 2015

18. (a) Choose the correct statement related to the matrices $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ *and* $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$.

(i) $A^3 = A, B^3 \neq B$ (ii) $A^3 \neq A, B^3 = B$

(iii) $A^3 = A, B^3 = B$ (iv) $A^3 \neq A, B^3 \neq B$

(1)

(b) If $M=\begin{bmatrix}7 & 5\\ 2 & 3\end{bmatrix}$, then verify the equation

$M^2 - 10M + 11I_2 = 0$. (2)

(c) Inverse of the matrix $\begin{bmatrix}0 & 1 & 2\\ 0 & 1 & 1\\ 1 & 0 & 2\end{bmatrix}$ (3)

19. Prove that

$$\begin{vmatrix}1 & x & x^3\\ 1 & y & y^3\\ 1 & z & z^3\end{vmatrix}=(x+y+z)(x-y)(y-z)(z-x)$$

OR

Prove that $\begin{vmatrix}1! & 2! & 3!\\ 2! & 3! & 4!\\ 3! & 4! & 5!\end{vmatrix}=4!$ (3)

20. Solve the system of linear equations:

$x+2y+z=8$; $2x+y-z=1$;

$x-y+z=2$ (3)

SAY 2014

21. a) If A, B are symmetric matrices of same order then AB–BA is always a ………..

A) Skew-Symmetric matrix

B) Symmetric matrix

C) Identity matrix

D) Zero matrix. (1)

b) For the matrix $A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$, verify that $A+A'$ is a symmetric matrix. (2)

22. a) Let A be a square matrix of order 2×2 then $|KA|$ is equal to

A) $K|A|$ B) $K^2|A|$

C) $K^3|A|$ D) $2K|A|$ (1)

b) Prove that

$$\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)^3$$

(3)

c) Examine the consistency of the system of equations: $5x+3y=5;\ 2x+6y=8$ (1)

(Scores:3)

MARCH 2014

23. Consider the matrices

$$A=\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix} \text{ and } B=\begin{bmatrix}a & b\\ c & d\end{bmatrix}.$$

24. If $AB=\begin{bmatrix}2 & 9\\ 5 & 6\end{bmatrix}$, find the values a, b, c and d. (3)

25. Consider a 2×2 $matrix\ A=[aij]$, where

$$aij=\frac{(1+2j)^2}{2}$$

a) Write A (2)

b) Find $A+A'$ (1)

26. Consider the matrix $A=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$

a) Show that $A^2-7A-2l=0$. (2)

b) Hence find A^{-1}. (2)

c) Solve the following system of equation using matrix method:

$2x+3y=4;\ 4x+5y=6$ (1)

SAY 2013

27. Consider the matrices $A=\begin{bmatrix}2 & -6\\ 1 & 2\end{bmatrix}$ and

$$A-3B=\begin{bmatrix}5 & -3\\ -2 & -1\end{bmatrix}$$

a) Find matrix B (1)

b) Find matrix AB (1)

c) Find the transpose of B. (1)

28. If a matrix $A=\begin{bmatrix}3x & x\\ -x & 2x\end{bmatrix}$ is a solution of the matrix equation $x^2-5x+7I=0$, find any one value of x. (3)

MARCH 2013

29. Let $A=\begin{bmatrix}2 & 4\\ 1 & -3\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 5\\ 0 & 2 & 6\end{bmatrix}$

a) Find AB. (2)

b) Is BA defined? Justify your answer. (1)

30. a) Find the values of x in which

$$\begin{vmatrix}3 & x\\ x & 1\end{vmatrix}=\begin{vmatrix}3 & 2\\ 4 & 1\end{vmatrix}$$ (1)

b) Using properties of determinants, show that the points $A(a,b+c), B(b,c+a)$ and $C(c,a+b)$ are collinear. (2)

c) Examine the consistency of following equations:

$5x-6y+4z=15;\ 7x+4y-3z=19;$
$2x+y+6z=46$ (2)

SAY 2012

31. i) Find the value of a and b if the matrix

$$\begin{bmatrix}0 & 3 & a\\ b & 0 & -2\\ 5 & 2 & 0\end{bmatrix} \text{is skew – symmetric.}$$ (1)

ii) Express $A=\begin{bmatrix}7 & 3 & -5\\ 0 & 1 & 5\\ -2 & 7 & 3\end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix. (2)

32. i) If A is a square matrix such that $A^2=A$, then $(I+A)^2-3A$ is equal to

a) A b) $I-A$

c) I d) 3A (1)

ii) Find $A^2-5A+6I$, if $A=\begin{bmatrix}2&5&8\\6&0&5\\0&-2&0\end{bmatrix}$ (2)

MARCH 2012

33. Consider the matrix $A=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$

a) Find A^2. (2)

b) Find K so that $A^2=kA-7I$ (1)

34. a) If $\begin{vmatrix}x&3\\5&2\end{vmatrix}=5$, then $x=\ldots\ldots\ldots$ (1)

b) Prove that

$$\begin{vmatrix}y+k&y&y\\y&y+k&y\\y&y&y+k\end{vmatrix}=k^2(3y+k).$$ (2)

c) Solve the following system of equations using matrix method:

$5x+2y=3;\ 3x+2y=5$ (2)

SAY 2011

35. Let $A=\begin{bmatrix}3&6&5\\6&7&8\end{bmatrix}$ and $C=\begin{bmatrix}1&2&-3\\4&5&6\end{bmatrix}$

a) Find 2A (1)

b) Find the matrix B such that $2A+B=3C$ (2)

36. Let $A=\begin{bmatrix}2&4\\-1&1\end{bmatrix}$

a) Apply the elementary operation $R_1\to\frac{R_1}{2}$ in the matrix A. (1)

b) Find the inverse of A using elementary transformations. (2)

MARCH 2011

37. a) Find the value of x and y from the following equation.

$$a\begin{bmatrix}x&5\\7&y-3\end{bmatrix}+\begin{bmatrix}3&-4\\1&2\end{bmatrix}=\begin{bmatrix}7&6\\15&14\end{bmatrix}$$ (1)

38. Given $P=\begin{bmatrix}2&-3\\-1&2\end{bmatrix}$. Find the inverse of P by elementary row operation. (3)

SAY 2010

39. Consider the matrices $A=\begin{bmatrix}2&1&3\\2&3&1\\1&1&1\end{bmatrix}$ and

$B=\begin{bmatrix}-1&2&3\\-2&3&1\\-1&1&1\end{bmatrix}$

a) Find $A+B$ (1)

b) Find $(A+B)(A-B)$ (2)

40. Let $A=\begin{bmatrix}0&2\beta&\gamma\\\alpha&\beta&-\gamma\\\alpha&-\beta&\gamma\end{bmatrix}$ be a 3×3 matrix.

a) Find A', the transpose of A. (1)

b) If $A'A=6I$, where I is the 3×3 identity matrix, find the values of α,β and γ (2)

MARCH 2010

41. a) Construct a 3×3 matrix A whose elements are given by $a_{ij}=2i-j$ (1)

b) If $B=\begin{bmatrix}1&2\\3&-1\\4&2\end{bmatrix}$, find AB (2)

42. $A=\begin{bmatrix}1 & 4 & -1\\ 2 & 5 & 4\\ -1 & -6 & 3\end{bmatrix}$. Write A as the sum of a symmetric matrix and a skew symmetric matrix. (3)

SAY 2009

43. If $A=\begin{bmatrix}1 & 3\\ -2 & 4\end{bmatrix}$

a) Find A^2 (1)

b) Show that $A^2-5A+10I=0$ (2)

c) Find A^{-1}. (1)

MARCH 2009

44. Let $B=\begin{bmatrix}2 & 3 & 1\\ 3 & -1 & 4\\ 1 & -4 & -3\end{bmatrix}$

a) Find B' (1)

b) Check whether $\frac{B-B'}{2}$ is a skew-symmetric matrix. (2)

45. Let $A=\begin{bmatrix}-1 & 2 & 4\\ 1 & 1 & 3\\ 3 & 2 & 3\end{bmatrix}$

a) Find $|A|$ (1)

b) Find $adj\ A$ (2)

c) Verify that $A(adj\ A)=|A|I$ (2)

MARCH 2008

46. Let $A=\begin{bmatrix}1 & 4 & 0\\ -1 & 2 & 2\\ 0 & 0 & 2\end{bmatrix}$

a) Is A singular? (1)

b) Find $adj\ A$ (2)

c) Find A^{-1} using $adj\ A$ and $|A|$ (2)

47. Let $\Delta=\begin{vmatrix}-a^2 & ab & ac\\ ba & -b^2 & bc\\ ac & bc & -c^2\end{vmatrix}$.

Let R_1,R_2,R_3 be the rows and C_1,C_2,C_3 be the columns of the above determinant.

a) Take a,b,c common from R_1,R_2,R_3 respectively. (1)

b) Take a,b,c common from C_1,C_2,C_3 respectively. (1)

c) Apply the operations $C_2\to C_2+C_1, C_3\to C_3+C_1$ (1)

d) Hence evaluate Δ. (2)

SAY 2007

48. Let $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 5 & 4\\ 1 & 6\end{bmatrix}$

a) What is the order of AB? (1)

b) Find A' and B' (1)

c) Verify that $(AB)'=B'A'$ (2)

49. Consider the system of equations:

$x+y+3z=5;\ x+3y-3z=1$

$-2x-4y-4z=-10$

Convert this system of equations in the standard form $AX=B$

a) Find A^{-1}

b) Hence or otherwise solve the system of equations.

MARCH 2007

50. Consider the following statement: $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, for all $n \in N$

a) Write P(1). (1)

b) If P(K) is true, then show that P(K+1) is also true. (2)

c) Solve the following system of equations, using matrix method: $2x+5y=1,\ 3x+2y=7$

(2)

51. If $\begin{bmatrix} 7 & 5 \\ -3 & 7 \end{bmatrix} \times A = \begin{bmatrix} 17 & -1 \\ 47 & -13 \end{bmatrix}$,then

a) Find the 2×2 matrix A. (2)

b) Find A^2 (1)

c) Show that $A^2+5A-6I=0$, where I is the identity matrix of order 2. (1)

d) What is A^{-1}? (1)

MARCH 2006

52. If A and B are square matrices of order 3 such that $|A|=-1$ and $|B|=3$, then the determinant of the matrix $3AB$ is …………

a) –9 b) –27

c) –81 d) 9 (1)

53. If $f(x) = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, show that

$f(x+y) = f(x).f(y)$ (2)

54. If a,b,c are positive, show that

$$\begin{vmatrix} 1 & \log_a b & \log_a C \\ \log_b a & 1 & \log_b C \\ \log_c a & \log_c b & 1 \end{vmatrix} = 0 \quad (2)$$ (3)

55. Solve the equations:

$x+y+z=3;\ x+2y+3z=4;\ x+4y+9z=6$

by matrix method. (5)

MARCH 2005

56. If A is a square matrix of order 3 then $|adj\ A|$ is

a) $|A|^3$ b) $|A^{-1}|$

c) $3|A|$ d) $|A|^2$ (1)

57. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of k so that

$A^2 = kA - 2I$ (2)

58. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, prove that

$A^n = \begin{bmatrix} \cos nx & \sin nx \\ -\sin nx & \cos nx \end{bmatrix}$ (3)

59. Using properties of determinants, prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$ (4)

60. Solve the equations:

$x+y+z=6;\ x+2z-7=0;\ 3x+y+z=12$

by matrix method. (5)

SAY 2004

61. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 8 & 5 \\ 2 & -1 & 3 \end{bmatrix}$

then,

a) AB cannot be defined.

b) AB is a 2 x 3 matrix

c) AB is a 3 x 3 matrix

d) None of these. (1)

62. If $A=\begin{bmatrix}4 & -1\\ -2 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 2 & 0\end{bmatrix}$ and A+KB is singular, then K= ....

a) $\frac{1}{2}$ b) $-\frac{3}{2}$

c) $\frac{3}{2}$ d) $-\frac{1}{2}$ (1)

63. Evaluate: $\begin{vmatrix}x^2-x+1 & x-1\\ x+1 & x+1\end{vmatrix}$. (2)

64. Let A = $\begin{bmatrix}3 & 8\\ 2 & 1\end{bmatrix}$, find A^{-1} and verify that $A^{-1}=\frac{1}{13}A-\frac{4}{13}I$, where I is 2 x 2 unit matrix. (3)

65. Prove that $\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix}=(a-b)(b-c)(c-a)$ (3)

66. Solve the lineare equations:

$x+y+z=6;\ x+2z-7=0;\ 3x+y+z=12$

with the help of matrices. (5)

MARCH 2004

67. If A= $\begin{bmatrix}p & 0\\ 0 & q\end{bmatrix}$ and AA^T = I, then

a) p = 1, q =1 b) p = 1, q = –1

c) p = ± 1, q =± 1 c) p = q = 0 (1)

68. If A= $\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$ and

B= $\begin{vmatrix}a_1 & b_1 & ma_1+nb_1+c_1\\ a_2 & b_2 & ma_2+nb_2+c_2\\ a_3 & b_3 & ma_3+nb_3+c_3\end{vmatrix}$, then B =

a) mA b) nA

c) mA+nA d) A (1)

69. Evaluate: $\begin{vmatrix}\cos 80 & -\cos 10\\ \sin 80 & \sin 10\end{vmatrix}$. (2)

70. show that

$$\begin{vmatrix}a+b+2c & a & b\\ c & b+c+2a & b\\ c & a & c+a+2b\end{vmatrix}=2(a+b+c)^3$$

(3)

71. If A = $\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$, find K so that $A^2=8A+KI$ (3)

72. Solve the system of equations by matrix method:

$x+y+z=3;\ y-z=0;\ 2x-y=1$ (5)

SAY 2003

73. If A = $\begin{bmatrix}5 & 3 & 10\end{bmatrix}$ and B = $\begin{bmatrix}2\\ 4\\ 6\end{bmatrix}$, then AB is:

a) $\begin{bmatrix}10 & 12 & 60\end{bmatrix}$ b) $\begin{bmatrix}10\\ 12\\ 60\end{bmatrix}$

c) $\begin{bmatrix}82\end{bmatrix}$ d) Not defined.

74. If the matrix $\begin{bmatrix} 1 & 4 & 3 \\ 6 & 8 & -5 \\ 2 & a & 6 \end{bmatrix}$, is singular, then the value of a is

a) 5 b) 7

c) 8 d) 10 (1)

75. If A= $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, find a matrix X such that $2A + X = 3B$. (2)

76. Without expanding prove that $a+b+c$ is a factor of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$. (2)

77. Let A = $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$, C = $\begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ verify that $(AB)C = A(BC)$. (3)

78. If I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, E = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bE)^3 = a^3 I + 3a^2 bE$,where a and b are constants. (3)

79. If α and β are the odd multiples of $\frac{\pi}{2}$, prove that AB = 0 where A = $\begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$ and B = $\begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$. (3)

80. Test whether the following system of equations is consistent:

$2x + y + z = 1; x - 2y - z = 1.5; 3y - 5z = 9$. (3)

81. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ (5)