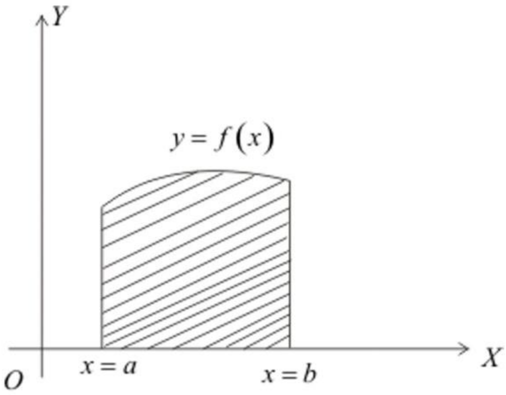
APPLICATIONS OF DEFINITE INTEGRALS

First area:

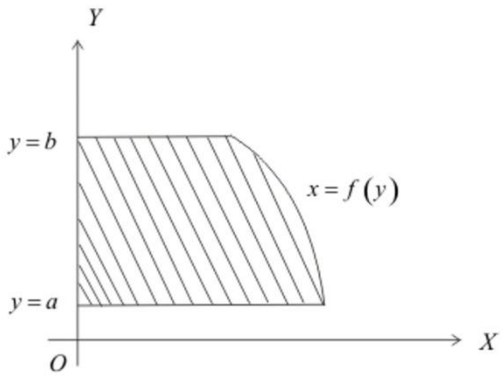
The area enclosed between the curve $y = f(x)$, the x axis and the ordinates at $x = a$ and $x = b$ is

$$\int_{x=a}^{x=b} y\,dx$$



Second area:

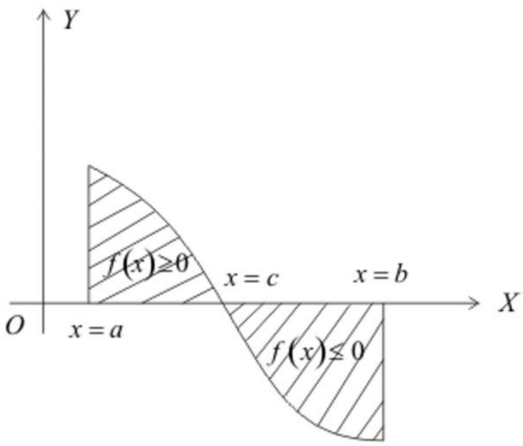
The area enclosed between the curve $x = f(y)$, the y axis and the ordinates at $y = a$ and $y = b$ is $\int_{y=a}^{y=b} x\,dy$



Third area:

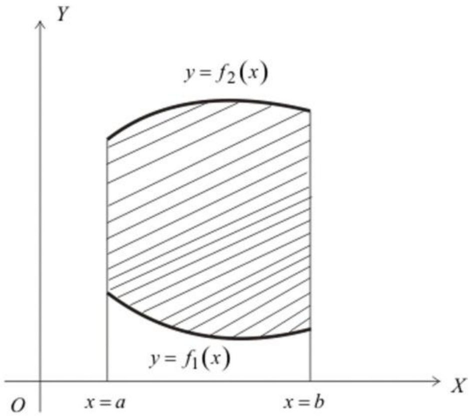
If $f(x) \geq 0,\ for\ a \leq x \leq c$ and $f(x) \leq 0,\ for\ c \leq x \leq b$, then the area enclosed between the curve $y = f(x)$, the x axis and the ordinates at $x = a$ and $x = b$ is

$$\int_{x=a}^{x=c} f(x)\,dx + \left|\int_{x=c}^{x=b} f(x)\,dx\right|.$$



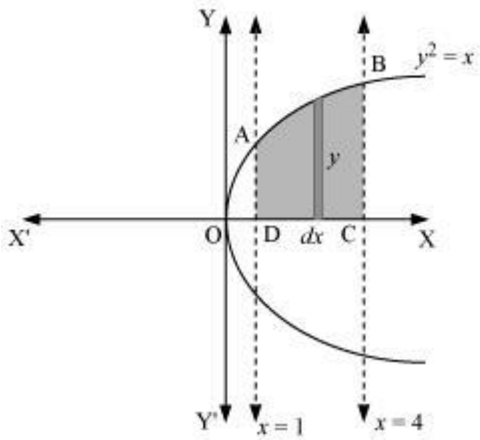
Fourth area:

The area enclosed between the curves $y = f_1(x)$ $y = f_2(x)$, the x axis and the ordinates at $x = a$ and $x = b$ is $\int_{x=a}^{x=b} \left[f_2(x) - f_1(x)\right]dx$.



Questions:

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis.

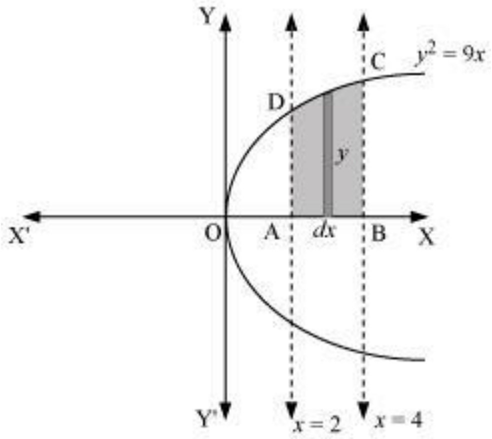


Area of ABCD $= \int_{x=1}^{x=4} y\,dx$

$$= \int_{x=1}^{x=4} \sqrt{x}\,dx$$

$$= \frac{2}{3}\left[x^{\frac{3}{2}}\right]_1^4 = \frac{2}{3}\left[4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] = \frac{2}{3}\left(2^3 - 1\right) = \frac{2}{3}(7) = \frac{14}{3} \text{ sq units.}$$

2. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.



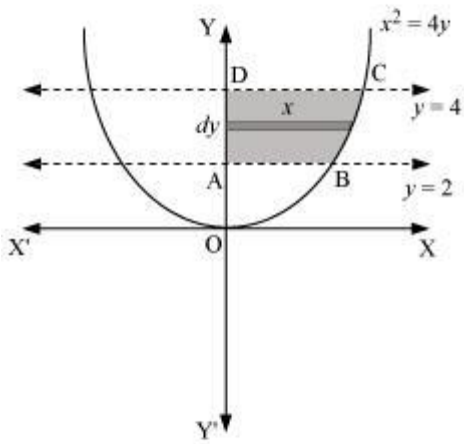
Area of ABCD $= \int_{x=2}^{x=4} y\,dx$

$$= \int_{x=2}^{x=4} 3\sqrt{x}\,dx$$

$$= 3\times\frac{2}{3}\left[x^{\frac{3}{2}}\right]_2^4 = 2\left[4^{\frac{3}{2}} - 2^{\frac{3}{2}}\right] = 2\left[2^3 - \left(\sqrt{2}\right)^3\right]$$

$$= 2\left[8 - 2\sqrt{2}\right] = \left(16 - 4\sqrt{2}\right) \text{ sq. units}$$

3. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.



Area of ABCD $= \int_{y=2}^{y=4} x\,dy$

$$= \int_{y=2}^{y=4} 2\sqrt{y}\, dy$$

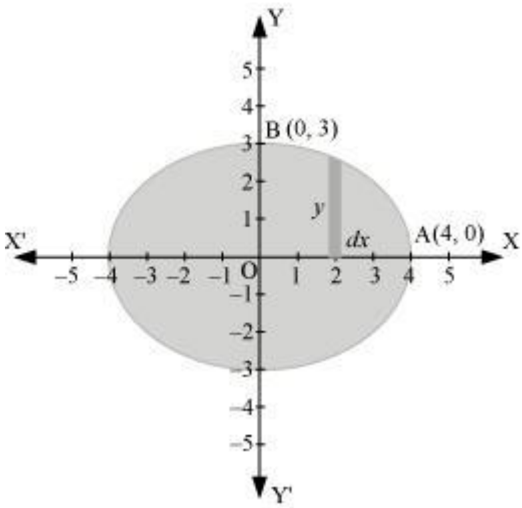
$$= 2\left[\frac{2}{3} y^{\frac{3}{2}}\right]_2^4$$

$$= \frac{4}{3}\left[x^{\frac{3}{2}}\right]_2^4 = \frac{4}{3}\left[4^{\frac{3}{2}} - 2^{\frac{3}{2}}\right] = \frac{4}{3}\left[2^3 - \left(\sqrt{2}\right)^3\right]$$

$$= \frac{4}{3}\left[8 - 2\sqrt{2}\right] = \left(\frac{32 - 8\sqrt{2}}{3}\right) \text{ sq units.}$$

4. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

The given equation of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ can be represented as



It can be observed that the ellipse is symmetrical about *x*-axis and *y*-axis.

∴ Area bounded by ellipse = 4 × Area of OAB

$$\text{Area } = 4 \times \int_{x=0}^{x=4} y\, dx$$

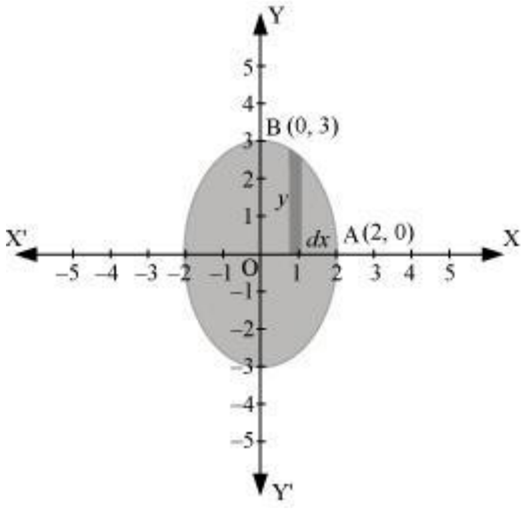
$$= 4 \times \frac{3}{4}\int_0^4 \sqrt{16 - x^2}\, dx \qquad \| \frac{y^2}{9} = 1 - \frac{x^2}{16}$$

$$= 3\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4 \qquad \| y^2 = \frac{9}{16}\left(16 - x^2\right) \Rightarrow y = \frac{3}{4}\sqrt{16 - x^2}$$

$$= 3\left[0 + 8\sin^{-1}(1) - \{0 + 0\}\right]$$

$$= 24 \times \frac{\pi}{2} = 12\pi \text{ sq. units.}$$

5. Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$



$$\text{Area } = 4\times\int_{x=0}^{x=2} y\,dx$$

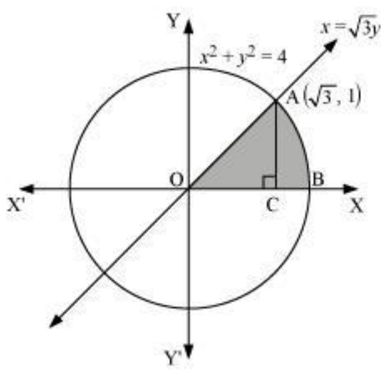
$$= 4\times\frac{3}{2}\int_0^4\sqrt{2^2-x^2}\,dx \qquad \| \frac{y^2}{9}=1-\frac{x^2}{4}$$

$$= 6\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2 \qquad \| y^2=\frac{9}{4}\left(4-x^2\right)\Rightarrow y=\frac{3}{2}\sqrt{4-x^2}$$

$$= 6\left[0+2\sin^{-1}(1)-\{0+0\}\right]$$

$$= 12\times\frac{\pi}{2}=6\pi \text{ sq. units.}$$

6. Find the area of the region in the first quadrant enclosed by x-axis, line $x=\sqrt{3}y$ and the circle $x^2+y^2=4$.



The point of intersection of the line and the circle in the first quadrant is $\left(\sqrt{3},1\right)$.

Area of the shaded portion = Area ΔOCA + Area ACB

$$= \int_0^{\sqrt{3}} (y \text{ of line})\,dx + \int_{\sqrt{3}}^{2} (y \text{ of circle})\,dx$$

$$= \int_0^{\sqrt{3}} \frac{1}{\sqrt{3}} x\,dx + \int_{\sqrt{3}}^{2} \sqrt{2^2 - x^2}\,dx$$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{2^2-x^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$$

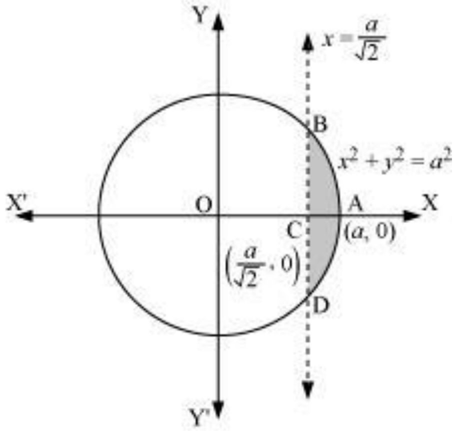
$$= \frac{1}{\sqrt{3}}\left[\frac{\left(\sqrt{3}\right)^2}{2} - 0\right] + \left[0 + 2\sin^{-1}(1) - \left\{\frac{\sqrt{3}}{2}\sqrt{2^2 - \left(\sqrt{3}\right)^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\right]$$

$$= \frac{\sqrt{3}}{2} + 2\times\frac{\pi}{2} - \left[\frac{\sqrt{3}}{2}\sqrt{1} + 2\times\frac{\pi}{3}\right]$$

$$= \frac{\sqrt{3}}{2} + \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \pi - \frac{2\pi}{3} = \frac{3\pi - 2\pi}{3} = \frac{\pi}{3}\, sq.units$$

7. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

The area of the smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line $x = \frac{a}{\sqrt{2}}$, is the area ABCDA.



Area ABCD = 2 × Area ABCA

$$= 2\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_{\frac{a}{\sqrt{2}}}^{a}$$

$$= 2\left[\frac{a}{2}(0) + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right) - \left\{\frac{\frac{a}{\sqrt{2}}}{2}\sqrt{a^2 - \left(\frac{a}{\sqrt{2}}\right)^2} + \frac{\left(\frac{a}{\sqrt{2}}\right)^2}{2}\sin^{-1}\left(\frac{\frac{a}{\sqrt{2}}}{a}\right)\right\}\right]$$

$$= 2\left[\frac{a^2}{2}\times\frac{\pi}{2} - \left\{\frac{a}{2\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} + \frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right\}\right]$$

$$= 2\left[\frac{\pi a^2}{4} - \left\{\frac{a}{2\sqrt{2}}\frac{a}{\sqrt{2}} + \frac{a^2}{2}\times\frac{\pi}{4}\right\}\right]$$

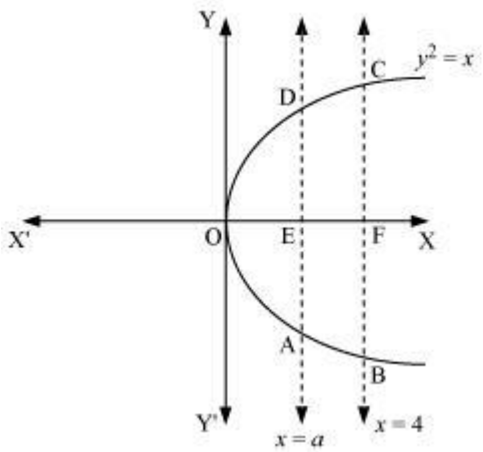
$$= \frac{\pi a^2}{2} - \frac{a^2}{2} - \frac{\pi a^2}{4} = \frac{a^2}{2}\left(\pi - 1 - \frac{\pi}{2}\right)$$

$$= \frac{a^2}{2}\left(\pi - 1 - \frac{\pi}{2}\right) = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right) \text{ sq. units.}$$

8. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a.

The line, $x = a$, divides the area bounded by the parabola and $x = 4$ into two equal parts.

∴ Area OAD = Area ABCD



It can be observed that the given area is symmetrical about x-axis.

⇒ Area OED = Area EFCD

$$\int_0^a \sqrt{x}\,dx = \int_a^4 \sqrt{x}\,dx$$

$$\left[x^{\frac{3}{2}}\right]_0^a = \left[x^{\frac{3}{2}}\right]_a^4$$

$$a^{\frac{3}{2}} = 4^{\frac{3}{2}} - a^{\frac{3}{2}}$$

$$2\times a^{\frac{3}{2}} = 2^3 = 8$$

$$a^{\frac{3}{2}} = 4 \Rightarrow a = 4^{\frac{2}{3}}$$

9. Find the area of the region bounded by the parabola and $y = x^2$ and $y = |x|$.

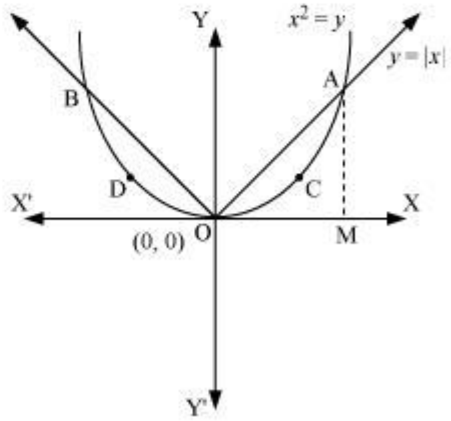
The given area is symmetrical about y-axis.

$y = x^2$ ………… (1) is an upward parabola.

Substituting $y=|x|$ in (1)

$$x^2=|x|$$
$$x^4=x^2 \Rightarrow x^4-x^2=0$$
$$\Rightarrow x^2\left(x^2-1\right)=0$$
$$\Rightarrow x=0 \ \ or \ \ x=\pm 1$$

The point of intersection of parabola, $y=x^2$, and line, $y=|x|$, is A (1, 1).



$$\therefore \text{ Required area } =2\left[\int_0^1\left(|x|-x^2\right)dx\right]=2\left[\int_0^1\left(x-x^2\right)dx\right]$$
$$=2\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1=2\left[\frac{1^2}{2}-\frac{1^3}{3}-0\right]$$
$$=2\left[\frac{1}{2}-\frac{1}{3}\right]=2\left(\frac{3-2}{6}\right)=\frac{2}{6}=\frac{1}{3} \text{ sq.units.}$$

10. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y – 2$

The area bounded by the curve, $x^2 = 4y$, and line, $x = 4y – 2$, is represented by the shaded area OBAO.

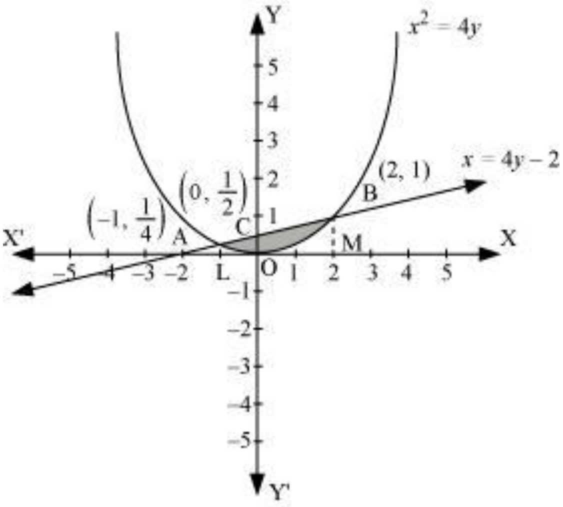
Let the curves be $x^2=4y$..........(1) and $x=4y-2$ ....(2)

Solving, we have: $x+2=4y$

Sub. in (1), $x^2=x+2 \Rightarrow x^2-x-2=0 \Rightarrow (x-2)(x+1)=0$

$$\Rightarrow x=2 \ \ and \ \ x=-1$$
$$when \ \ x=2, \ 2+2=4y \Rightarrow y=\frac{4}{4}=1$$
$$when \ \ x=-1, \ -1+2=4y \Rightarrow y=\frac{1}{2}$$

$\therefore$ co-ordinates of A and B are: $\left(-1, \frac{1}{2}\right)$ *and* $(2,1)$



$\therefore$ The area of the shaded region $= \int_{-1}^{2} (y \text{ of line} - y \text{ of parabola})\, dx$

$$= \int_{-1}^{2} \left(\frac{x+2}{4} - \frac{x^2}{4} \right) dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^{2}$$

$$= \frac{1}{4}\left[\frac{2^2}{2} + 2(2) - \frac{2^3}{3} - \left\{ \frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3} \right\} \right]$$

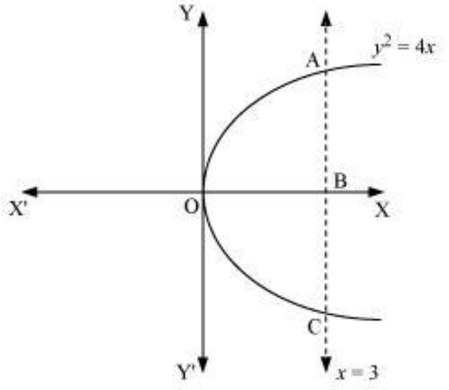
$$= \frac{1}{4}\left[2 + 4 - \frac{8}{3} - \left\{ \frac{1}{2} - 2 + \frac{1}{3} \right\} \right]$$

$$= \frac{1}{4}\left[6 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} \right] = \frac{1}{4}\left[8 - \frac{9}{3} - \frac{1}{2} \right]$$

$$= \frac{1}{4}\left[5 - \frac{1}{2} \right] = \frac{1}{4} \times \frac{9}{2} = \frac{9}{8} \text{ sq. units.}$$

11. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$

The region bounded by the parabola, $y^2 = 4x$, and the line, $x = 3$, is the area OACO.



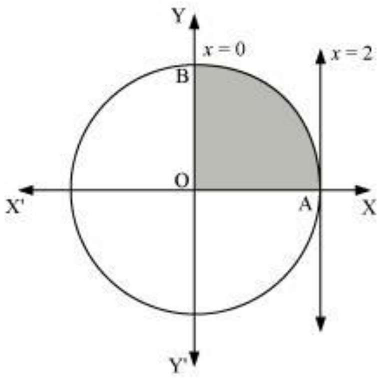
$$\therefore \text{ The required } = 2\times\int_0^3 y\,dx = 2\int_0^3 2\sqrt{x}\,dx = 4\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^3$$

$$= 4\times\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^3 = \frac{8}{3}\left[3^{\frac{3}{2}} - 0\right] = \frac{8}{3}\left[\left(\sqrt{3}\right)^3\right] = \frac{8}{3}\times 3\sqrt{3} = 8\sqrt{3} \text{ sq.units}$$

12. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x=0 and x=2 is

A. π $\frac{\pi}{2}$ **B.** $\frac{\pi}{3}$ **C.** $\frac{\pi}{4}$ **D.**

The area bounded by the circle and the lines, $x = 0$ and $x = 2$, in the first quadrant is represented as



$$\therefore \text{ The required } = \int_0^2 \sqrt{2^2 - x^2}\,dx = \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= \left[0 + 2\sin^{-1}(1) - (0+0)\right]$$

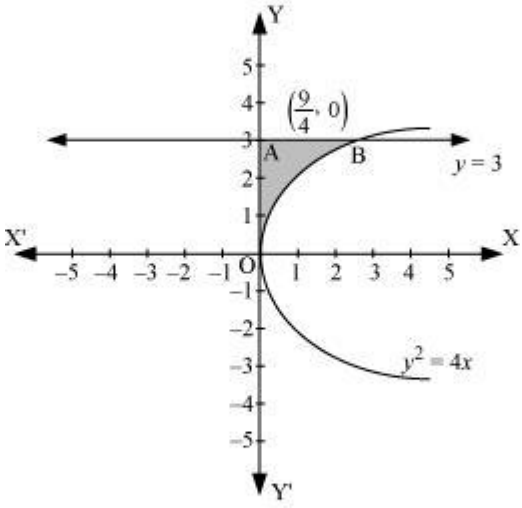
$$= 2\times\frac{\pi}{2} = \pi \text{ sq. units.}$$

Ans: (A)

13. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

A. 2 **B.** $\frac{9}{4}$ **C.** $\frac{9}{3}$ **D.** $\frac{9}{2}$

The area bounded by the curve, $y^2 = 4x$, y-axis, and $y = 3$ is represented as



Area of the shaded region $= \int_0^3 x\,dy = \int_0^3 \frac{y^2}{4}\,dy = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$

$= \frac{1}{12}\left(3^3 - 0^3\right) = \frac{1}{12}(27) = \frac{9}{4}$ sq.units.

Ans: B.

Exercise 8.2

1. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$

   $4x^2 + 4y^2 = 9$..........(1) is a circle passing through the origin and having radius $\frac{3}{2}$ units and

   $x^2 = 4y$ ....(2) is an upward parabola.

   In (1), we have, $4(4y) + 4y^2 = 9 \Rightarrow 4y^2 + 16y - 9 = 0$

   $4y^2 + 18y - 2y - 9 = 0 \Rightarrow 2y(2y+9) - 1(2y+9) = 0$

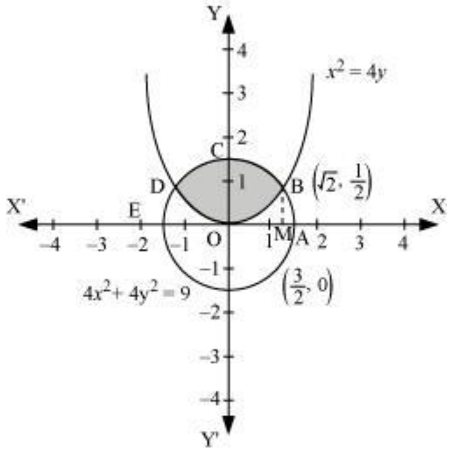
   $(2y+9)(2y-1) = 0 \Rightarrow 2y + 9 = 0 \;\; or \;\; 2y - 1 = 0$

   $y = -\frac{9}{2} \;\; or \;\; y = \frac{1}{2}$

   But $y = -\frac{9}{2}$ is inadmissible. $\therefore$ $y = \frac{1}{2}$

   When $y = \frac{1}{2}$, $x^2 = 4\left(\frac{1}{2}\right) = 2 \Rightarrow x = \pm\sqrt{2}$

   $\therefore$ the points of intersection of the circle and parabola are $\left(\sqrt{2}, \frac{1}{2}\right)$ and $\left(-\sqrt{2}, \frac{1}{2}\right)$.



The required area $= 2\times\int_0^{\sqrt{2}}\left[\sqrt{\left(\frac{3}{2}\right)^2 - x^2} - \left(\frac{x^2}{4}\right)\right]dx$

$$= 2\left[\frac{x}{2}\sqrt{\left(\frac{3}{2}\right)^2 - x^2} + \frac{\left(\frac{3}{2}\right)^2}{2}\sin^{-1}\left(\frac{x}{\frac{3}{2}}\right) - \frac{1}{4}\times\frac{x^3}{3}\right]_0^{\sqrt{2}}$$

$$= 2\left[\frac{\sqrt{2}}{2}\sqrt{\left(\frac{3}{2}\right)^2 - \sqrt{2}^2} + \frac{\left(\frac{3}{2}\right)^2}{2}\sin^{-1}\left(\frac{\sqrt{2}}{\frac{3}{2}}\right) - \frac{1}{4}\times\frac{\left(\sqrt{2}\right)^3}{3} - 0\right]$$

$$= 2\left[\frac{\sqrt{2}}{2}\sqrt{\frac{9}{4} - 2} + \frac{9}{8}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) - \frac{1}{4}\times\frac{2\sqrt{2}}{3}\right]$$

$$= 2\left[\frac{\sqrt{2}}{4} - \frac{2\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\right]$$

$$= 2\left[\frac{3\sqrt{2}}{12} - \frac{2\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\right] = \left[\frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\right]$$

$$= 2\times\frac{1}{2}\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\right] = \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) \text{ sq. units.}$$

2. Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$

$(x-1)^2 + y^2 = 1$ ....(1) is a circle passing having centre (1,0) and radius 1 unit and

$x^2 + y^2 = 1$ ....(2) is a circle passing through the origin and having radius 1 unit.

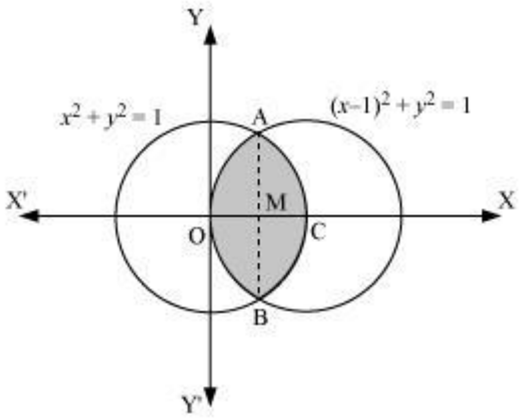
From (2), $y^2 = 1 - x^2$ ....(3)

Sub. in (1), we have, $(x-1)^2+1-x^2=1 \Rightarrow x^2-2x+1+1-x^2=1$

$-2x=0-1 \Rightarrow x=\frac{1}{2}$

When $x=\frac{1}{2}$, in (3), $y^2=1-\left(\frac{1}{2}\right)^2=1-\frac{1}{4}=\frac{3}{4} \Rightarrow y=\pm\frac{\sqrt{3}}{2}$

$\therefore$ the points of intersection of the circles are $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$.



$\therefore$ the required area $=2\times\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}\,dx+2\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}\,dx$

$$=2\times\left[\frac{(x-1)}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+2\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$

$$=2\times\left[\frac{\left(\frac{1}{2}-1\right)}{2}\sqrt{1-\left(\frac{1}{2}-1\right)^2}+\frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right)-\left\{0+\frac{1}{2}\sin^{-1}(0-1)\right\}\right]$$

$$+2\left[0+\frac{1}{2}\sin^{-1}1-\left\{\frac{1}{4}\sqrt{1-\frac{1}{4}}+\frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$$

$$=2\left[\frac{-1}{4}\sqrt{1-\frac{1}{4}}+\frac{1}{2}\sin^{-1}\left(-\frac{1}{2}\right)-\frac{1}{2}\sin^{-1}(-1)\right]$$

$$+2\left[\frac{1}{2}\left(\frac{\pi}{2}\right)-\frac{1}{4}\frac{\sqrt{3}}{2}-\frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$=2\left[\frac{-1}{4}\frac{\sqrt{3}}{2}+\frac{1}{2}\left(-\frac{\pi}{6}\right)-\frac{1}{2}\left(-\frac{\pi}{2}\right)\right]+2\left[\frac{\pi}{4}-\frac{1}{4}\frac{\sqrt{3}}{2}-\frac{\pi}{12}\right]$$

$$= 2\left[\frac{-\sqrt{3}}{8} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\sqrt{3}}{8} - \frac{\pi}{12}\right]$$

$$= 2\left[\frac{\pi}{2} - \frac{\pi}{6} - \frac{2\sqrt{3}}{8}\right] = 2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]$$

$$= \left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right] \text{sq. units.}$$

3. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$

$y = x^2 + 2$ ....(1) is an upward parabola and

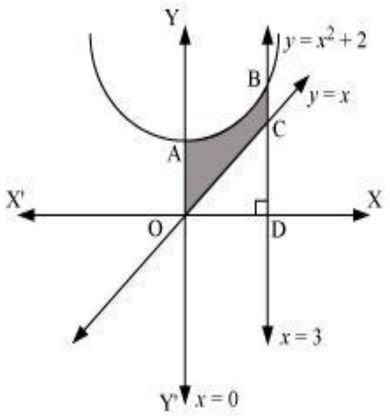
$when\ x = 0,\ y = 0$

$when\ x = 3,\ y = 3^2 + 2 = 11$

And $y = x$ ....(2) is an identity function.

$when\ x = 0,\ y = 0$

$when\ x = 3,\ y = 3$



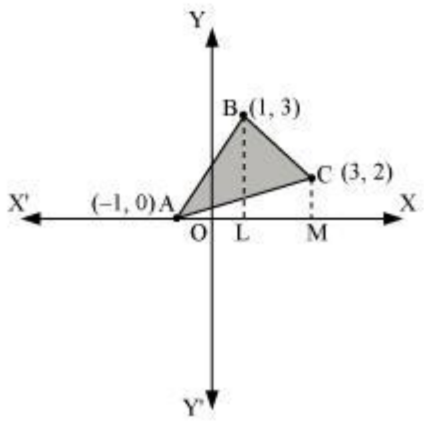
$\therefore$ the required area $= \int_0^3 \left(x^2 + 2 - x\right) dx$

$$= \left[\frac{x^3}{3} + 2x - \frac{x^2}{2}\right]_0^3$$

$$= \left[\frac{3^3}{3} + 2(3) - \frac{(3)^2}{2} - 0\right] = 9 + 6 - \frac{9}{2}$$

$$= 15 - \frac{9}{2} = \frac{30 - 9}{2} = \frac{21}{2} \text{ sq. units}$$

4. Using integration finds the area of the region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).



Equation of line segment AB is $\frac{x+1}{1+1}=\frac{y-0}{3-0}\Rightarrow\frac{x+1}{2}=\frac{y}{3}\Rightarrow y=\frac{3}{2}(x+1)$

Equation of line segment BC is $\frac{x-1}{3-1}=\frac{y-3}{2-3}\Rightarrow\frac{x-1}{2}=\frac{y-3}{-1}\Rightarrow y-3=-\frac{1}{2}(x-1)$

$$\Rightarrow y=-\frac{1}{2}(x-1)+3=-\frac{1}{2}(x-1)+\frac{6}{2}=\frac{1}{2}[-x+1+6]=\frac{1}{2}(7-x)$$

Equation of line segment AC is $\frac{x+1}{3+1}=\frac{y-0}{2-0}\Rightarrow\frac{x+1}{4}=\frac{y}{2}\Rightarrow y=\frac{1}{2}(x+1)$

Area of $\Delta ABC=\int_{-1}^{1}(y\ of\ \ AB)dx+\int_{1}^{3}(y\ of\ \ BC)dx+\int_{-1}^{3}(y\ of\ \ AC)dx$

$$=\int_{-1}^{1}\frac{3}{2}(x+1)dx+\int_{1}^{3}-\frac{1}{2}(7-x)dx-\int_{-1}^{3}\frac{1}{2}(x+1)dx$$

$$=\frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1}+\frac{1}{2}\left[7x-\frac{x^2}{2}\right]_{1}^{3}-\frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$=\frac{3}{2}\left[\frac{1^2}{2}+1-\left\{\frac{(-1)^2}{2}+(-1)\right\}\right]+\frac{1}{2}\left[7(3)-\frac{(3)^2}{2}-\left\{7(1)-\frac{(1)^2}{2}\right\}\right]$$

$$-\frac{1}{2}\left[\frac{3^2}{2}+3-\left\{\frac{(-1)^2}{2}+(-1)\right\}\right]$$

$$=\frac{3}{2}\left[\frac{1}{2}+1-\left\{\frac{1}{2}-1\right\}\right]+\frac{1}{2}\left[21-\frac{9}{2}-\left\{7-\frac{1}{2}\right\}\right]-\frac{1}{2}\left[\frac{9}{2}+3-\left\{\frac{1}{2}-1\right\}\right]$$

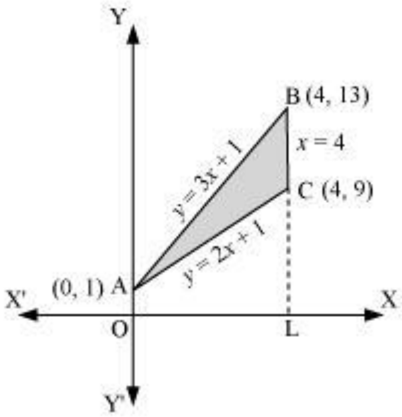
$$=\frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}+1\right]+\frac{1}{2}\left[21-\frac{9}{2}-7+\frac{1}{2}\right]-\frac{1}{2}\left[\frac{9}{2}+3-\frac{1}{2}+1\right]$$

$$=\frac{3}{2}[2]+\frac{1}{2}[14-4]-\frac{1}{2}[4+4]$$

$=3+5-4=4$ sq. units.

5. Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

On solving these equations, we obtain the vertices of triangle as:
A(0, 1), B(4, 13), and C (4, 9).



Area of $\Delta ABC = \int_0^4 (y \text{ of } AB - y \text{ of } AC)\,dx$

$$= \int_0^4 \left[(3x+1)-(2x+1)\right]dx$$

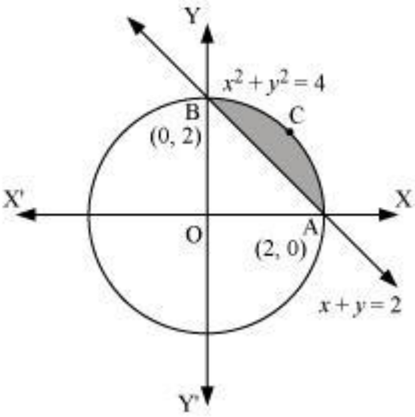
$$= \int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4$$

$$= \frac{16}{2} - 0 = 8 \text{ sq. units}$$

6. Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

A. $2(\pi - 2)$ B. $\pi - 2$ C. $2\pi - 1$ D. $2(\pi + 2)$

The smaller area enclosed by the circle, $x^2 + y^2 = 4$, and the line, $x + y = 2$, is represented by the shaded area ACBA as



The required area $= \int_0^2 \left[\sqrt{2^2 - x^2} - (2 - x)\right]dx$

$$= \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right) - 2x + \frac{x^2}{2}\right]_0^2$$

$$= \left[0 + \frac{4}{2}\sin^{-1}(1) - 2(2) + \frac{(2)^2}{2}\right]$$

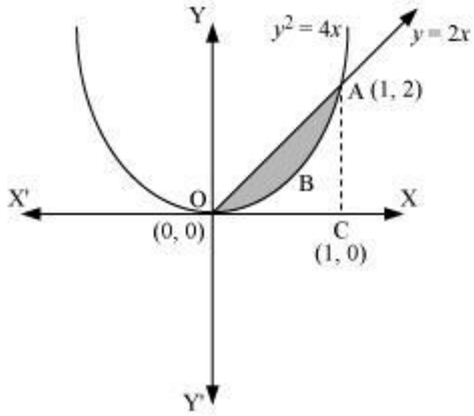
$$= 2 \times \frac{\pi}{2} - 4 + 2$$

$= (\pi - 2)$ sq. units.   Thus, the correct answer is B.

7. Area lying between the curve $y^2 = 4x$ and $y = 2x$ is

A. $\frac{2}{3}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{3}{4}$

The area lying between the curve, $y^2 = 4x$ and $y = 2x$, is represented by the shaded area OBAO as



The points of intersection of these curves are O (0, 0) and A (1, 2).

We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).

$\therefore$ Required area $= \int_0^1 \left(2x - 2\sqrt{x}\right) dx$

$$= \left[2\frac{x^2}{2} - 2\times\frac{2}{3}x^{\frac{3}{2}}\right]_0^1$$

$$= \left[1 - \frac{4}{3}\right] = \left|\frac{3-4}{3}\right| = \left|\frac{-1}{3}\right| = \frac{1}{3}\,sq.units$$

Thus, the correct answer is B.