CHAPTER 10
VECTOR ALGEBRA

SAY 2018

1. Consider the vectors $\vec{a} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - \hat{k}$
   a) Find the projection of $\vec{a}$ on $\vec{b}$. (2)
   b) If $\vec{a}$ is perpendicular to $\vec{c}$, then projection of $\vec{a}$ on $\vec{c} = \ldots\ldots\ldots$ (1)
   c) Write a vector $\vec{d}$ such that the projection of $\vec{a}$ on $\vec{c} = |\vec{a}|$. (1)

2. a) In the figure ABCD is a parallelogram. If $\overrightarrow{AB} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{AD} = \hat{i} + \hat{j} + 2\hat{k}$. Find $\overrightarrow{AC}$ *and* $\overrightarrow{DB}$. (2)
   b) If $\vec{a}$ on $\vec{b}$ are adjacent sides of any parallelogram and $\vec{c}$ and $\vec{d}$ are its diagonals, show that $\left|\vec{c} \times \vec{d}\right| = 2\left|\vec{a} \times \vec{b}\right|$. (2)

MARCH 2018

3. a) Prove that for any vectors, $\vec{a}, \vec{b}, \vec{c}$, $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$. (3)
   b) Show that if $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are coplanar, then $\vec{a}, \vec{b}, \vec{c}$ are also coplanar. (1)

4. a) If $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$
   i) Find $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$. (2)
   ii) Find a unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$. (2)
   b) Consider the points A(1,2,7), B(2,6,3), C(3,10,–1).
   i) Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$. (1)
   ii) Prove that A, B, C are collinear. (1)

SAY 2017

5. a) The value of $|\vec{x}|$. If $\vec{b}$ is a unit vector and $\left(2\vec{x} - 2\vec{b}\right).\left(\vec{x}+\vec{b}\right) = 30$
   a) $\sqrt{6}$ b) 6
   c) 4 d) 12 (1)
   b) If $\vec{a} = \vec{i} + 3\vec{j}$ and $\vec{b} = 3\vec{j} + \vec{k}$, then find a unit vector which is perpendicular to both $\vec{a}$ and $\vec{b}$ (2)

6. a) Cosine of the angle between the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is
   a) 1/3 b) 2/3
   c) 1/2 d) 1 (1)
   b) If $\vec{a}, \vec{b}, \vec{c}$ are three vector such that $|\vec{a}| = 3$, $\left|\vec{b}\right| = 4$, $|\vec{c}| = 4$ and each one of them is perpendicular to the sum of the other two, then find $\left|\vec{a}+\vec{b}+\vec{c}\right|$ (4)

MARCH 2017

7. a) The projection of the vector $2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\hat{i} + \hat{j} + \hat{k}$ is
   a) $\frac{3}{\sqrt{3}}$ b) $\frac{7}{\sqrt{3}}$
   c) $\frac{3}{\sqrt{17}}$ d) $\frac{7}{\sqrt{17}}$ (1)
   b) Find the area of a parallelogram whose adjacent sides are the vector $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j}$. (2)

8. a) The angle between the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is
   a) 60^0 b) 30^0
   c) 45^0 d) 90^0 (1)
   b) If $\vec{a}, \vec{b}, \vec{c}$ are unit vector such that $\vec{a}+\vec{b}+\vec{c} = 0$ find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ (4)

SAY 2016

9. a) The projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$ is

i) 1 ii) 0 iii) 2 iv) -1 (1)

b) Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ (2)

10. a) $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$ is equal to

i) $\vec{0}$ ii) $|\vec{a}|^2-|\vec{b}|^2$

iii) $\vec{a}\times\vec{b}$ iv) $2(\vec{a}\times\vec{b})$ (1)

b) If $\vec{a}$ and $\vec{b}$ are any two vectors, then prove that $(\vec{a}\times\vec{b})^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b}\\ \vec{a}.\vec{b} & \vec{b}.\vec{b}\end{vmatrix}$ (2)

c) Using vectors, show that the points A(1,2,7), B(2,6,3) and C(3,10,–1) are collinear. (2)

MARCH 2016

11. a) The angle between the vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}|=|\vec{b}|=\sqrt{2}$ and $\vec{a}.\vec{b}=1$ is

i) $\frac{\pi}{2}$ ii) $\frac{\pi}{3}$

iii) $\frac{\pi}{4}$ iv) 0 (1)

b) Find the unit vector along $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}+\hat{k}$ (2)

12. a) If the points A and B are (1,2,-1) and (2,1,-1) respectively, then $\overrightarrow{AB}$ is

i) $\hat{i}+\hat{j}$ ii) $\hat{i}-\hat{j}$

iii) $2\hat{i}+\hat{j}-\hat{k}$ iv) $\hat{i}+\hat{j}+\hat{k}$ (1)

b) Find the value of λ for which the vectors $2\hat{i}-4\hat{j}+5\hat{k}, \hat{i}-\lambda\hat{j}+\hat{k}$ and $3\hat{i}+2\hat{j}-5\hat{k}$ are coplanar. (2)

c) Find the angle between the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$. (2)

SAY 2015

13. Consider the vectors $\vec{a}=2\hat{i}+2\hat{j}-5\hat{k}$ and $\vec{b}=-\hat{i}+7\hat{k}$.

a) Find $\vec{a}+\vec{b}$ (1)

b) Find a unit vector in the direction of $\vec{a}+\vec{b}$ (2)

14. Consider the triangle ABC with vertices A(1,2,3), B(–1,0,4) and C(0,1,2)

a) Find $\overrightarrow{AB}, \overrightarrow{AC}$ (1)

b) Find $\angle A$. (2)

c) Find the area of triangle ABC. (2)

MARCH 2015

15. (a) If $\vec{a},\vec{b},\vec{c},\vec{d}$ respectively are the position vectors representing the vertices A,B,C,D of a parallelogram, then write $\vec{d}$ in terms of $\vec{a},\vec{b}$ and $\vec{c}$. (1)

b) Find the projection vector of $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ along the vector $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$. (2)

Also write $\vec{b}$ as the sum of a vector along $\vec{a}$

and a vector perpendicular to $\vec{a}$. (1)

c) Find the area of a parallelogram for which the vectors $\vec{a} = 2\hat{i} + \hat{j}$ and $3\hat{i} + \hat{j} + 4\hat{k}$ are adjacent sides. (2)

OR

a) Write the magnitude of a vector $\vec{a}$ in terms of dot product. (1)

b) If $\vec{a}, \vec{b}$ and $\vec{a} + \vec{b}$ are unit vectors, then prove that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$. (2)

c) If $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $m\hat{i} + 3\hat{j} - \hat{k}$ are perpendicular to each other, then find m. (1)
Also find the area of the rectangle having these two vectors as sides. (2)

SAY 2014

16. Consider the vectors
$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k};\ \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

a) Find $\vec{a}.\vec{b}$ (1)

b) Find the angle between $\vec{a}$ and $\vec{b}$. (2)

c) Find the area of parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$. (2)

MARCH 2014

17. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 6\hat{i} + 2\hat{j} + 3\hat{k}$.

a) Find a unit vector in the direction of $\vec{a} + \vec{b}$. (1)

b) Find the angle between $\vec{a}$ and $\vec{b}$. (2)

18. Consider the triangle ABC with vertices A(1,1,1), B(1,2,3) and C(2,3,1),

a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$ (2)

b) Find $\overrightarrow{AB} \times \overrightarrow{AC}$ (2)

c) Hence find the area of the triangle ABC. (1)

(Score: 2)

SAY 2013

19. a) Find the angle between the vectors
$\vec{a} = 3\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ (3)

b) The adjacent sides of a parallelogram are
$\vec{a} = 3\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - \hat{k}$

i) Find $\vec{a} \times \vec{b}$ (1)

ii) If the area of the parallelogram is $\sqrt{42}$ square units, find the value of λ. (2)

MARCH 2013

20. The position vectors of the vertices of ΔABC are
$3\hat{i} - 4\hat{j} - 4\hat{k},\ 2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} - 3\hat{j} - 5\hat{k}$.

a) Find $\overrightarrow{AB}, \overrightarrow{BC}$ and $\overrightarrow{CA}$. (2)

b) Prove that Δ ABC is a right angled triangle. (1)

SAY 2012

21. a) For any three vector $\vec{a}, \vec{b}, \vec{c}$ show that
$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$ (1)

b) Given A(1,1,1) B(1,2,3) and C(2,3,1) are the vertices if ΔABC. Find the area of ΔABC. (3)

MARCH 2012

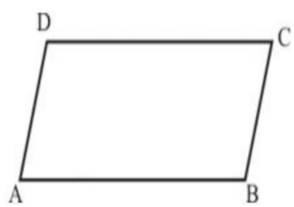
22. Consider the vectors $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 6\hat{i} + 3\hat{j} + 2\hat{k}$

a) Find $\vec{a}.\vec{b}$ and $\vec{a} \times \vec{b}$ (3)

b) Verify that $\left|\bar{a} \times \bar{b}\right|^2 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2 - \left(\bar{a}.\bar{b}\right)^2$ (2)

SAY 2011

23. ABCD is a parallelogram with A as the origin. $\vec{b}$ and $\vec{d}$ are the position vectors of B and D respectively.



a) What is the position vector of C? (1)

b) What is the angle between $\overrightarrow{AB}$ and $\overrightarrow{AD}$? (1)

c) Find $\overrightarrow{AC}$ (1)

d) If $\left|\overrightarrow{AC}\right| = \left|\overrightarrow{BD}\right|$, show that ABCD is a rectangle. (3)

MARCH 2011

24. a) With the help of suitable figure, for any three vectors $\bar{a}, \bar{b}, \bar{c}$. Show that

$$\left(\bar{a} + \bar{b}\right) + \bar{c} = \bar{a} + \left(\bar{b} + \bar{c}\right) \quad (2)$$

b) If $\bar{a} = \hat{i} - \hat{j} + \hat{k}$, $\bar{b} = 2\hat{i} - 2\hat{j} - \hat{k}$, what is the projection of $\bar{a}$ on $\bar{b}$? (2)

25. a) If $\bar{a} = 3\hat{i} - \hat{j} - 5\hat{k}$ $\bar{b} = \hat{i} - 5\hat{j} + 3\hat{k}$ show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular. (2)

b) Given the position vector of three points $A\left(\hat{i} - \hat{j} + 2\hat{k}\right)$, $B\left(4\hat{i} + 5\hat{j} + 8\hat{k}\right)$, $C\left(3\hat{i} + 3\hat{j} + 6\hat{k}\right)$

i) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$ (1)

ii) Prove that A,B,C are collinear points. (1)

SAY 2010

26. Let A(2,3,4), B(4,3,2) and C(5,2,–1) be three points.

a) Find $\overrightarrow{AB}$, $\overrightarrow{BC}$. (2)

b) Find the projection of $\overrightarrow{BC}$ on $\overrightarrow{AB}$. (2)

c) Find the area of the triangle ABC. (2)

MARCH 2010

27. a) Choose the correct answer from the bracket. Let ABCD be a parallelogram whose sides AB and AD are represented by the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ respectively. If $\vec{a}$ is a unit vector parallel to $\overrightarrow{AC}$, then $\vec{a}$ equals ………….. (1)

$$\left[\frac{1}{7}(3i - 6j - 3k), \frac{1}{7}(3i + 6j - 2k), \frac{1}{7}(3i + 5j - 3k), \frac{1}{3}(3i + 6j + 2k)\right]$$

b) Find a vector coplanar with the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ (1)

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c) If G is the centroid of a triangle ABC, then $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}$ =……….. (1)

JUNE 2009

28. a) If A(1,2,4) and B (2,-1,3) are two points.

i) Find $\overrightarrow{AB}$ (1)

ii) Find unit vector along $\overrightarrow{AB}$ (1)

b) Show that the points with position vectors $2\hat{i}+6\hat{j}+3\hat{k};\hat{i}+2\hat{j}+7\hat{k};3\hat{i}+10\hat{j}-\hat{k}$ are collinear. (2)

29. a) i) Find λ for which $\vec{a}=\lambda\hat{i}-\hat{j}+5\hat{k}$ and $\vec{b}=3\hat{i}+4\hat{j}-\hat{k}$ are orthogonal.

ii) *not in the present syllabus*.

b) P,Q, R. S are points with position vectors $2\hat{i}+4\hat{j}+6\hat{k}$, $3\hat{i}+5\hat{j}+4\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$ and $5\hat{i}+8\hat{j}+5\hat{k}$ respectively.

i) Find the vectors $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{PS}$. (1)

ii) Show that P,Q,R and S are coplanar. (2)

c) *not in the present syllabus*.

OR

a) i) Find the projection of $\hat{i}+\hat{j}+\hat{k}$ in the direction of $\hat{i}+\hat{j}$. (1)

ii) Not in the present syllabus.

b) Not in the present syllabus.

c) Find the area of the parallelogram determined by the vectors $\vec{a}=\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=3\hat{i}-4\hat{j}+5\hat{k}$. (2)

MARCH 2009

30. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}$, $\vec{c}=3\hat{i}+5\hat{j}-2\hat{k}$, $\vec{d}=-\hat{j}+\hat{k}$

a) Find $\vec{b}-\vec{a}$

b) Find the unit vector along $\vec{b}-\vec{a}$ (1)

c) Prove that $\vec{b}-\vec{a}$ and $\vec{d}-\vec{c}$ are parallel vectors. (2)

MARCH 2008

31. a) D,E,F are the mid-points of sides of ΔABC . Show that for any point O, $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}$ (2)

b) Prove that the points whose position vectors are given by $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ form a right-angled triangle. (3)

32. Consider the points A(0,-2,1), B(1,-1,-2) and C(-1,1,0) lying in a plane.

i) Compute $\overrightarrow{AB}$ and $\overrightarrow{AC}$ (2)

ii) Find $\overrightarrow{AB}\times\overrightarrow{AC}$ (1)

iii) Find a unit vector perpendicular to the plane.

SAY 2008

33. Consider the vectors $\vec{a}=2\hat{i}+2\hat{j}-5\hat{k}$ and $\vec{b}=-\hat{i}+7\hat{k}$

a) Find $\vec{a}+\vec{b}$ (1)

b) Find a unit vector in the direction of $\vec{a}+\vec{b}$ (2)

34. Consider the triangle ABC with vertices A(1,2,3),B(–1,0,4) and (0,1,2)

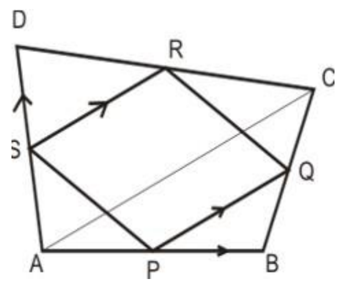
a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$ (1)

b) Find $\angle A$. (2)

c) Find the area of triangle ABC. (2)

MARCH 2007

35. Consider the following quadrilateral ABCD in which P,Q,R,S are the mid points of the sides.



a) Find $\overrightarrow{PQ}$ *and* $\overrightarrow{SR}$ (2)

b) Show that PQRS is a parallelogram. (1)

c) If $\vec{a}$ is any vector, prove that

$\vec{a} = (\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k}$ (2)

36. Consider the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} + 7\hat{j} + 3\hat{k}$.

a) Find $\vec{a}.\vec{b}$ (1)

b) Find the angle between $\vec{a}$ and $\vec{b}$ (1)

c) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar vectors, find λ. (1)

d) Using the values of λ, evaluate $\vec{a} \times (\vec{b} \times \vec{c})$ (2)

EXTRA QUESTIONS

1. Find λ if the vectors $2\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} - 4\hat{j} + \lambda\hat{k}$ are orthogonal.

2. Find the value of x if the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = x\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ are coplanar.

3. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} - 3\hat{k}$ are the position vectors of three points A, B and C. Prove that A,B and C are collinear.

4. Prove that $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

5. If $|\vec{a}| = 2, |\vec{b}| = 5, |\vec{a}\times\vec{b}| = 8$, then find $\vec{a}.\vec{b}$

6. Find the area of the parallelogram whose diagonals are represented by the vectors $-2\hat{i} + \hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - \hat{k}$

7. Find a unit vector perpendicular to $3\hat{i} - 2j + 6\hat{k}$

8. Find the unit vector in the direction of sum of the vectors $\vec{a}+\vec{b}$, where $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

9. Find the unit vector perpendicular to both the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} - \hat{k}$.

10. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, show that $(\vec{a}+\vec{b})^2 = (\vec{a}-\vec{b})^2$

11. If $|\vec{a}| = 10, |\vec{b}| = 2, \vec{a}.\vec{b} = 12,$ then find $|\vec{a}\times\vec{b}|$

12. Show that $\left[\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\ \ \vec{a}\times\vec{b}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

13. Find the angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$ *and* $\vec{b} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

14. Prove that the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$

15. If $\vec{a}$ *and* $\vec{b}$ are any two vectors, prove that $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2$

16. With the help of vectors, the area of triangle PQR whose vertices are P(1,3,2), Q(2,–1,1) and R(–1,2,3)

17. Find the sine of the angle between the vectors $2\hat{i} - \hat{j} - \hat{k}$ and $4\hat{i} + 7\hat{j} + 3\hat{k}$

18. Find the projection of a vector $2\hat{i}-3\hat{j}+2\hat{k}$ on the vector $\hat{i}+2\hat{j}+3\hat{k}$
19. Find the volume of the parallelepiped whose coterminous edges are given by the vectors $\hat{i}+2\hat{j}+\hat{k}$, $2\hat{i}-\hat{j}-2\hat{k}$ and $\hat{i}+2\hat{j}-3\hat{k}$.
20. Find the volume of the tetrahedron whose coterminous edges are given by the vectors $2\hat{i}-3\hat{j}+\hat{k}$, $\hat{i}-\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$.

    [*Hint: Volume of a tetrahedron* $=\frac{1}{6}\times$ *Volume of a parallelepiped.*]
21. Show that the vectors $2\hat{i}-4\hat{j}+2\hat{k}$ and $3\hat{i}-6\hat{j}+3\hat{k}$ are collinear.
22. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}, \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c}=18$.
23. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\left|\vec{a}\right|=5, \left|\vec{b}\right|=12 \, and \left|\vec{c}\right|=13$, and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.
24. Find λ when the projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ *on* $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units.
25. Prove that $\vec{a}.\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+\vec{b}+\vec{c}\right)=0$
26. Show that $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are coplanar.
27. Find the value of $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\left(\hat{i}\times\hat{k}\right).\hat{j}$.
28. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, show that $\sin\left(\frac{\theta}{2}\right)=\frac{1}{2}\left|\vec{a}-\vec{b}\right|$.
29. Prove that $\vec{a}.\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+2\vec{b}+3\vec{c}\right)=\left[\vec{a}\ \vec{b}\ \vec{c}\ \right]$