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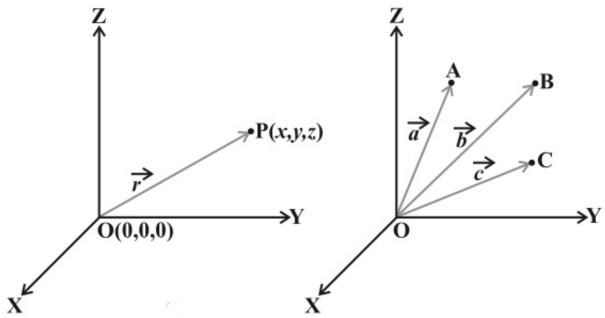
Chapter 11

Three Dimensional Geometry

We can identify each and every point of space with the help of three mutually perpendicular coordinate axes OX, OY and OZ.

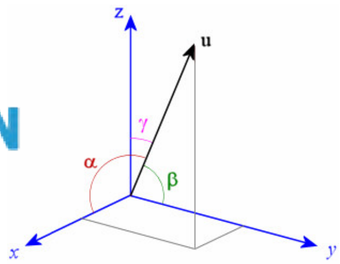


In this chapter we shall study the following concepts.

1. Direction cosines and ratios
2. Equation of line: given point and parallel vector
3. Equation of line: given 2 points
4. Angle between two lines: Vector Form
5. Angle between two lines: Cartesian Form
6. Angle between two lines: Direction ratios or cosines
7. Shortest distance between two skew lines
8. Shortest distance between two parallel lines
9. Equation of plane: In Normal Form
10. Equation of plane: Prependicular to Vector and Passing Through a Point
11. Equation of plane: Passing Through 3 Non Collinear Points
12. Equation of plane: Intercept Form
13. Equation of plane: Passing Through Intersection Of Planes

14. Coplanarity of 2 lines
15. Angle between two planes
16. Distance of point from a plane
17. Angle between a Line and a Plane

11.1 Direction cosines and ratios



- If a directed line L passing through the origin makes angles α, β and γ with x, y and z-axes, respectively, called **direction angles**.
- The cosine of these angles, namely, $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called **direction cosines** of the directed line L. The direction cosines are usually denoted by l, m, and n. i.e.,
$$l = \cos\alpha, m = \cos\beta, n = \cos\gamma$$
- If l, m, n are the direction cosines of a line L, then $l^2 + m^2 + n^2 = 1$
- Let a, b, c are proportional to the direction cosines l, m, n, then a, b, c are called the **direction ratios**.
- If a, b, c are direction ratios of a line L, then $a\hat{i} + b\hat{j} + c\hat{k}$ is a **vector parallel to the line** L.
- If l, m, n are direction cosines of a line L, then $l\hat{i}+m\hat{j}+n\hat{k}$ is a **unit vector parallel** to the line L.
- If l, m, n be the direction cosines and a, b, c ne the dirction ratios of a line, then
$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}}, n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$
- If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points on a space, then the direction ratios of the line segement joining the points P and Q are:
$$a = x_2 - x_1, b = y_2 - y_1, c = z_2 - z_1$$
The direction cosines of the line segement joining the points P and Q are:
$$l = \frac{x_2 - x_1}{|PQ|}, m = \frac{y_2 - y_1}{|PQ|}, n = \frac{z_2 - z_1}{|PQ|}$$
where $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Problems

1. If a line makes angle $90°, 60°$ and $30°$ with the positive direction of x, y and z-axis respectively, find its direction cosines.

2. If a line makes angle $90°, 135°$ and $45°$ with the positive direction of x, y and z-axis respectively, find its direction cosines.

3. Find the direction cosines of x, y and z-axis.

4. Find the direction cosines of a line which makes equal angles with the coordinate axes.

5. If a line has direction ratios $2, -1, -2$, determine its direction cosines.

6. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines ?

7. Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.

8. Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, -4)$, $(-1, 1, 2)$ and $(-5, -5, -2)$.

9. Show that the points $A(2, 3, -4), B(1, -2, 3)$ and $C(3, 8, -11)$ are collinear.

10. Show that the points $(2, 3, 4), (-1, -2, 1), (5, 8, 7)$ are collinear.

11.2 Equation of line: A given point and a parallel vector

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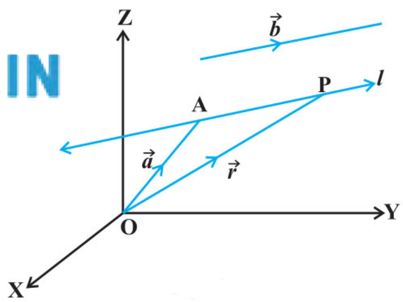


- **Vector Form:** The vector equation of a straight line passing through a fixed point A with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

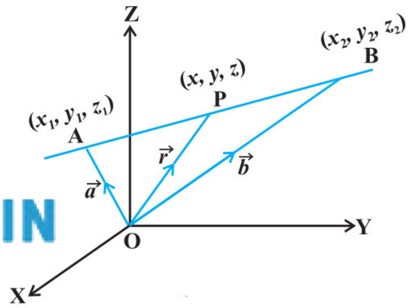
- **Cartesian Form:** The cartesian equation of a straight line passing through a fixed point $A(x_1, y_1, z_1)$ and parallel to a given vector $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Problems

1. Find the Vector and the Cartesian equations of the line through the point $(5, 2, 4)$ and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.
2. Find the equation of the line which passes through the point $(1, 2, 3)$ and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$
3. The Cartesian equation of a line is $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$. Find the vector equation for the line.
4. The Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Find the vector equation for the line.
5. Find the equation of a line parallel to x-axis and passing through the origin.
6. Find the equation of the line in vector and in cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$.
7. Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.
8. Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

11.3 Equation of line: Given 2 points



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- **Vector Form:** The vector equation of a straight line passing through two fixed points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively is

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

- **Cartesian Form:** The cartesian equation of a straight line passing through two fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

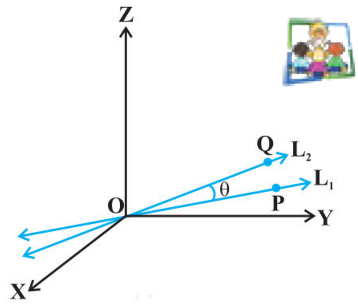
Problems

1. Find the vector equation for the line passing through the points $(-1, 0, 2)$ and $(3, 4, 6)$.

2. Find the vector and the cartesian equations of the lines that passes through the origin and $(5, -2, 3)$.

3. Find the vector and the cartesian equations of the line that passes through the points $(3, -2, -5), (3, -2, 6)$.

11.4 Angle between two lines: Vector Form



- Let $L_1 : \vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $L_2 : \vec{r} = \vec{a_2} + \mu\vec{b_2}$ are two straight lines in space. Then the acute angle(θ) between the lines L_1 and L_2 is given by

$$\cos\theta = \left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|$$

Problems

1. Find the angle between the pair of lines given by $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

2. Find the angle between the pair of lines given by $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

3. Find the angle between the pair of lines given by $\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda(\hat{i} + 2\hat{j} - \hat{j} - 2\hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} + 56\hat{k} + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$

11.5 Angle between two lines: Cartesian Form

- Let $L_1 : \dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}$ and $L_2 : \dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}$ are two straight lines in space. Then the acute angle(θ) between the lines L_1 and L_2 is given by

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

- If lines L_1 and L_2 are parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- If the lines L_1 and L_2 are perpendicular, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Problems

1. Find the angle between the pair of lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.
2. Find the angle between the following pair of lines $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

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3. Find the angle between the following pair of lines $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$ and $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$
4. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.
5. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.
6. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular. Find the value of k.

11.6 Angle between two lines: Direction ratios or cosines

- The acute angle(θ) between the lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is given by
$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$
- The acute angle(θ) between the lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 is given by
$$\cos\theta = \left|\frac{l_1l_2 + m_1m_2 + n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\sqrt{l_2^2+m_2^2+n_2^2}}\right|$$
- If lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- If lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

- If lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 are parallel, then $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$
- If lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 are perpendicular, then $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

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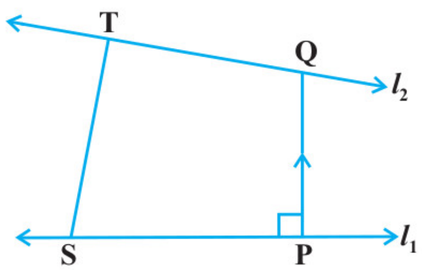
Problems

1. Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.
2. Show that the line through the points $(1, -1, 2), (3, 4, -2)$ is perpendicular to the line through the points $(0, 3, 2)$ and $(3, 5, 6)$.
3. Show that the line joining the origin to the point $(2, 1, 1)$ is perpendicular to the line determined by the points $(3, 5, -1), (4, 3, -1)$.
4. Show that the line through the points $(4, 7, 8), (2, 3, 4)$ is parallel to the line through the points $(-1, -2, 1), (1, 2, 5)$.
5. Find the angle between the lines whose direction ratios are a, b, c and $b-c, c-a, a-b$.
6. If the coordinates of the points A, B, C, D be $(1, 2, 3), (4, 5, 7), (-4, 3, -6)$ and $(2, 9, 2)$ respectively, then find the angle between the lines AB and CD.
7. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1 n_2 - m_2 n_1,\ n_1 l_2 - n_2 l,\ m_1 n_2 - m_2 n_1\ ,\ l_1 m_2 - l_2 m_1$

11.7 Shortest distance between two skew lines

In a space, there are lines which are neither intersecting nor parallel. Such pair of lines are called **Skew lines**.



- **Vector Form:** Let $l_1 : \vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $l_2 : \vec{r} = \vec{a_2} + \mu \vec{b_2}$ are two skew lines in space. Then the shortest distance between the lines is given by

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

- **Cartesian Form:** Let $l_1 : \dfrac{x-x_1}{a_1} = \dfrac{y-y_1}{b_1} = \dfrac{z-z_1}{c_1}$ and $l_2 : \dfrac{x-x_2}{a_2} = \dfrac{y-y_2}{b_2} = \dfrac{z-z_2}{c_2}$ are two skew lines in space. Then the shortest distance between the lines is given by

$$d = \left| \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & b_2 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}} \right|$$

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Problems

1. Find the shortest distance between the lines $\dfrac{x+1}{7} = \dfrac{y+1}{-6} = \dfrac{z+1}{1}$ and $\dfrac{x-3}{1} = \dfrac{y-5}{-2} = \dfrac{z-7}{1}$.
2. Find the shortest distance between the lines $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$
3. Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k})$
4. Find the shortest distance between the lines $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$
5. Find the shortest distance between the lines $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

11.8 Shortest distance between two parallel lines

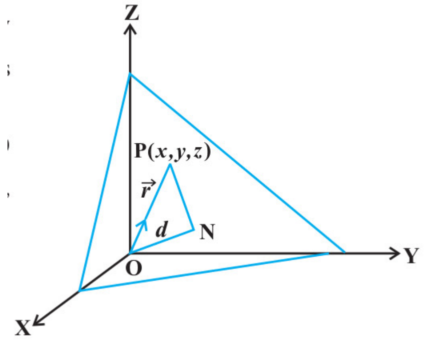
Let $l_1 : \vec{r} = \vec{a_1} + \lambda\vec{b}$ and $l_2 : \vec{r} = \vec{a_2} + \mu\vec{b}$ are two parallel lines in space. Then the shortest distance between the lines is given by

$$d = \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$$

Problems

1. Find the distance between the lines $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

11.9 Equation of plane: In Normal Form



- **Vector Form:** Vector equation of a plane at a distance d from the origin and $\hat{n}$ is the unit vector normal to the plane through the origin is

$$\vec{r} \cdot \hat{n} = d$$

- **Cartesian Form:** Equation of a plane which is at a distance of d from the origin and the direction cosines of the normal to the plane as l, m, n is

$$lx + my + nz = d$$

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- For a plane $ax + by + cz = d$
  - Direction ratios of normal$= a, b, c$
  - Direction cosines of normal: $l = \dfrac{a}{\sqrt{a^2+b^2+c^2}}, m = \dfrac{b}{\sqrt{a^2+b^2+c^2}}, n = \dfrac{c}{\sqrt{a^2+b^2+c^2}}$
  - Distance from origin $= \dfrac{d}{\sqrt{a^2+b^2+c^2}}$

Problems

1. Find the vector equation of the plane which is at a distance of $\dfrac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$. Also find its cartesian form.
2. Find the vector equation of the plane which is at a distance of 7 from the origin and its normal vector from the origin is $3\hat{i} + 5\hat{j} - 6\hat{k}$.
3. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.
   (a) $z = 2$
   (b) $x + y + z = 1$
   (c) $2x + 3y - z = 5$
   (d) $5y + 8 = 0$

4. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin.

5. Find the Cartesian equation of the following planes:

   (a) $\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 2$

   (b) $\vec{r}.(2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$

   (c) $\vec{r}.((s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k}) = 2$

6. Find the direction cosines of the unit vector perpendicular to the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) + 1 = 0$

7. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$.

8. In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

   (a) $2x + 3y + 4z - 12 = 0$

   (b) $3y + 4z - 6 = 0$

   (c) $x + y + z = 1$

   (d) $5y + 8 = 0$

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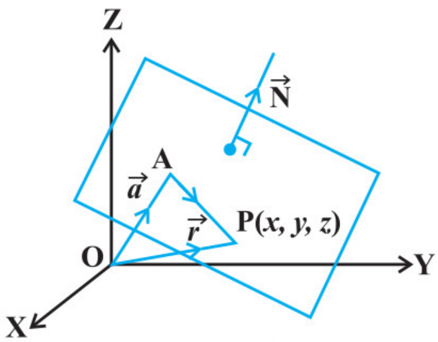
11.10 Equation of plane: Prependicular to Vector and Passing Through a Point



- **Vector Form:** Vector equation of a plane passing through the point A whose position vector is $\vec{a}$ and perpendicular to the vector $\vec{N}$ is

$$(\vec{r} - \vec{a}).\vec{N} = 0$$

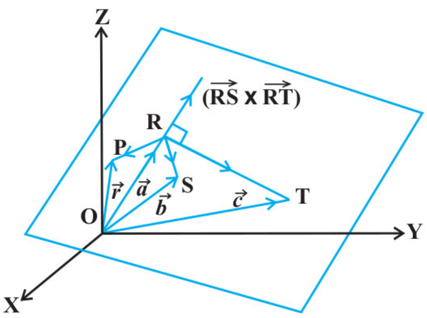
- **Cartesian Form:** The Cartesian Equation of a plane passing through $A(x_1, y_1, z_1)$ and perpendiculat to a line with direction ratios a, b, c is

$$a(x_1 - x_2) + b(y_2 - y_1) + c(z_2 - z_1)$$

Problems

1. Find the vector and cartesian equations of the plane which passes through the point $(5, 2, -4)$ and perpendicular to the line with direction ratios $2, 3, -1$.
2. Find the vector and cartesian equations of the planes
   (a) that passes through the point $(1, 0, -2)$ and the normal to the plane is $\hat{i} + \hat{j} - \hat{k}$
   (b) that passes through the point $(1, 4, 6)$ and the normal to the plane is $\hat{i} - 2\hat{j} + \hat{k}$
3. Find the vector equation of the line passing through $(1, 2, 3)$ and perpendicular to the plane $\vec{r}.(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.
4. Find the equation of the plane passing through (a, b, c) and parallel to the plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 2$
5. If O be the origin and the coordinates of P be $(1, 2, -3)$, then find the equation of the plane passing through P and perpendicular to OP.

11.11 Equation of plane: Passing Through 3 Non Collinear Points



- **Vector Form:** Vector equation of a plane passing through three points R, S, T whose position vectors are $\vec{a}, \vec{b}, \vec{c}$ is

$$(\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$$

- **Cartesian Form:** The Cartesian Equation of a plane passing through $R(x_1, y_1, z_1)$, $R(x_2, y_2, z_2)$,$R(x_3, y_3, z_3)$ is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Problems

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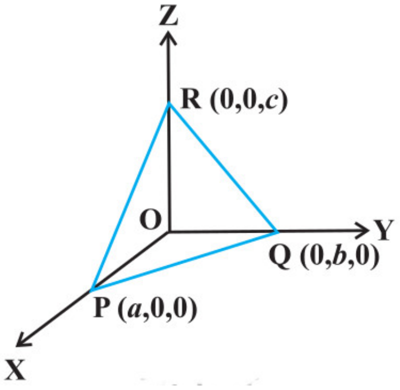
1. Find the vector equations of the plane passing through the points $R(2, 5, -3)$, $S(-2, -3, 5)$ and $T(5, 3, -3)$
2. Find the equations of the planes that passes through three points.
   (a) $(1, 1, -1), (6, 4, -5), (-4, -2, 3)$
   (b) $(1, 1, 0), (1, 2, 1), (2, 2, -1)$

11.12 Equation of plane: Intercept Form



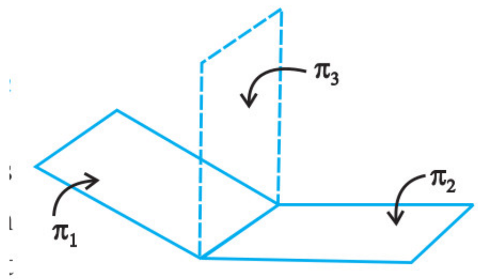
- Equation of a plane with intercepts a, b, c on x, y and $z-$ axis respectively is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Problems

1. Find the equation of the plane with intercepts $2, 3$ and 4 on the x, y and z-axis respectively.
2. Find the intercepts cut off by the plane $2x + y - z = 5$
3. Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

11.13 Equation of plane: Passing Through Intersection Of Planes



- **Vector Form:** Vector equation of a plane passing through the intersection of planes $\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ and also through the point (x_1, y_1, z_1) is

$$\vec{r}.(\vec{n_1} + \lambda\vec{n_2}) = d_1 + \lambda d_2$$

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- **Cartesian Form:** The Cartesian Equation of a plane passing through the intersection of planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ and also through the point (x_1, y_1, z_1) is

$$(a_1x + b_1y + c_1z - d_1) + \lambda(a_2x + b_2y + c_2z - d_2) = 0$$

Problems

1. Find the equation of the plane through the intersection of the planes $3x-y+2z-4=0$ and $x+y+z-2=0$ and the point $(2,2,1)$.
2. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ and through the point $(2,1,3)$.
3. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=6$, $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k})=-5$ and through the point $(1,1,1)$.
4. Find the equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$, $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to $x-$ axis.
5. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$.

11.14 Coplanarity of 2 lines

- **Vector Form:** Two lines $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ are coplanar if
$$(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})=0$$
- **Cartesian Form:** Two lines $\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$ and $\dfrac{x-x_2}{a_2}=\dfrac{y-y_2}{b_2}=\dfrac{z-z_2}{c_2}$ are coplanar if
$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

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Problems

1. Show that the lines $\dfrac{x+3}{-3}=\dfrac{y-1}{1}=\dfrac{z-5}{5}$ and $\dfrac{x+1}{-1}=\dfrac{y-2}{2}=\dfrac{z-5}{5}$ are coplanar.

11.15 Angle between two planes

- **Vector Form:** The Angle(θ) between two planes $\vec{r}\cdot\vec{n_1}=d_1$ and $\vec{r}\cdot\vec{n_2}=d_2$ is given by
$$\cos\theta=\left|\frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|$$
- **Cartesian Form:** The Angle(θ) between two planes $a_1x+b_1y+c_1z=d_1$ and $a_2x+b_2y+c_2z=d_2$ is given by
$$\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$
- If Planes are parallel, then $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$
- If the Planes are perpendicular, then $a_1a_2+b_1b_2+c_1c_2=0$

Problems

1. Find the angle between the two planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$ using vector method.

2. Find the angle between the planes whose vector equations are $\vec{r}.(2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$, $\vec{r}.(3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$

3. Find the angle between the two planes $3x - 6y + 2z = 7$ and $2x + 2y - 2z = 5$.

4. In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

   (a) $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$

   (b) $2x + y + 3z - 2 = 0$ and $x - 2y + 5 = 0$

   (c) $2x - 2y + 4z + 5 = 0$ and $3x - 3y + 6z - 1 = 0$

   (d) $2x - y + 3z - 1 = 0$ and $2x - y + 3z + 3 = 0$

   (e) $4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$

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11.16 Distance of point from a plane

- **Vector Form:** The distance of a point with position vector $\vec{a}$ from the plane $\vec{r}\cdot\vec{n} = d$ is given by

$$Distance = \left|\frac{\vec{a}\cdot\vec{n} - d}{|\vec{n}|}\right|$$

- **Cartesian Form:** The distance of the point (x_1, y_1, z_1) from the plane $ax+by+cz = d$ is given by

$$Distance = \left|\frac{ax_1 + by_1 + cz_1 - d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

Problems

1. Find the distance of a point $(2, 5, -3)$ from the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$

2. In the following cases, find the distance of each of the given points from the corresponding given plane.

   (a) $(0, 0, 0)$; $3x - 4y + 12z = 3$

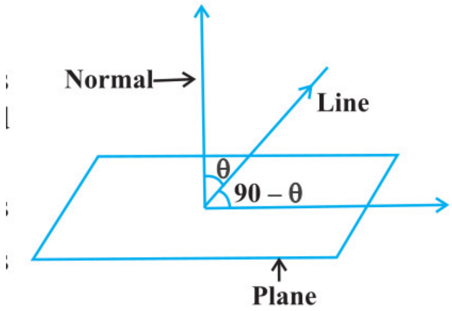
   (b) $(3, -2, 1)$; $2x - y + 2z + 3 = 0$

   (c) $(2, 3, -5)$; $x + 2y - 2z = 9$

   (d) $(-6, 0, 0)$; $2x - 3y + 6z - 2 = 0$

3. Find the distance between the point $P(6, 5, 9)$ and the plane determined by the points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$.

11.17 Angle between a Line and a Plane



- The angle(Φ) between a line $\vec{r} = \vec{a} + \lambda\vec{b}$ and the plane $\vec{r} \cdot \vec{n} = d$ is given by

$$\sin\Phi = \left|\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}\right|$$

Problems

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1. Find the angle between the line $\dfrac{x+1}{2} = \dfrac{y}{3} = \dfrac{z-3}{6}$ and the plane $10x+2y-11z = 3$.