CHAPTER 11

THREE DIMENSIONAL GEOMETRY

SAY 2018

1. Consider the plane $2x-3y+z=5$.
   a) Find the equation of the plane passing through the point (1,1,3) and parallel to the above plane. (2)
   b) Find the distance between the above parallel planes. (2)

2. a) Show that the lines
$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-3}{1}$ and
$\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-4}{2}$ are coplanar. (2)
   b) Find the equation of the plane that contain above lines. (2)
   c) Show that the above line intersect at the Point (3,1,4) (2)

MARCH 2018

3. a) Find the equation of a plane which makes x, y, z intercepts respectively as 1,2,3. (2)
   b) Find the equation of a plane passing through the point (1,2,3) which is parallel to above plane. (2)

4. a) Find the angle between the lines:
$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ and
$\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$ (2)
   b) Find the shortest distance between the pair of lines:
$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$
$\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$ (2)

SAY 2017

4. a) Which of the following is a plane perpendicular to $x+3y+4z=7$?
   a) $4x+3y+z=7$ b) $4x-z=7$
   c) $3x+4y+z=0$ d) $x+y+z=0$ (1)
   b) Find the shortest distance between the lines
$\vec{r}=\hat{i}-2\hat{j}+3\hat{k}+t\left(-\hat{i}-2\hat{j}-2\hat{k}\right)$ and
$\vec{r}=\hat{i}-\hat{j}-\hat{k}+s\left(\hat{i}-\hat{j}-\hat{k}\right)$ (3)

5. a) Distance of the point (1,0,0) from the plane $x+2y+2z=0$
   a) $\frac{2}{3}$ b) $\frac{1}{3}$
   c) $\frac{1}{2}$ d) 1 (1)
   b) Find the Cartesian equation of a line passing through (1,2,-4) and perpendicular to the lines
$\frac{x-2}{2}=\frac{y-1}{-1}=\frac{z-1}{1}$ and $\frac{x-5}{1}=\frac{y}{1}=\frac{z-2}{1}$ (3)

MARCH 2017

6. a) Th
e line $x-1=y=z$ is perpendicular to the lines
$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$
$\vec{r}=\hat{i}+\hat{j}+\hat{k}+\mu\left(\hat{i}+\hat{j}+\hat{k}\right)$ (3)
   b) Distance of the point (0,0,1) from the plane

$x+y+z=3$

a) $\frac{1}{\sqrt{3}}$ b) $\frac{2}{\sqrt{3}}$

c) $\sqrt{3}$ d) $\frac{\sqrt{3}}{2}$ (1)

b) Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to $x-y+z=0$ (3)

SAY 2016

7. a) The equation of the line which passes through the point (1,2,3) and parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$ is

i) $3\hat{i}+2\hat{j}-2\hat{k}+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

ii) $2\hat{i}-5\hat{k}+\lambda\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$

iii) $\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(-2\hat{i}+4\hat{j}-2\hat{k}\right)$

iv) $\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$ (1)

b) Find the angle between the pair of lines $\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $\vec{r}=7\hat{i}-6\hat{k}+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$ (3)

8. a) The distance of the plane $x+y+z+1=0$ from the point $(1,1,1)$ is

i) 4 units ii) $\frac{1}{\sqrt{3}}$ units

iii) $\frac{4}{\sqrt{3}}$ units iv) $\frac{1}{4\sqrt{3}}$ units (1)

b) Find the equation of the plane passing through (1,0,-2) and perpendicular to each of the planes $2x+y-z=2$ and $x-y-z=3$. (3)

MARCH 2016

9. Find the shortest distance between the lines $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu\left(3\hat{i}-5\hat{j}+2\hat{k}\right)$ and $\vec{r}=\hat{i}+\hat{j}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$. (4)

a) Equation of the plane with intercepts 2,3,4 on the x,y and z axis respectively is (1)

i) $2x+3y+4z=1$ ii) $2x+3y+4z=12$

iii) $6x+4y+3z=1$ iv) $6x+4y+3z=12$

b) Find the Cartesian equation of the plane passing through the points $A(2,5,-3)$ $B(-2,-3,5)$ and $C(5,3,-3)$. (3)

SAY 2015

10. a) Find the value of 'p' if the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{p}=\frac{z}{3}$ are perpendicular. (1)

b) Find the shortest distance between the lines: $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and $\vec{r}=\left(4\hat{i}+5\hat{j}+6\hat{k}\right)+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$ (3)

11. Consider a plane whih passes through the point $(5,2,-4)$ and perpendicular to the line $\vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(2\hat{i}+3\hat{j}-\hat{k}\right)$

a) Write the equation in Cartesian form. (1)

b) Find its distance from the point $(1,2,-1)$ (1)

c) Find the angle made by it with the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ (2)

MARCH 2015

12. a) Write the Cartesian equation of the straight line through the point (1,2,3) and along the vector $3\hat{i}+\hat{j}+2\hat{k}$. (1)

b) Write a general point on this straight line. (1)

c) Find the point of intersection of this straight

line with the plane $2x+3y-z+2=0$ (2)

d) Find the distance from (1,2,3) to the plane $2x+3y-z+2=0$. (1)

SAY2014

13. a) If a_1,b_1,c_1 and a_2,b_2,c_2 are the direction ratios of two lines, then write the condition of its perpendicularity. (1)

b) Find the angle between the lines:
$\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$. (3)

14. Consider the planes $2x+y-2z=5$ and $3x-6y-2z=7$

a) Find their normal vector. (2)

b) Find the angle between these two planes. (2)

MARCH 2014

15. The foot of the perpendicular drawn from origin to a plane is(4,-2,5).

a) How far is the plane from the origin? (1)

b) Find a unit vector perpendicular to that plane. (1)

c) Obtain the equation of the plane in general form. (1)

16. Given straight line
$\vec{r}=\left(3\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+2\hat{k}\right)$ and
$\vec{r}=\left(5\hat{j}-2\hat{k}\right)+\mu\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$

a) Find the angle between the lines (2)

b) Obtain a unit vector perpendicular to both the lines. (2)

c) Form the equation of the line perpendicular to the given lines and passing through the point(1,1,1). (1)

SAY 2013

17. Fill in the blanks: (1 x 3=3)

a) If l,m,n are the direction cosines of a line then $l^2+m^2+n^2=$..... (1)

b) The distance from the origin to the plane $2x-3y+4z-6=0$ is ……

c) If θ is the angle between the planes $2x+y-2z=5$ and $3x-6y-2z=7$ then $\cos\theta=$..... ..

18. Consider the vector equation of two planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=3$ *and* $\vec{r}.\left(\hat{i}-\hat{j}-\hat{k}\right)=4$
Find the vector equation of the plane through the intersection of the above two planes and the point(1,2,-1). (3)

MARCH 2013

19. Consider the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$.

a) Express the equations of the lines into vector form. (1)

b) Find the shortest distance between the lines. (3)

20. a) Find the equation of the plane through the points (3, –1,2), (5,2,4) and (–1,–1,6) (2)

b) Find the perpendicular distance from the point (6,5,9) to this plane. (2)

SAY 2012

21. a) Find the angle between the lines having direction ratios 1,1,2 and $\sqrt{3}-1,-\sqrt{3}-1,4$ (2)

b) If the lines $\frac{x-1}{3}=\frac{y-1}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular, find the value of λ. (2)

22. a) The foot of the perpendicular from the origin to a planes is P(4,–2,5). Write $\overrightarrow{OP}$ (1)

b) Find the equation of the above plane in vector form and Cartesian form. (3)

2012 MARCH

23. Consider the points A (3,–4,–5) and B (2,–3,1)

a) Find the vector and Cartesian equations of the line passing through the points A and B. (2)

b) Find the point where the line crosses the XY plane (2)

24. a) Find the Cartesian equation of the plane passing through the point (1,2,–3) and perpendicular to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. (2)

b) Find the angle between the above plane and the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z}{6}$ (2)

SAY 2011

25. a) Consider the lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$.

a) Write the direction ratios of this line (1)

b) Find the angle between these two lines. (3)

b) Find the vector and Cartesian equations of the plane that asses through the point (1, 0, –2) and normal to $\hat{i} + \hat{j} - \hat{k}$. (3)

OR

a) Wtrite the vector equation of a line passing through the points (–3, 1, 2) and (2, 3, 4). (1)

b) Find the shortest distance between the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ (4)

MARCH 2011

26. a) Find the equation of the plane with intercepts 2,3 and 4 on X,Y and Z axes respectively. (1)

b) Find the distance of the point (–1,–2,3) from the plane $\vec{r}.(2i - 3j + 4k) = 4$ (3)

27. Consider the lines $\frac{x-3}{2} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+5}{1} = \frac{y+2}{1} = \frac{z-3}{2}$

a) Find the angle between them. (1)

b) Find the shortest distance between them. (3)

OR

Consider the points A(2,2,–1) B(3,4,2) and C (7,0,6)

a) Find AB. (1)

b) Fin the vector and Cartesian equation of the plane passing through these points. (3)

SAY 2010

28. a) Find the vector equation of the plane passing through the intersection of the two planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r}.(2i + 3j + 4k) = -5$ and through the point (1,1,1). (3)

b) Express the vector equation $\vec{r}.(5\hat{i} + 3\hat{j} + 4\hat{k}) = 2$ of a plane in cartesian form and hence find its perpendicular distance from the origin. (2)

OR

Given the plane $5x - 2y + 4z - 9 = 0$

a) Find the foot of the perpendicular drawn from the origin to the plane. (3)

b) Write the vector equation and Cartesian equation of this perpendicular. (2)

MARCH 2010

29. a) The coordinates of the foot of the perpendicular from (1,2,1) on the x-axis is …… (1)

b) The ratio in which the line segment joining the points (–2,4,7) and (3,–5,8) is divided by the yz plane is …….. (1)

c) If A,B,C are angles which a line makes with the co-ordinate axes, then the value of $\sin^2 A + \sin^2 B + \sin^2 C =$ …… (1)

30. a) Out of syllabus.

b) Find the shortest distance between the skew-lines whose vector equations are:

$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right);$

$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ (4)

c) Find the angle between the planes $2x - y + z = 4$ *and* $x + 3y - z = 9$ (2)

OR

a) Out of syllabus.

b) Find the equation of the plane passing through the points (1,–1,1) and (2,3,0) and perpendicular to the plane $x + 2y + 3z = 8$. (4)

c) Find the equation of the line passing through the point (2,4,5) and perpendicular to the plane $3x + 5y - 2z = 11$ (2)

SAY 2009

31. a) Find the co-ordinates of the point which divides the join of (4, 1, –3) and (2, –3,5) in the ratio 3:1 internally. (2)

b) i) Find the direction ratios of the line through the points P(1, –1, 2) and Q(3, 4, –2). (1)

ii) If R(0, 3, 2) and S(3, 5, 6), show that PQ is perpendicular to RS. (2)

32. a) A line passes through the point (3,–2,5) and parallel to the vector $2\hat{i} + \hat{j} - 2\hat{k}$.

i) What is the vector equation of the line? (1)

ii) What is the Cartesian equation of the line? (1)

b) Find the shortest distance between the skew lines whose vector equations are

$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right);$

$\vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$ (4)

MARCH 2009

33. a) Out of syllabus

b) Out of sllabus

OR

34. Consider the planes $3x - 4y + 5z = 10$ and $2x + 2y - 3z = 4$

a) Write the equation of the plane through the line of intersection of the above planes. (1)

b) Write the direction ratio of the line $x = 2y = 3z$ (1)

c) If the above line is parallel to the obtained plane. Show that the plane is $x - 20y + 27z = 14$ (4)

MARCH 2008

35. Consider the points (–1,2,4) and (1,0,5).

i) Find the direction consines of the line joining the two points. (2)

ii) Out of syllabus

36. i) Out of syllabus

ii) Out of syllabus.
iii) Out of syllabus.

37. a) Find the equation of the plane through the point (1,2,3), perpendicular to the planes $x - y + z = 2$ and $2x + y - 3z = 5$ (3)

b) Find the distance between the parallel planes $x + 2y + 2z - 8 = 0$ and $6y - 3x - 6z = 57$ (3)

"ENTHUSIASM IS A POWER that can give Dreames to the Dreamless, Life to the Lifeless, and Hope to the Hopeless".