Chapter 12

# LINEAR PROGRAMMING PROBLEMS

The linear programming problem is one that is concerned with finding the optimal values (maximum and minimum) of a linear functions of several variables (called object function) subject to the conditions that the variables are non-negative and satisfy a set of linear inequalities (linear constraints). Variables are sometimes called decision variables.

1. **Objective function:** Function which is to be maximized or minimized subject to specified constraints on the variables is called an object function.
2. **Constraints:** The linear inequalities or equations on the variables of an LPP which describe the condition under which the maximization or minimization is to be accomplished are called constraints.
3. **Solution:** Values of the variables of an LPP, which satisfy the constraints is called solution.
4. **Feasible solution:** Values of the variables of an LPP is called a feasible solution of the LPP, it satisfies the constraints and non-negativity restrictions of the problems.
5. **Infeasible solution:** Values of the variables of an LPP are called an infeasible solution of the LPP, if the system of constraints has no point which satisfies all the constraints and non-negativity restrictions.
6. **Feasible region:** The common region determined by all the constraints including non-negative constraints is called the feasible region.
7. **Optimal feasible region:** If the feasible solution of an LPP optimizes the objective function, then the solution is called optimal feasible solution.
8. **Convex polygon:** A closed plane figure bounded by three or more line segments is called convex polygon.

**Graphical methods for solving LPP**

**9. Corner point method**

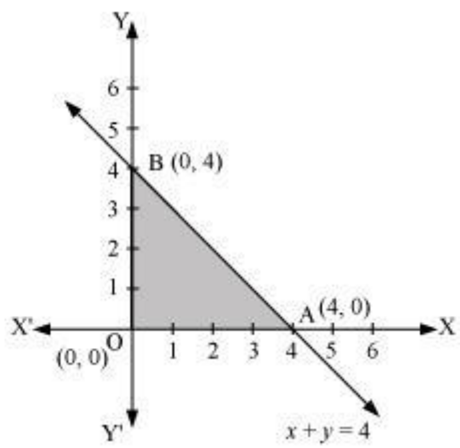
i. Find the feasible region of the LPP and determines its corner points or vertices.
ii. Evaluate the object function at each corner point. Let M and m respectively be the maximum and minimum values at these points.
iii. If the feasible region is bounded, M and m be the maximum and minimum values of the object function.
iv. If the feasible region is unbounded,

a. M is the maximum value of the object function, if the open half plane determined by has no point in common with the feasible region. Otherwise, the object function has no maximum value.
b. m is the minimum value of the object function, if the open half plane determined by has no point in common with the feasible region. Otherwise, the object function has no minimum value.

**Exercise 12.1**

1. Maximise Z = $3x + 4y$ Subject to the constraints: $x + y \leq 4$, $x \geq 0$, $y \geq 0$.

The feasible region determined by the constraints, $x + y \leq 4$, $x \geq 0$, $y \geq 0$, is as follows.



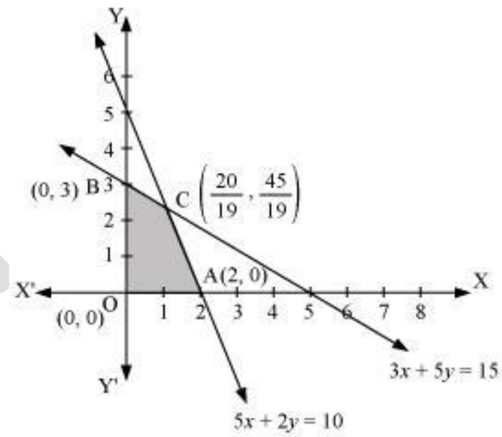

The corner points of the feasible region are O (0, 0), A (4, 0), and B (0, 4). The values of Z at these points are as follows.

| Corner point | Z = $3x + 4y$ | |
|---|---|---|
| O(0, 0) | 0 | |
| A(4, 0) | 12 | |
| B(0, 4) | 16 | → Maximum |

Therefore, the maximum value of Z is 16 at the point B (0, 4).

2. Minimise Z = −3$x$ + 4$y$ subject to $x+2y\le 8, 3x+2y\le 12, x\ge 0, y\ge 0$.

The feasible region determined by the system of constraints $x+2y\le 8, 3x+2y\le 12$, $x \ge 0$, and $y \ge 0$, is as follows.




The corner points of the feasible region are O (0, 0), A (4, 0), B (2, 3), and C (0, 4).

The values of Z at these corner points are as follows.

| **Corner point** | **Z = −3$x$ + 4$y$** | |
|---|---|---|
| 0(0, 0) | 0 | |
| A(4, 0) | −12 | → Minimum |
| B(2, 3) | 6 | |
| C(0, 4) | 16 | |

Therefore, the minimum value of Z is −12 at the point (4, 0).

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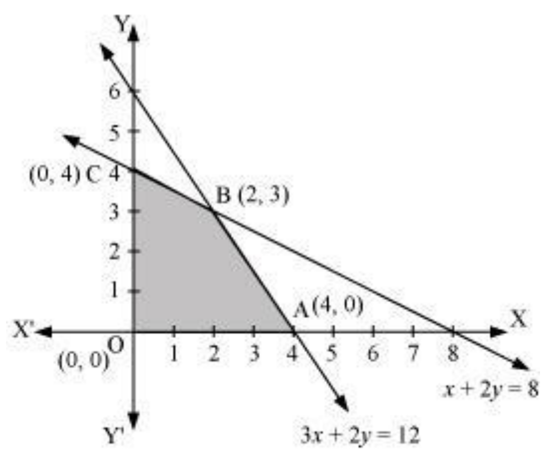
3. Maximise Z = 5$x$ + 3$y$ subject to $3x+5y\le 15,\ 5x+2y\le 10, x\ge 0, y\ge 0$.

The feasible region determined by the system of constraints, $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, and $y \ge 0$, are as follows.




The corner points of the feasible region are O (0, 0), A (2, 0), B (0, 3), and $3x+5y\le 15,\ 5x+2y\le 10, x\ge 0, y\ge 0$
.

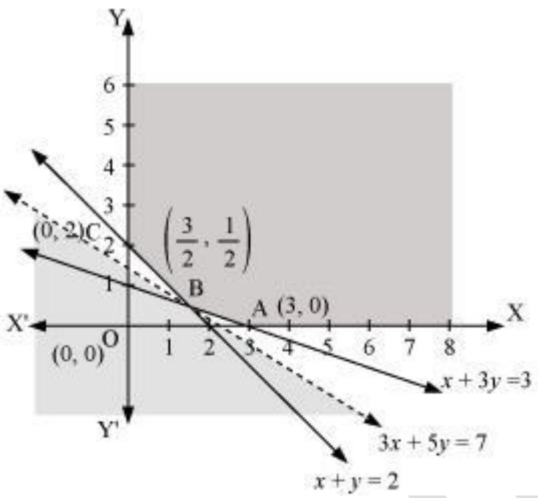
The values of Z at these corner points are as follows.

| **Corner point** | **Z = 5$x$ + 3$y$** | |
|---|---|---|
| 0(0, 0) | 0 | |
| A(2, 0) | 10 | |
| B(0, 3) | 9 | |
| $C\left(\frac{20}{19},\frac{45}{19}\right)$ | $\frac{235}{19}$ | → Maximum |

Therefore, the maximum value of Z is $\frac{235}{19}$ at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$.

4. Minimise Z = 3$x$ + 5$y$ such that $x+3y \geq 3,\ x+y \geq 2,\ x, y \geq 0$ .

The feasible region determined by the system of cons $x+3y \geq 3, x+y \geq 2$ and $x$, $y \geq 0$, is as follows.




It can be seen that the feasible region is unbounded.

The corner points of the feasible region are A (3, 0), $B\left(\frac{3}{2}, \frac{1}{2}\right)$ and C (0, 2).

The values of Z at these corner points are as follows:

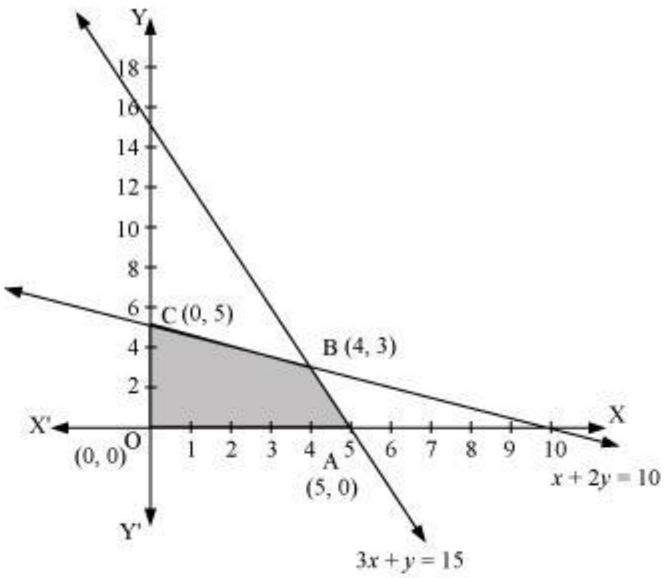
| **Corner point** | **Z = 3$x$ + 5$y$** | |
|---|---|---|
| A(3, 0) | 9 | |
| B$\left(\frac{3}{2}, \frac{1}{2}\right)$ | 7 | → Smallest |
| C(0, 2) | 10 | |

As the feasible region is unbounded, therefore, 7 may or may not be the minimum value of Z.
For this, we draw the graph of the inequality, 3$x$ + 5$y$ < 7, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $3x + 5y < 7$ Therefore, the minimum value of Z is 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$.

5. Maximise Z = $3x + 2y$ subject to $x + 2y \le 10, 3x + y \le 15, x, y \ge 0$ .

The feasible region determined by the constraints, $x + 2y \le 10$, $3x + y \le 15$, $x \ge 0$, and $y \ge 0$, is as follows.




The corner points of the feasible region are A (5, 0), B (4, 3), and C (0, 5).

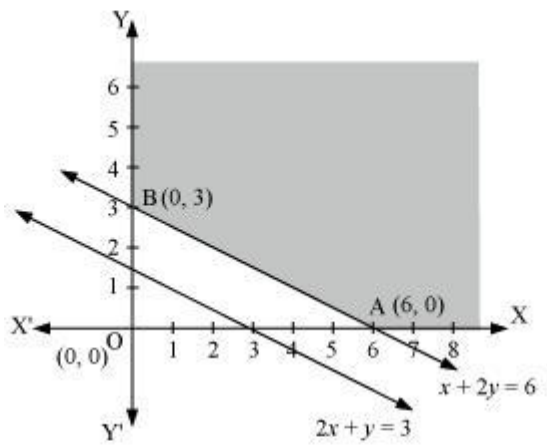
The values of Z at these corner points are as follows.

| **Corner point** | **Z = 3*x* + 2*y*** | |
|---|---|---|
| A(5, 0) | 15 | |
| B(4, 3) | 18 | → Maximum |
| C(0, 5) | 10 | |

Therefore, the maximum value of Z is 18 at the point (4, 3).

6. Minimise Z = $x + 2y$ subject to $2x + y \ge 3,\ x + 2y \ge 6,\ x, y \ge 0$ .

The feasible region determined by the constraints, $2x + y \ge 3$, $x + 2y \ge 6$, $x \ge 0$, and $y \ge 0$, is as follows.




The corner points of the feasible region are A (6, 0) and B (0, 3).

The values of Z at these corner points are as follows.

| Corner point | Z = $x + 2y$ |
|---|---|
| A(6, 0) | 6 |
| B(0, 3) | 6 |

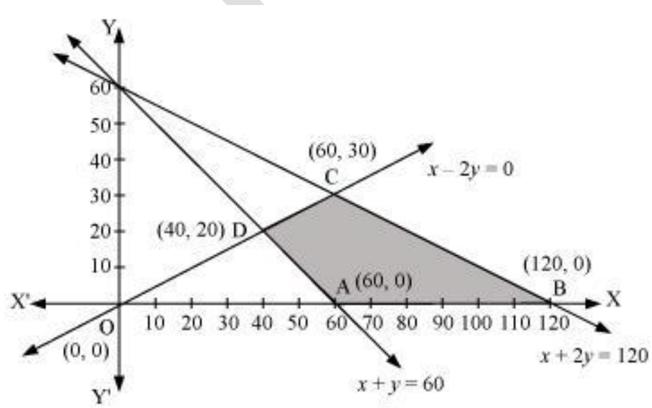
It can be seen that the value of Z at points A and B is same. If we take any other point such as (2, 2) on line $x + 2y = 6$, then Z = 6

Thus, the minimum value of Z occurs for more than 2 points.

Therefore, the value of Z is minimum at every point on the line, $x + 2y = 6$

7. Minimise and Maximise $Z = 5x + 10y$subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$.

The feasible region determined by the constraints, $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, and $y \geq 0$, is as follows.



The corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30), and D (40, 20).

The values of Z at these corner points are as follows.

| **Corner point** | **Z = 5*x* + 10*y*** | |
|---|---|---|
| A(60, 0) | 300 | → Minimum |
| B(120, 0) | 600 | → Maximum |
| C(60, 30) | 600 | → Maximum |
| D(40, 20) | 400 | |

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

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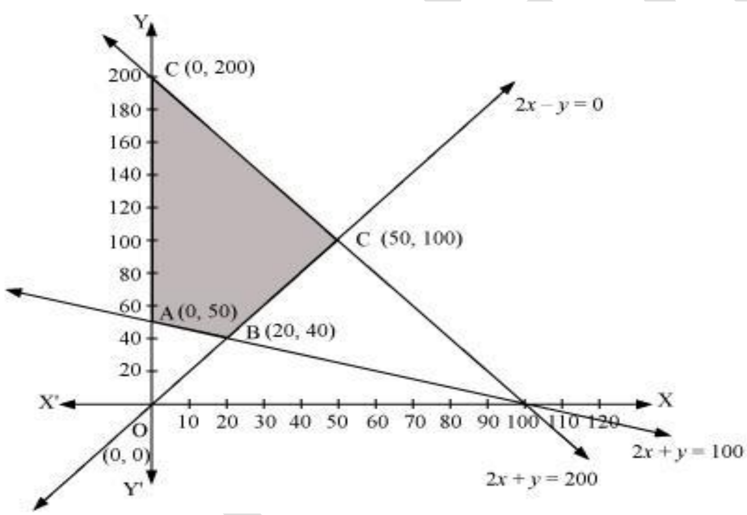
8. Minimise and Maximise $Z = x + 2y$ subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$.

   The feasible region determined by the constraints, $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$, is as follows.




The corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100), and D(0, 200).

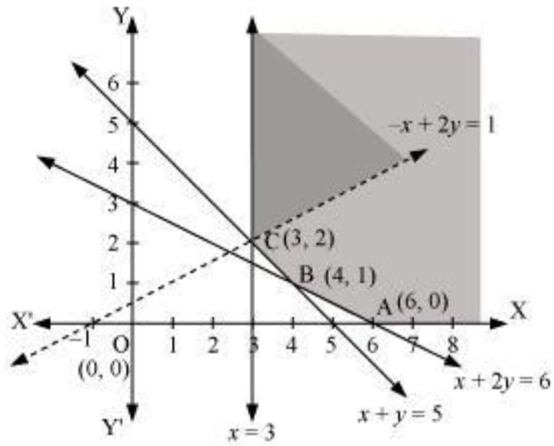
The values of Z at these corner points are as follows.

| **Corner point** | **Z = *x* + 2*y*** | |
|---|---|---|
| A(0, 50) | 100 | → Minimum |
| B(20, 40) | 100 | → Minimum |
| C(50, 100) | 250 | |
| D(0, 200) | 400 | → Maximum |

The maximum value of Z is 400 at (0, 200) and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

9. Maximise Z = − $x$ + 2$y$, subject to the constraints: $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$ and $y \geq 0$ .

The feasible region determined by the constraints, $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$ and $y \geq 0$ is as follows.




It can be seen that the feasible region is unbounded.

The values of Z at corner points A (6, 0), B (4, 1), and C (3, 2) are as follows.

| **Corner point** | **Z = −*x* + 2*y*** |
|---|---|
| A(6, 0) | Z = − 6 |
| B(4, 1) | Z = − 2 |
| C(3, 2) | Z = 1 |

As the feasible region is unbounded, therefore, Z = 1 may or may not be the maximum value.

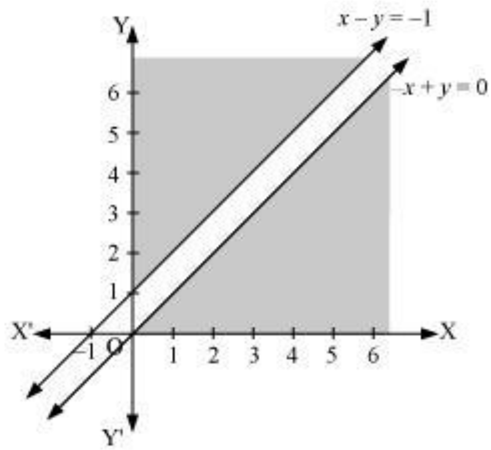
For this, we graph the inequality, $-x + 2y > 1$, and check whether the resulting half plane has points in common with the feasible region or not.

The resulting feasible region has points in common with the feasible region.

Therefore, Z = 1 is not the maximum value. Z has no maximum value.

10. Maximise Z = $x$ + $y$, subject to $x - y \leq -1,\ -x + y \leq 0, x, y \geq 0$.

The region determined by the constraints, $x - y \leq -1,\ -x + y \leq 0, x, y \geq 0$ is as follows.




There is no feasible region and thus, Z has no maximum value.

**Exercise 12.2**

1. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units /kg of vitamin A and 5 units /kg of vitamin B while food Q contains 4 units /kg of vitamin A and 2 units /kg of vitamin B. Determine the minimum cost of the mixture?

Let the mixture contain $x$ kg of food P and $y$ kg of food Q. Therefore,
$x \geq 0$ and $y \geq 0$. The given information can be compiled in a table as follows.

| | Vitamin A (units/kg) | Vitamin B (units/kg) | Cost (Rs/kg) |
|---|---|---|---|
| **Food P** | 3 | 5 | 60 |
| **Food Q** | 4 | 2 | 80 |
| **Requirement (units/kg)** | 8 | 11 | |

The mixture must contain at least 8 units of vitamin A and 11 units of vitamin B. Therefore, the constraints are $3x + 4y \geq 8$ ; $5x + 2y \geq 11$

Total cost, Z, of purchasing food is, $Z = 60x + 80y$

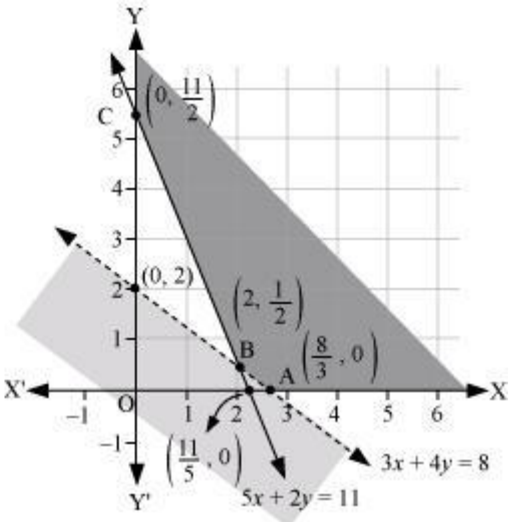
The mathematical formulation of the given problem is Minimise $Z = 60x + 80y$ ……………… (1)

subject to the constraints, $3x + 4y \geq 8$ ……………….… (2)

$5x + 2y \geq 11$ ……………………….. (3)

$x, y \geq 0$ …………………..… (4)

The feasible region determined by the system of constraints is as follows.




It can be seen that the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3},0\right), B\left(2,\frac{1}{2}\right)$ *and* $C\left(0,\frac{11}{2}\right)$.

The values of Z at these corner points are as follows.

| **Corner point** | **Z = 60*x* + 80*y*** | |
|---|---|---|
| $A\left(\frac{8}{3},0\right)$ | 160 | $\rightarrow$ Minimum |
| $B\left(2,\frac{1}{2}\right)$ | 160 | |
| $C\left(0,\frac{11}{2}\right)$ | 440 | |

As the feasible region is unbounded, therefore, 160 may or may not be the minimum value of Z.

For this, we graph the inequality, $60x + 80y < 160$ or $3x + 4y < 8$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $3x + 4y < 8$

Therefore, the minimum cost of the mixture will be Rs 160 at the line segment joining the points $\left(\frac{8}{3},0\right)$ and $\left(2,\frac{1}{2}\right)$.

2. One kind of cake requires 200g flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes?

Let there be $x$ cakes of first kind and $y$ cakes of second kind. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

| | **Flour (g)** | **Fat (g)** |
|---|---|---|
| **Cakes of first kind, *x*** | 200 | 25 |
| **Cakes of second kind, *y*** | 100 | 50 |
| **Availability** | 5000 | 1000 |

$$200x + 100y \leq 5000 \Rightarrow 2x + y \leq 50$$
$$25x + 50y \leq 1000 \Rightarrow x + 2y \leq 40$$

Total numbers of cakes, Z, that can be made are, Z = $x + y$

The mathematical formulation of the given problem is

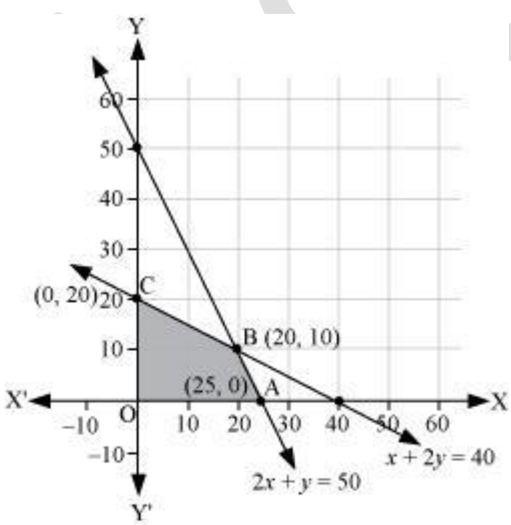
Maximize Z = $x + y$ … (1), subject to the constraints,

$2x + y \leq 50$ …………….. (2)

$x + 2y \leq 40$ ……………..(3)

$x, y \geq 0$ …………………….(4)

The feasible region determined by the system of constraints is as follows.




The corner points are A (25, 0), B (20, 10), O (0, 0), and C (0, 20).

The values of Z at these corner points are as follows.

| Corner point | Z = $x + y$ | |
|---|---|---|
| A(25, 0) | 25 | |
| B(20, 10) | 30 | → Maximum |
| C(0, 20) | 20 | |
| O(0, 0) | 0 | |

Thus, the maximum numbers of cakes that can be made are 30 (20 of one kind and 10 of the other kind).

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3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(ii) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

(i) Let the number of rackets and the number of bats to be made be $x$ and $y$ respectively.

The machine time is not available for more than 42 hours.

$1.5x + 3y \leq 42$ ……………………. (1)

The craftsman's time is not available for more than 24 hours.

$3x + y \leq 24$ ………………….. (2)

The factory is to work at full capacity. Therefore,

$1.5x + 3y = 42$

$3x + y = 24$

On solving these equations, we obtain

$x = 4$ and $y = 12$

Thus, 4 rackets and 12 bats must be made.

(i) The given information can be complied in a table as follows.

| | Tennis Racket | Cricket Bat | Availability |
|---|---|---|---|
| **Machine Time (h)** | 1.5 | 3 | 42 |
| **Craftsman's Time (h)** | 3 | 1 | 24 |

$\therefore 1.5x + 3y \le 42$

$3x + y \le 24$

$x, y \ge 0$

The profit on a racket is Rs 20 and on a bat is Rs 10.

$\backslash Z = 20x + 10y$

The mathematical formulation of the given problem is

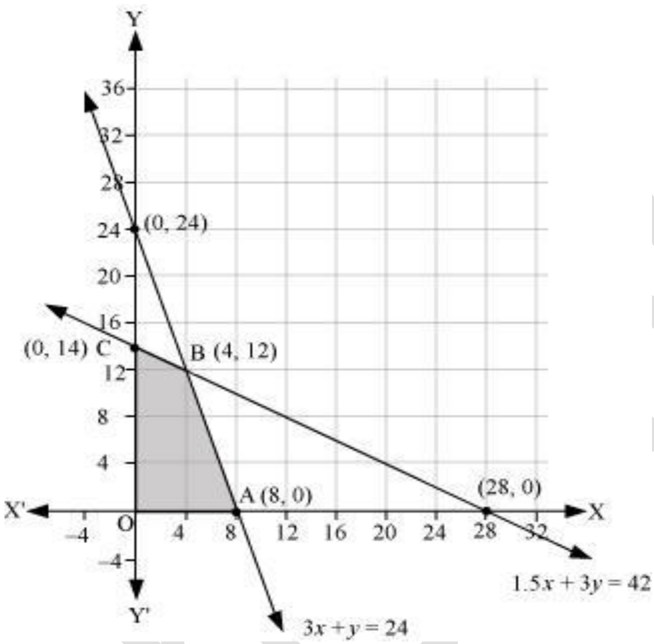
Maximize $Z = 20x + 10y$ ……………(1), subject to the constraints,

$1.5x + 3y \le 42$ ……………..… (2)

$3x + y \le 24$ …………..….. (3)

$x, y \ge 0$ ……………….. (4)

The feasible region determined by the system of constraints is as follows.




The corner points are A (8, 0), B (4, 12), C (0, 14), and O (0, 0).

The values of Z at these corner points are as follows.

| **Corner point** | **Z = 20*x* + 10*y*** | |
|---|---|---|
| A(8, 0) | 160 | |
| B(4, 12) | 200 | → Maximum |
| C(0, 14) | 140 | |
| O(0, 0) | 0 | |

Thus, the maximum profit of the factory when it works to its full capacity is Rs 200.

4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit, of Rs 17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day?

Let the manufacturer produce $x$ packages of nuts and $y$ packages of bolts. \ $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

| | **Nuts** | **Bolts** | **Availability** |
|---|---|---|---|
| **Machine A (h)** | 1 | 3 | 12 |
| **Machine B (h)** | 3 | 1 | 12 |

The profit on a package of nuts is Rs 17.50 and on a package of bolts is Rs 7. Therefore, the constraints are $x+3y \leq 12$ *and* $3x+y \leq 12$ and . .

Total profit, Z = $17.5x + 7y$

The mathematical formulation of the given problem is Maximise Z = $17.5x + 7y$ ………….. (1)

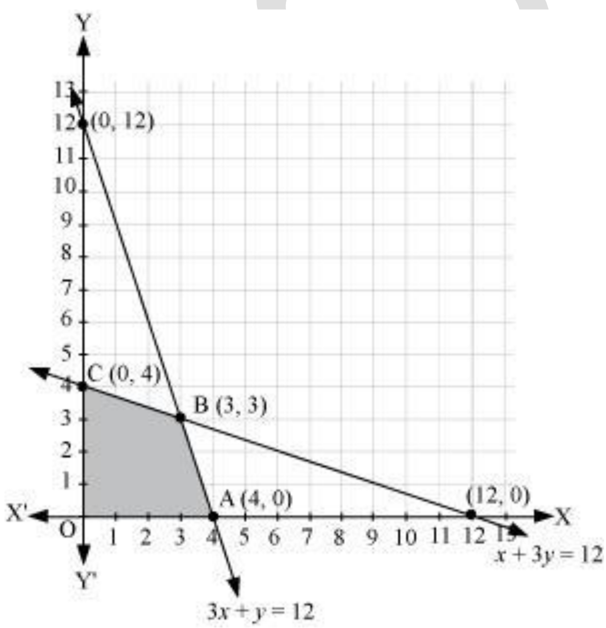
subject to the constraints,

$x + 3y \leq 12$ …………….. (2)

$3x + y \leq 12$ …………… (3)

$x, y \geq 0$ ……………… (4)

The feasible region determined by the system of constraints is as follows.




The corner points are A (4, 0), B (3, 3), and C (0, 4).

The values of Z at these corner points are as follows.

| **Corner point** | **Z = 17.5*x* + 7*y*** | |
|---|---|---|
| O(0, 0) | 0 | |
| A(4, 0) | 70 | |
| B(3, 3) | 73.5 | → Maximum |
| C(0, 4) | 28 | |

The maximum value of Z is Rs 73.50 at (3, 3).

Thus, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit of Rs 73.50.

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5. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.

Let the factory manufacture $x$ screws of type A and $y$ screws of type B on each day. Therefore,
$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

| | **Screw A** | **Screw B** | **Availability** |
|---|---|---|---|
| **Automatic Machine (min)** | 4 | 6 | 4 × 60 =120 |
| **Hand Operated Machine (min)** | 6 | 3 | 4 × 60 =120 |

The profit on a package of screws A is Rs 7 and on the package of screws B is Rs 10. Therefore, the constraints are $4x+6y \leq 240$, $6x+3y \leq 240$

Total profit, Z = $7x + 10y$

The mathematical formulation of the given problem is Maximize Z = $7x + 10y$ …….. (1)

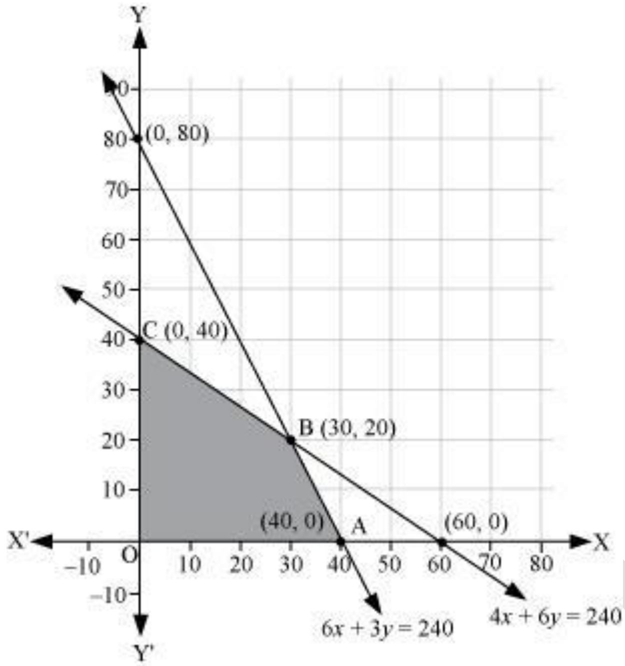
subject to the constraints,

$4x+6y \leq 240$ ………… (2)

$6x + 3y \le 240$ .............. (3)

$x, y \ge 0$ .............. (4)

The feasible region determined by the system of constraints is




The corner points are A (40, 0), B (30, 20), and C (0, 40).

The values of Z at these corner points are as follows.

| Corner point | Z = 7*x* + 10*y* | |
|---|---|---|
| A(40, 0) | 280 | |
| B(30, 20) | 410 | → Maximum |
| C(0, 40) | 400 | |

The maximum value of Z is 410 at (30, 20).

Thus, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.

6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from

a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit?

Let the cottage industry manufacture $x$ pedestal lamps and $y$ wooden shades. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

| | Lamps | Shades | Availability |
|---|---|---|---|
| **Grinding/Cutting Machine (h)** | 2 | 1 | 12 |
| **Sprayer (h)** | 3 | 2 | 20 |

The profit on a lamp is Rs 5 and on the shades is Rs 3. Therefore, the constraints are $2x + y \leq 12$ *and* $3x + 2y \leq 20$

Total profit, Z = $5x + 3y$

The mathematical formulation of the given problem is Maximize Z = $5x + 3y$ …………..… (1)

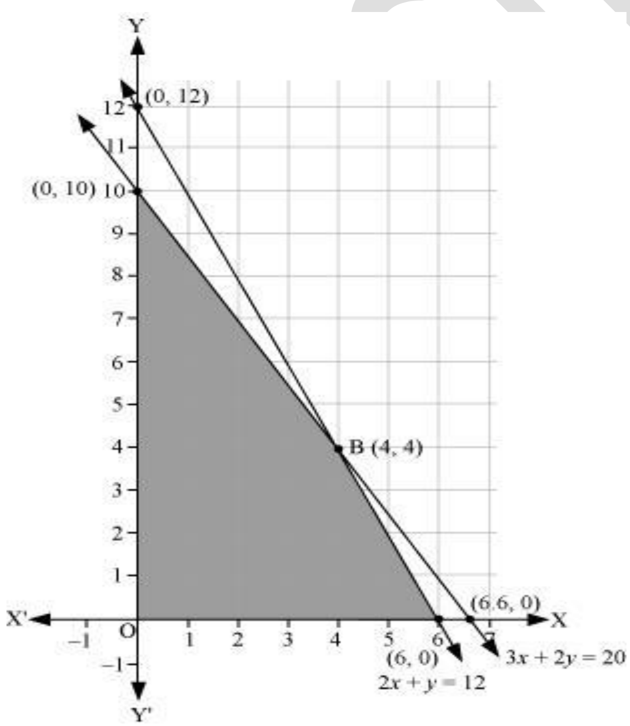
subject to the constraints,

$2x + y \leq 12$ ………………..… (2)

$3x + 2y \leq 20$ ………………..… (3)

$x, y \geq 0$ ……..………… (4)

The feasible region determined by the system of constraints is as follows.




The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of Z at these corner points are as follows

| Corner point | Z = 5*x* + 3*y* | |
|---|---|---|
| A(6, 0) | 30 | |
| B(4, 4) | 32 | → Maximum |
| C(0, 10) | 30 | |

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits.

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7. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours of assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

Let the company manufacture $x$ souvenirs of type A and $y$ souvenirs of type B. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

| | Type A | Type B | Availability |
|---|---|---|---|
| **Cutting (min)** | 5 | 8 | 3 × 60 + 20 =200 |
| **Assembling (min)** | 10 | 8 | 4 × 60 = 240 |

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6. Therefore, the constraints are $5x+8y \leq 200$, $10x+8y \leq 240 \Rightarrow 5x+4y \leq 120$

Total profit, Z = 5$x$ + 6$y$

The mathematical formulation of the given problem is

Maximize Z = 5$x$ + 6$y$ ……………… (1)

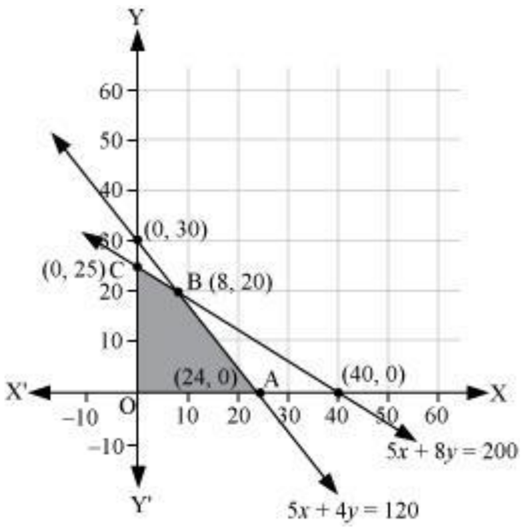
subject to the constraints,

$5x+8y \leq 200$ ……………… (2)

$5x+4y \leq 120$ …………..… (3)

$x, y \geq 0$ ………………… (4)

The feasible region determined by the system of constraints is as follows.




The corner points are A (24, 0), B (8, 20), and C (0, 25).

The values of Z at these corner points are as follows.

| **Corner point** | **Z = 5*x* + 6*y*** | |
|---|---|---|
| A(24, 0) | 120 | |
| B(8, 20) | 160 | → Maximum |
| C(0, 25) | 150 | |

The maximum value of Z is 200 at (8, 20).

Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.

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8. A merchant plans to sell two types of personal computers − a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.

Let the merchant stock $x$ desktop models and $y$ portable models. Therefore,

$x \geq 0$ and $y \geq 0$

The cost of a desktop model is Rs 25000 and of a portable model is Rs 4000. However, the merchant can invest a maximum of Rs 70 lakhs.

$$\therefore \begin{array}{l} 25000x + 4000y \leq 7000000 \\ i.e.,\ 5x + 8y \leq 1400 \end{array}$$

The monthly demand of computers will not exceed 250 units. $x + y \leq 250$

The profit on a desktop model is Rs 4500 and the profit on a portable model is Rs 5000.

Total profit, Z = 4500$x$ + 5000$y$

Thus, the mathematical formulation of the given problem is

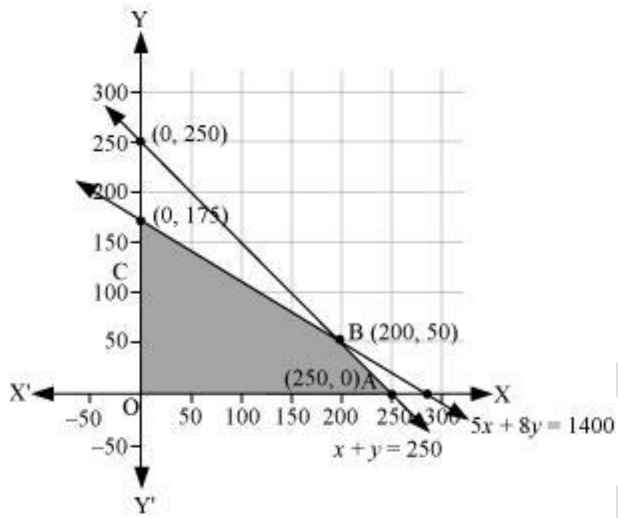
Maximum Z = 4500$x$ + 5000$y$ ……….(1), subject to the constraints,

$5x+8y\le 1400$ .............(2)

$x+y\le 250$..............(3)

$x, y\ge 0$ .............(4)

The feasible region determined by the system of constraints is as follows.




The corner points are A (250, 0), B (200, 50), and C (0, 175).

The values of Z at these corner points are as follows.

| **Corner point** | **Z = 4500$x$ + 5000$y$** | |
|---|---|---|
| A(250, 0) | 1125000 | |
| B(200, 50) | 1150000 | → Maximum |
| C(0, 175) | 875000 | |

The maximum value of Z is 1150000 at (200, 50).

Thus, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 1150000.

8. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods $F_1$and $F_2$ are available. Food $F_1$ costs Rs 4 per unit food and $F_2$ costs Rs 6 per unit. One unit of food $F_1$ contains 3 units of vitamin A and 4 units of minerals. One unit of food $F_2$ contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements?

Let the diet contain $x$ units of food $F_1$ and $y$ units of food $F_2$. Therefore, $x \geq 0$ and $y \geq 0$. The given information can be complied in a table as follows.

| | Vitamin A (units) | Mineral (units) | Cost per unit (Rs) |
|---|---|---|---|
| **Food $F_1$ ($x$)** | 3 | 4 | 4 |
| **Food $F_2$ ($y$)** | 6 | 3 | 6 |
| **Requirement** | 80 | 100 | |

The cost of food $F_1$ is Rs 4 per unit and of Food $F_2$ is Rs 6 per unit. Therefore, the constraints are

$3x + 6y \geq 80$ ; $4x + 3y \geq 100$ ; $x, y \geq 0$. Total cost of the diet, $Z = 4x + 6y$

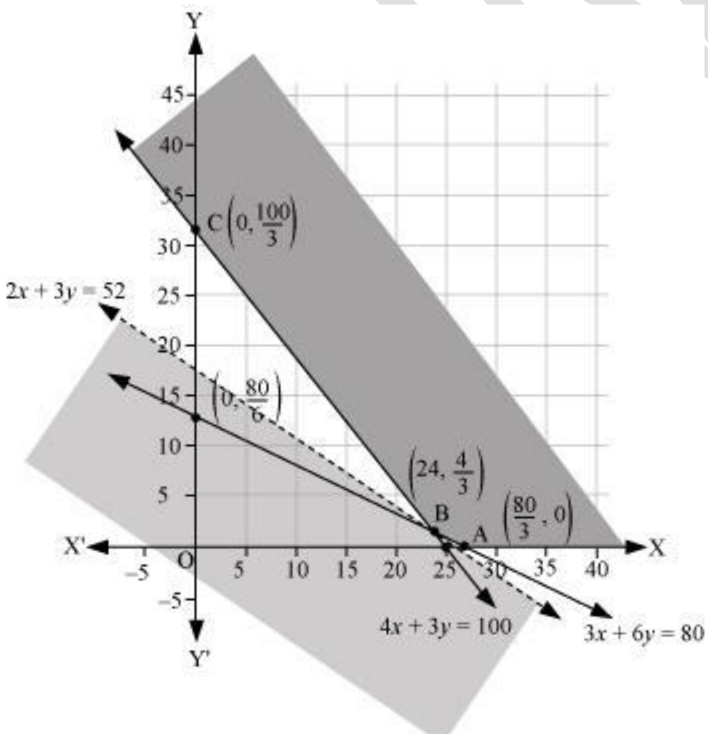
The mathematical formulation of the given problem is

Minimise $Z = 4x + 6y$ …………………..… (1) subject to the constraints,

$3x + 6y \geq 80$ …………………….. (2)

$4x + 3y \geq 100$ …………………... (3)

$x, y \geq 0$ …………………….. (4). The feasible region determined by the constraints is as follows.




It can be seen that the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3}, 0\right)$; $B\left(2, \frac{1}{2}\right)$ *and* $C\left(0, \frac{11}{2}\right)$ .

The corner points are $A\left(\frac{80}{3},0\right)$; $B\left(24,\frac{4}{3}\right)$ *and* $C\left(0,\frac{100}{3}\right)$ .

The values of Z at these corner points are as follows.

| Corner point | Z = 4*x* + 6*y* | |
|---|---|---|
| $A\left(\frac{80}{3},0\right)$ | $\frac{320}{3}=106.67$ | |
| $B\left(24,\frac{4}{3}\right)$ | 104 | → Minimum |
| $C\left(0,\frac{100}{3}\right)$ | 200 | |

As the feasible region is unbounded, therefore, 104 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $2x + 3y < 52$

Therefore, the minimum cost of the mixture will be Rs 104.

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9. There are two types of fertilizers $F_1$ and $F_2$. $F_1$ consists of 10% nitrogen and 6% phosphoric acid and $F_2$ consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If $F_1$ cost Rs 6/kg and $F_2$ costs Rs 5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Let the farmer buy $x$ kg of fertilizer $F_1$ and $y$ kg of fertilizer $F_2$. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

| | Nitrogen (%) | Phosphoric Acid (%) | Cost (Rs/kg) |
|---|---|---|---|
| **$F_1$ (*x*)** | 10 | 6 | 6 |
| **$F_2$ (*y*)** | 5 | 10 | 5 |
| **Requirement (kg)** | 14 | 14 | |

$F_1$ consists of 10% nitrogen and $F_2$ consists of 5% nitrogen. However, the farmer requires at least 14 kg of nitrogen.

∴ 10% of $x$ + 5% of $y \geq 14$

$$\frac{x}{10}+\frac{y}{20}\geq 14 \Rightarrow 2x+y\geq 280$$

$F_1$ consists of 6% phosphoric acid and $F_2$ consists of 10% phosphoric acid. However, the farmer requires at least 14 kg of phosphoric acid.

∴ 6% of $x$ + 10% of $y \geq 14$

$$\frac{6x}{100}+\frac{10y}{100}\geq 14$$
$$3x+56y\geq 700$$

Total cost of fertilizers, Z = $6x + 5y$

The mathematical formulation of the given problem is

Minimize Z = $6x + 5y$ … (1)

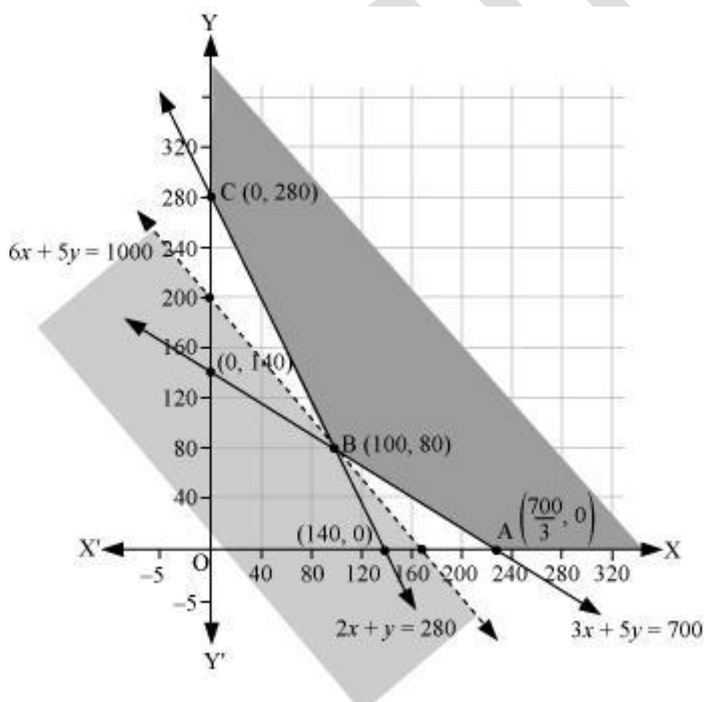
subject to the constraints,

$2x + y \geq 280$ …………….. (2)

$3x + 5y \geq 700$ ………….. (3)

$x, y \geq 0$ ………………… (4)

The feasible region determined by the system of constraints is as follows.




It can be seen that the feasible region is unbounded.

The corner points are $A\left(\frac{700}{3},0\right);\ B(100,80)\ and\ C(0,280)$.

The values of Z at these points are as follows.

| Corner point | Z = 6*x* + 5*y* | |
|---|---|---|
| $A\left(\frac{700}{3},0\right)$ | 1400 | |
| B(100, 80) | 1000 | → Minimum |
| C(0, 280) | 1400 | |

As the feasible region is unbounded, therefore, 1000 may or may not be the minimum value of Z. For this, we draw a graph of the inequality, $6x + 5y < 1000$, and check whether the resulting half plane has points in common with the feasible region or not. It can be seen that the feasible region has no common point with $6x + 5y < 1000$. Therefore, 100 kg of fertiliser $F_1$ and 80 kg of fertilizer $F_2$ should be used to minimize the cost. The minimum cost is Rs 1000.

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10. The corner points of the feasible region determined by the following system of linear inequalities:

$$2x + y \le 10;\ x + 3y \le 15,\ xy \ge 0\ are\ (0,0),(5,0),(3,4)\ and\ (0,5).$$

Let Z = $px + qy$, where $p, q > 0$. Condition on $p$ and $q$ so that the maximum of Z occurs at both (3, 4) and (0, 5) is

(A) $p = q$ (B) $p = 2q$ (C) $p = 3q$ (D) $q = 3p$

The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points, (3, 4) and (0, 5).

∴ Value of Z at (3, 4) = Value of Z at (0, 5)

$\Rightarrow p(3) + q(4) = p(0) + q(5)$

$\Rightarrow 3p + 4q = 5q$

$\Rightarrow q = 3p$

Hence, the correct answer is D.

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