Chapter-7

System of Particles and Rotational Motion

Motion of a Rigid Body

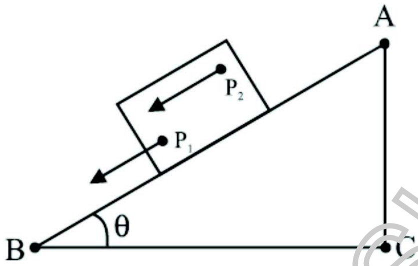
1. What is meant by a **rigid body**?

Ans: A rigid body has a definite shape and size. The distance between any two particles of the rigid body does not change even on the application of a force.

2. What are the different types of motion of rigid bodies?

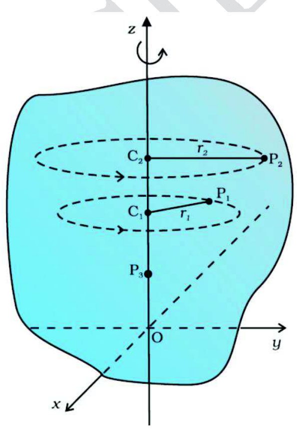
Ans:

(i) Translational Motion



In pure translational motion at any instant of time all the particles of the body have the same velocity.

(ii) Rotation

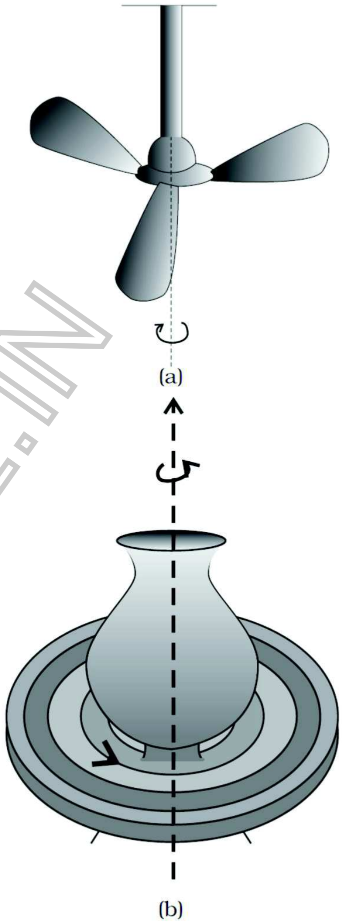


In the rotation of a rigid body about a fixed axis every particle of the body moves in circles. The circular path of the particle will be perpendicular to the axis of rotation.

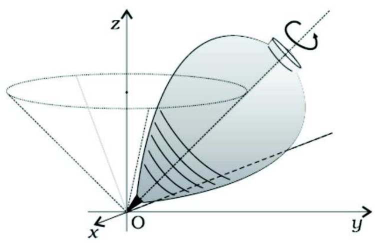
In rotation the angular velocity (ω) of all particles of the body remains constant.

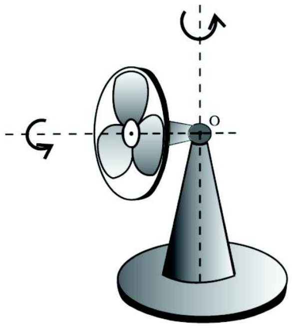
Eg: - Ceiling fan

Potter's wheel



(iii) Precession



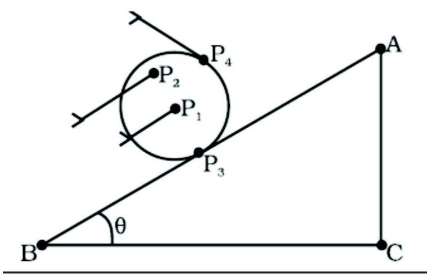


In this type of motion, the axis of rotation moves.

Eg: - Spinning of a top

An oscillating table fan or a pedestal fan.

(iv) Rolling motion



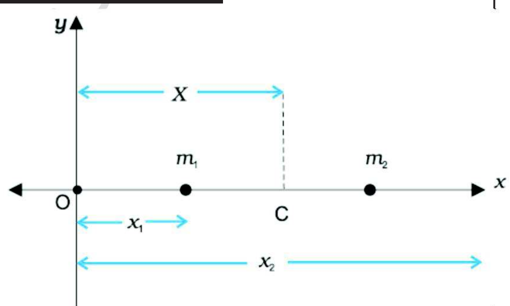
Rolling Motion is a combination of translational motion and rotation.

Centre of Mass

3. Define centre of mass. Give the expressions to find centre of mass.

Ans: It is the point at which the entire mass of the body can be assumed to be concentrated.

In one dimension



The distance to the centre of mass

$$X = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

If $m_1 = m_2 = m$

$$X = \frac{mx_1 + mx_2}{2m}$$

$$X = \frac{x_1 + x_2}{2}$$

For a system of **n** particle,

$$X = \frac{m_1x_1 + m_2x_2 + \cdots m_nx_n}{m_1 + m_2 + \cdots\cdots m_n}$$

$$= \frac{\sum m_ix_i}{\sum m_i}$$

ie $X = \dfrac{\sum m_ix_i}{M}$

In two dimension

Consider three particles lying in a plane having co-ordinates (x_1, y_1), (x_2, y_2), (x_3, y_3). The co-ordinates of the centre of mass (X, Y) is given by

$$X = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$Y = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

If $m_1 = m_2 = m_3$,

$$X = \frac{\cancel{m}(x_1 + x_2 + x_3)}{3\cancel{m}}$$

$$Y = \frac{\cancel{m}(y_1 + y_2 + y_3)}{3\cancel{m}}$$

$$X = \frac{x_1 + x_2 + x_3}{3}$$

$$Y = \frac{y_1 + y_2 + y_3}{3}$$

Thus for three particles of same mass the centre of mass coincides with the centroid of the triangle formed by the particles.

In three dimension

If the particles lie in space, then coordinates of centre of mass (X, Y, Z) is given by,

$$X = \frac{\sum m_i x_i}{M}$$

$$Y = \frac{\sum m_i y_i}{M}$$

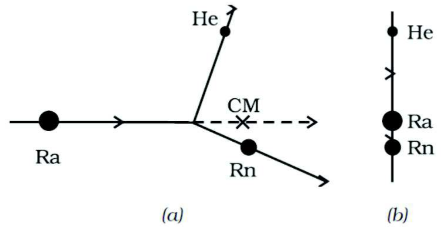
$$Z = \frac{\sum m_i z_i}{M}$$

4P. In the **HCl** molecule, the separation b/n the nuclei of the two atoms is **1.27A^{0}**, calculate the approximate location of the **centre of mass** of the molecule. Given **Cl** atom is nearly **35.5** times as massive as a **H** atom.

Ans:

5. Give some examples of motion of centre of mass.

Ans: **(i) Decay of a radioactive nucleus**

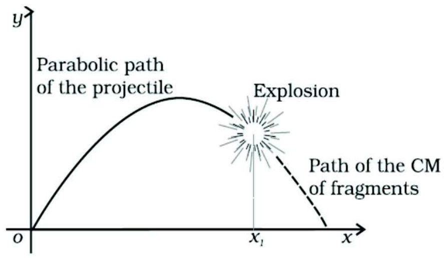


Consider the splitting of the heavy nucleus Radium [**Ra**] into lighter ones Helium [**He]** and Radon [**Rn**]. The centre of mass of the **Ra** nucleus was initially at rest. After decay the two nuclei **He** and **Rn** move in opposite directions such that the centre of mass is again at rest.

Explanation: -

Decay is caused by internal forces. Internal forces cannot change the state of C.M.

(ii) **Explosion of a shell**



Consider a shell moving along a parabolic path. During its motion it explodes. Each of the fragments moves along their own parabolic paths such that the C.M will follow the same parabolic path.

Explanation

The shell is moving under the external force of gravity. The explosion is caused by internal forces, which cannot change the path of centre of mass.

Vector Product of Two Vectors

6. Define Vector product of two vectors.

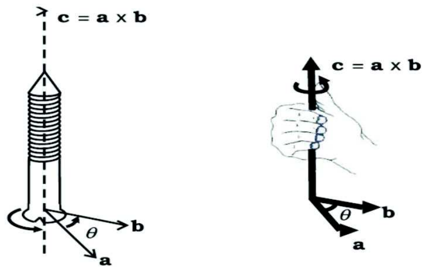
Ans:

$$\vec{A}\times\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta\ \hat{n}$$

$\hat{n}$ is a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$. Its direction (Direction of $\vec{A}\times\vec{B}$) is given by **right hand rule**.

7. State **right hand rule.**

Ans:



If you curl the fingers of your right hand from $\vec{A}$ to $\vec{B}$, then the thumb will give the direction of $\vec{A}\times\vec{B}$.

8. Give the component form of $\vec{A}\times\vec{B}$

If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along the x, y and z directions, then

$$\hat{i}\times\hat{j} = \left|\hat{i}\right|\left|\hat{j}\right|\sin 90\ \hat{k}$$

$$= 1\times1\times1\times\hat{k} = \hat{k}$$

Similarly,

$$\hat{j}\times\hat{k} = \hat{i} \text{ and } \hat{k}\times\hat{i} = \hat{j}$$

$$\hat{i}\times\hat{i} = \left|\hat{i}\right|\left|\hat{i}\right|\sin 0^\circ = 0$$

Similarly,

$$\hat{j}\times\hat{j} = 0$$

$$\hat{k}\times\hat{k} = 0$$

In component form,

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \text{ and}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$$= \hat{i}\begin{vmatrix}A_y & A_z\\ B_y & B_z\end{vmatrix} - \hat{j}\begin{vmatrix}A_x & A_z\\ B_x & B_z\end{vmatrix} + \hat{k}\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix}$$

$$= \hat{i}\left[A_yB_z - A_zB_y\right] - \hat{j}\left[A_xB_z - B_xA_z\right]$$

$$+\hat{k}\left[A_xB_y - B_xA_y\right]$$

9. Find the scalar and vector products of $\vec{A}=2\hat{i}+7\hat{j}-\hat{k}$ and $\vec{B} = 4\hat{i} - 3\hat{j} - 4\hat{k}$.

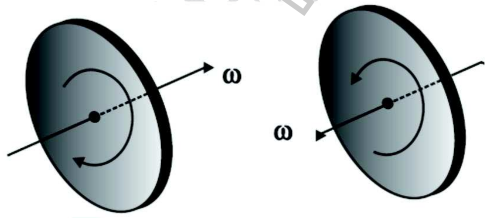
Ans:

10. Find the scalar and vector products of
$\vec{A}=3\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{B} = -2\hat{i} +\hat{j}-3\hat{k}$.

Ans:

11. Give the **direction angular velocity**.

Ans: The direction of angular velocity is given by right hand rule.



The relation between linear velocity and angular velocity is

v = rω

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

12. Define **angular acceleration.**

Ans: The rate of change of angular velocity is called angular acceleration.

Angular acceleration,

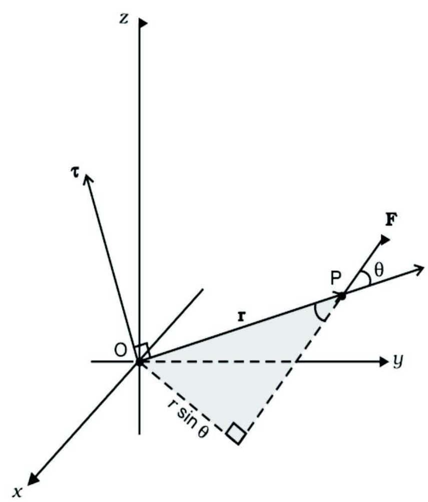
$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

SI unit of angular acceleration is **rad/ s^2.**

13. Define **torque (τ)**.

Torque is the force which produces turning effect.

<u>**Definition**:</u> - Torque or moment of force about an axis of rotation is measured as the product of the magnitude of the force and the perpendicular distance b/n the line of action of the force and the axis of rotation .



τ = Force × ⊥ distance

$\tau = F \times ON \qquad \sin\theta = \frac{ON}{r}$

$= F \times d \qquad ON = r\sin\theta$

$= F \times r\sin\theta = rF\sin\theta$

In vector form,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The direction of torque is given by right had rule.

SI unit of torque is **Nm**

Dimension = $[MLT^{-2}]\ [L]$

$= [ML^2T^{-2}]$

Note: - **(i)** Torque has the same dimension of work or energy.

(ii) $\tau = rF\sin\theta$.

When $\theta = 90^0$

$\tau = rF$ (This is the max. value or torque)

(iii) Torque is analogous to force in linear motion.

14. Express **torque in component form**.

Ans:

$$\text{Let } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

15. A force $\vec{F} = 5\hat{i} - 2\hat{j} + 3\hat{k}$ acts on a particle whose position vector is $2\hat{i} + 3\hat{j} + 5\hat{k}$. Find the torque about the origin.

Ans:

16. Door handles are fixed at the free edge. Why?

Ans: We have torque= force × perpendicular distance.

By fixing door handles at the free edge, we increase the perpendicular distance to the point at which the force is applied. So for a given applied force, torque will be maximum.

17. Define **angular momentum** of a particle.

Ans: Angular momentum of a rotating particle about an axis of rotation is the moment of linear momentum of the particle about that axis.

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = rp\sin\theta$$

$$= r(mv)\sin\theta$$

$$= mvr\sin\theta$$

The direction of angular momentum is $\perp$ to the plane containing $\vec{r}$ and $\vec{p}$. The direction of angular momentum is given by right had rule.

$$\text{If } \theta = 90^0$$

$$L = mvr\ 90^0$$

$$= mvr$$

$$= m(r\omega)r$$

$$L = mr^2\omega$$

SI unit is **Kgm²/S**

Dimension is $\mathbf{ML^2T^{-1}}$

18. Derive the relation b/n torque and angular momentum.

Ans: We have, $\vec{L} = \vec{r} \times \vec{p}$

Differentiating both sides with respect to time

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$
$$= \vec{r} \times \vec{F} + \vec{v} \times \vec{p}$$
$$= \vec{\tau} + \vec{v} \times (m\vec{v})$$
$$= \vec{\tau} + m(\vec{v} \times \vec{v})$$
$$= \vec{\tau} + m(0) = \vec{\tau}$$

That is the rate of change of angular momentum is equal to applied torque. This is Newton's second law in rotational motion.

19. State the **Law of Conservation of Angular momentum.**

Ans:

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

If $\vec{\tau}_{ext} = 0$

$$\frac{d\vec{L}}{dt} = 0$$

$$\Rightarrow \vec{L} = \text{constant}$$

The law of conservation of angular momentum states that, "**if there is no external torque acting on a system of particles, then their total angular momentum remains constant**."

20. Explain **equilibrium of a rigid body**.

Ans: <u>Condition for **translational equilibrium**</u>

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

We have, $= \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt}$

If the body is in translational equilibrium, v = Constant

$$\therefore \frac{d\vec{v}}{dt} = 0$$

$$\therefore \vec{F}_{ext} = m \times 0$$
$$= 0$$

This a rigid body is said to be at translational equilibrium if the net external force acting on it is zero.

<u>Condition for **rotational equilibrium**</u>

For a rotating body,

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt} \qquad \boxed{\vec{P} = m\vec{v}}$$
$$= \frac{d}{dt}(I\vec{\omega}) \qquad \boxed{\vec{L} = I\vec{\omega}}$$
$$= I\frac{d\vec{\omega}}{dt}$$

The body is in rotational equilibrium if **ω** = Constant

$$\therefore \frac{d\vec{\omega}}{dt} = 0$$

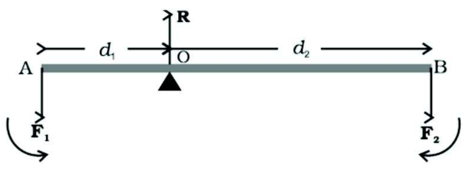
$$\Rightarrow \vec{\tau}_{ext} = I \times 0 = 0$$

Thus a rigid body is said to be at rotational equilibrium if the net external torque acting on it is zero.

<u>Principle of moments</u>

21. Explain the principle of moments.

<u>**Ans**:</u> A lever is a system of mechanical equilibrium.



For translational equilibrium of the body,

$R - F_1 - F_2 = 0 \Rightarrow R = F_1 + F_2$

For rotational equilibrium,

Net torque = 0

$d_1F_1 - d_2F_2 = 0$

$\mathbf{d_1F_1 = d_2F_2}$

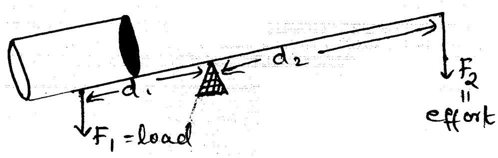
$\boxed{\textbf{load arm} \times \textbf{load} = \textbf{effort arm} \times \textbf{effort}}$

This is the principle of moments.

Mechanical advantage,

$$MA = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

To increase MA, d_1 is to be decreased.



22. Distinguish b/n centre of mass and centre of gravity.

Ans: Centre of mass is a point at which the whole mass of a body can be assumed to be concentrated.

Centre of gravity is the point at which the effective weight of body acts.

If 'g' remains constant throughout the space of body considered, then CM and centre of gravity are identical otherwise different.

Moment of Inertia

23. Define moment of inertia.

Ans: Moment of inertia of a particle of mass 'm' rotating about an axis at a distance **r** from it is given by

$$\mathbf{I = mr^2}$$

In the case of a rigid body, which consist of a large no of particles,

$$I = m_1r_1^2 + m_2r_2^2 + \cdots\cdots + m_nr_n^2$$

$$\boxed{I = \sum m_i r_i^2}$$

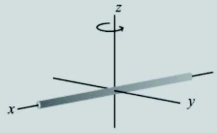
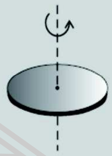
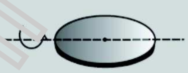
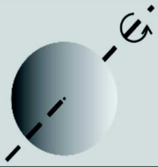
SI unit = Kgm^2, is a scalar quantity

Dimension is $\mathbf{ML^2}$

24. Give the physical significance of moment of inertia.

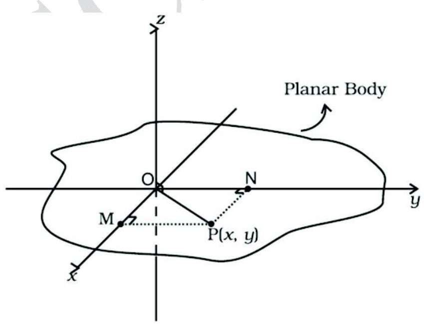
Ans: The inability of a rotating body to produce any change in its state is called its rotational inertia or moment of inertia. Moment of inertia depends not only on the mass but also on its distribution about the axis of rotation. Moment of inertia in rotational motion plays the same role as mass does in linear motion.

Table 7.1 Moments of Inertia of some regular shaped bodies about specific axes

Z	Body	Axis	Figure	I
(1)	Thin circular ring, radius *R*	Perpendicular to plane, at centre		MR^2
(2)	Thin circular ring, radius *R*	Diameter		$MR^2/2$
(3)	Thin rod, length L	Perpendicular to rod, at mid point		$ML^2/12$
(4)	Circular disc, radius *R*	Perpendicular to disc at centre		$MR^2/2$
(5)	Circular disc, radius *R*	Diameter		$MR^2/4$
(6)	Hollow cylinder, radius *R*	Axis of cylinder		MR^2
(7)	Solid cylinder, radius *R*	Axis of cylinder		$MR^2/2$
(8)	Solid sphere, radius *R*	Diameter		$2MR^2/5$

25. State the **Perpendicular axes theorem.**

Ans:



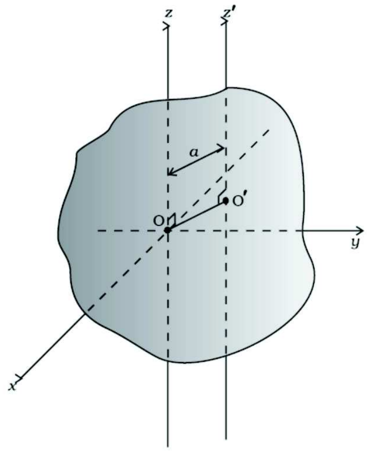
It states that "the M.I of a plane lamina about an axis ⊥r to its plane is equal to the sum of moments of inertia about two mutually ⊥r axes in its plane and intersecting each other at the point where the ⊥r axis meets the lamina.

$$I_z = I_x + I_y$$

26P. Find the moment of inertia of a ring about any diameter. Given that M.I of ring about the ⊥ axis through the centre is $\mathbf{MR^2}$.

<u>**Soln: -**</u>

27. State **Parallel axes theorem.**



Ans:

The theorem states that "the M.I of a body about any axis is equal to the sum of the M.I of the body about a parallel axis passing through the C.M of the body and the product of the mass of the body and square of the distance between the two parallel axes."

$$\mathbf{I = I_{cm} + Ma^2}$$

28P. Find the M.I of a sphere about a tangent to the sphere, given its M.I about any of its diameters to be $\frac{2}{5}MR^2$.

Ans:

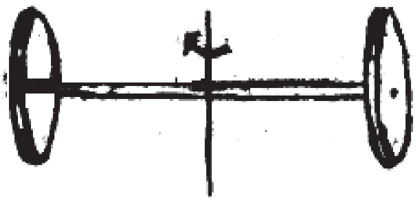
29P. Given M.I of a disc about any of its diameters to be $\frac{MR^2}{4}$ find its M.I about an axis normal to the disc and passing through a point on its edge.

Ans:

30P. (a) Find the moment of inertia of a thin metre scale about a perpendicular axis through the centre. Take **M** as the mass of the scale.

(b) Find the moment of inertia of a thin circular disc of mass **m** and radius **R** about one of its diameters.

(c) If a student fixes two discs each of radius **R** at the ends of meter scale and rotates the system about an axis perpendicular to the length of the scale as in figure. What will be the moment of inertia of the system?



Soln: -

31P. The moment of inertia of a thin ring of radius **R** about an axis passing through any diameter is $\frac{MR^2}{2}$.

(a) What is the radius gyration of a ring about a $\perp$ axis passing through the centre?

(b) A ring has a diameter **0.20 m** and mass **1 Kg**. Calculate its M.I about an axis passing through a tangent perpendicular to its plane.

Soln: -

32. Define **radius of gyration(K).**

Ans. The radius of gyration of a body about an axis may be defined as the distance from the axis to a mass point, whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.

In other words, If the moment of inertia of a body can be expressed as, **$I = MK^2$**, then **K** is known as the radius of gyration.

$$K = \sqrt{\frac{I}{M}}$$

33. Find the radius of gyration of the following rolling bodies about the axis passing through the centre of mass (i) Ring (ii) Disc (iii) Solid sphere (iv) Hollow sphere (v) Solid cylinder (vi) Hollow cylinder

Ans: Radius of gyration, $K = \sqrt{\frac{I}{M}}$

(i) Ring

$$I = MR^2$$

$$K = \sqrt{\frac{I}{M}}$$

$$= \sqrt{\frac{MR^2}{M}}$$

$$= \sqrt{R^2} = R$$

(ii) Disc

$$I = \frac{MR^2}{2}$$

$$K = \sqrt{\frac{I}{M}}$$

$$= \sqrt{\frac{\frac{MR^2}{2}}{M}}$$

$$= \sqrt{\frac{R^2}{2}} = \frac{R}{\sqrt{2}}$$

(iii) Solid sphere

$$I = \frac{2MR^2}{5}$$

$$K = \sqrt{\frac{I}{M}}$$

$$= \sqrt{\frac{\left(\frac{2MR^2}{5}\right)}{M}}$$

$$= \sqrt{\frac{2R^2}{5}} = \sqrt{\frac{2}{5}}R$$

(iv) Hollow sphere

$$I = \frac{2MR^2}{3}$$

$$K = \sqrt{\frac{I}{M}}$$

$$= \sqrt{\frac{\left(\frac{2MR^2}{3}\right)}{M}}$$

$$= \sqrt{\frac{2R^2}{3}} = \sqrt{\frac{2}{3}}R$$

(v) Hollow cylinder

$$I = MR^2$$

$$K = \sqrt{\frac{I}{M}}$$

$$= \sqrt{\frac{MR^2}{M}}$$

$$= \sqrt{R^2} = R$$

(vi) Solid cylinder

$$I = \frac{MR^2}{2}$$

$$K = \sqrt{\frac{I}{M}}$$

$$= \sqrt{\frac{\frac{MR^2}{2}}{M}}$$

$$= \sqrt{\frac{R^2}{2}} = \frac{R}{\sqrt{2}}$$

34. Give the **Kinematic equations of rotational motion.**

Ans:

Angular velocity after any time

Consider a body of mass 'm' rotating about an axis with uniform angular acceleration 'α'. Let ω_0 be the initial angular velocity. Then the angular velocity after a time 't' be ω_t

$$\omega_t = \omega_0 + \alpha t$$

Angular displacement after any time

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

Angular velocity after some angular displacement

$$\omega_t^2 = \omega_0^2 + 2\alpha\theta$$

35. Give the relation between **torque and angular acceleration**.

Ans: Torque $\tau = I\,\alpha$, where **I** is the moment of inertia and **α** is the angular acceleration.

36. Give the expression for the work done in rotating a body.

Ans: For constant torque

$$W = \tau\,\theta$$

For variable torque

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

37. Give the expression for power in rotational motion.

Ans: $P = \tau\omega$

Table 7.2 Comparison of Translational and Rotational Motion

	Linear Motion	Rotational Motion about a Fixed Axis
1	Displacement x	Angular displacement θ
2	Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
3	Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$
4	Mass M	Moment of inertia I
5	Force $F = Ma$	Torque $\tau = I\alpha$
6	Work $dW = F\,ds$	Work $W = \tau\, d\theta$
7	Kinetic energy $K = Mv^2/2$	Kinetic energy $K = I\omega^2/2$
8	Power $P = F\,v$	Power $P = \tau\omega$
9	Linear momentum $p = Mv$	Angular momentum $L = I\omega$

38P. A wheel of mass **1000Kg** and radius **1m** is rotating at the rate of **420 rpm**. What is the constant torque required to stop the wheel in **14 rotations**, assuming the mass to be concentrated at the rim of the wheel?

Soln:

39P. To maintain a rotor at a uniform angular speed of **200 rad/s** an engine needs to transmit a torque of **180 Nm**. What is the power required by the engine. Assume that the engine is **100 %** efficient?

Ans:

40. Derive the expression for kinetic energy of rotation.

Ans: Consider a rigid body of n particles,

K.E of the 1st particle = $\frac{1}{2} m_1 v_1^2$

K.E of the 2nd particle = $\frac{1}{2} m_2 v_2^2$

K.E of the nth particle = $\frac{1}{2} m_n v_n^2$ Total kinetic energy of the rotating body is given by,

$$K.E_r = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \cdots\cdots + \frac{1}{2} m_n v_n^2$$

We have $v_1 = r_1 \omega$, $v_2 = r_2 \omega$, $v_n = r_n \omega$

KE_r

$$= \frac{1}{2} m_1 (r_1\omega)^2 + \frac{1}{2} m_2 (r_2\omega)^2 + \cdots\cdots + \frac{1}{2} m_n (r_n\omega)^2$$

$$= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \cdots\cdots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$= \frac{1}{2} \omega^2 \left[m_1 r_1^2 + \frac{1}{2} m_2 r_2^2 + \frac{1}{2} m_3 r_3^2 + \cdots + \frac{1}{2} m_n r_n^2 \right]$$

$= \frac{1}{2} \omega^2$ I , I is the total M.I of the rigid body.

$$\boxed{K.E_r = \frac{1}{2} I \omega^2}$$

41P. A solid cylinder of mass **20 Kg** rotates about its axis with angular speed **100 rad/s**. The radius of the cylinder is **0.25m**. What is the K.E associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

Soln:

42. Give the expression for the law of Conservation of angular momentum in terms of moment of inertia and angular velocity.

We have,

$$\tau = \frac{dL}{dt}$$

$$\text{If } \tau = 0, \ \frac{dL}{dt} = 0$$

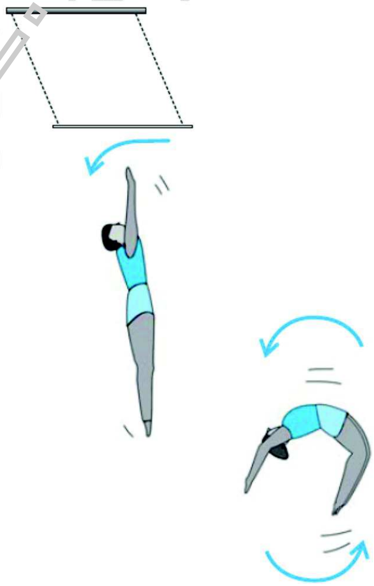
$$\Rightarrow L = \text{constant}$$

$$\boxed{I\,\omega = \text{constant}}$$

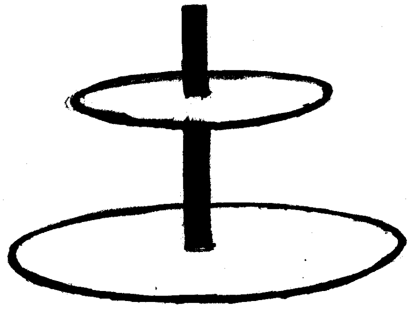
$$\text{i.e., } \boxed{I_1\,\omega_1 = I_2\,\omega_2}$$

43. Give some applications of the law of conservation of angular momentum.

Ans: The law conservation of angular momentum is applicable in the case ballot dancers, high board divers, skaters etc....

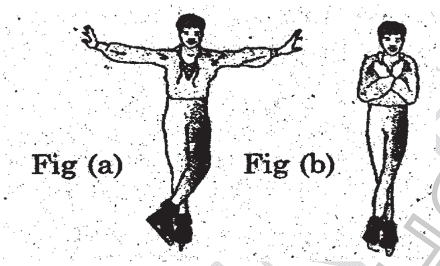


44P. A disc of moment of inertia I_1 is rotating freely with angular speed ω_1 and a second non-rotating disc with moment of inertia I_2 is dropped on it as shown in the figure. The two then rotate as one unit. Find the angular speed of rotation of the system.



Soln:

45Q. The figure below shows two spinning poses of a ballet dancer.



In which spinning pose does the ballet dancer have less angular velocity? Justify your answer.

Ans:

46P. A child stands at the centre of a turn table with his two arms out stretched. The turn table is set rotating with an angular speed of 40 rpm. How much is the angular speed of the child if he folds his hands back and there by reduces his moment of inertia to $\frac{2}{5}$ times the initial value.

Soln:

47P. Remya stands at the centre of a turn table with her two arms outstretched. The table is set rotating with an angular speed of 40 revolutions/ minute.

(a) What will happen to the moment of inertia if she folds her hands back?

(b) If the angular speed is increased to 100 rev/min, what will be the new moment of inertia?

Soln:

48P. If the radius of earth is suddenly reduced to half its value, what will be its effect on the duration of the day?

Ans:

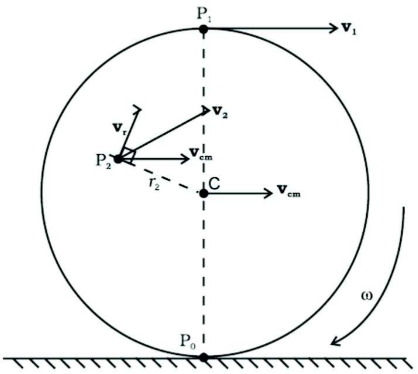
49P. Determine the duration of a day on earth if it suddenly shrinks to $1/8^{th}$ of its present **size** while the mass of the earth remains unchanged. (Assume earth is a perfect sphere)

Soln:

Rolling motion

50. Explain rolling motion. Obtain the expression for the velocity of centre of mass.

Ans: Rolling motion in the combination of translation and rotation.



At any instant a particle of a rolling body has two velocities

Translational velocity V_{cm} and

rotational velocity, $V_r = r\,\omega$

For the particle P_0, its rotational velocity = R ω
Since that particle is instantaneously at rest its translational velocity must be equal but opposite.
i.e., $\mathbf{V_{cm} = R\,\omega}$

$$\boxed{V_{cm} = R\,\omega}$$

Velocity the particle P_2
$= V_{cm} + V_r$
$= V_{cm} + R\omega$
$= V_{cm} + R\omega$
$= V_{cm} + V_{cm}$
$= 2V_{cm}$, this is the value of maximum velocity.
Thus in a rolling body, the lowermost particle has minimum speed and the topmost particle has maximum speed.

51. Derive an expression for the **kinetic energy** of a rolling body

Ans:

Kinetic energy of a rolling body is the sum of translational kinetic energy and rotational kinetic energy.

$$KE = \frac{1}{2}MV_{cm}{}^2 + \frac{1}{2}I_{cm}\omega^2$$

But for any rolling body $\mathbf{V_{cm}=R\omega}$

$\Rightarrow R = \dfrac{V_{cm}}{\omega}$ and we know $\mathbf{I_{cm}=MK^2}$, where **K** is the radius of gyration.

$$KE = \frac{1}{2}MV_{cm}{}^2 + \frac{1}{2}\left(MK^2\right)\left[\frac{V_{cm}}{R}\right]^2$$

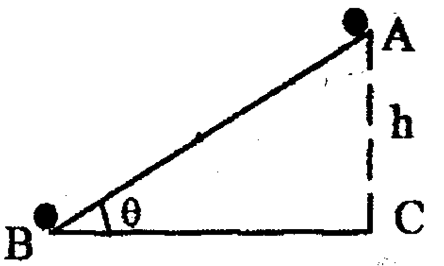
$$= \frac{1}{2}MV_{cm}{}^2 + \frac{1}{2}\left(MK^2\right)\frac{V_{cm}{}^2}{R^2}$$

$$= \frac{1}{2}MV_{cm}{}^2\left[1+\frac{K^2}{R^2}\right]$$

$$\boxed{KE = \frac{1}{2}MV_{cm}{}^2\left[1+\frac{K^2}{R^2}\right]}$$

52. Derive an expression for the **velocity** (speed) of the rolling body, when it reaches the bottom of the inclined plane.

Ans:



At the top of the inclined plane, the body is at rest and its energy is all potential which is equal to **Mgh**. At the bottom of the inclined plane, the energy is all kinetic and is given by,

$$KE = \frac{1}{2}MV_{cm}{}^2\left[1+\frac{K^2}{R^2}\right]$$

Then by conservation of energy,

$$Mgh = \frac{1}{2} M V_{cm}^{2} \left[1 + \frac{K^2}{R^2}\right]$$

$$\Rightarrow gh = \frac{1}{2} V_{cm}^{2} \left[1 + \frac{K^2}{R^2}\right]$$

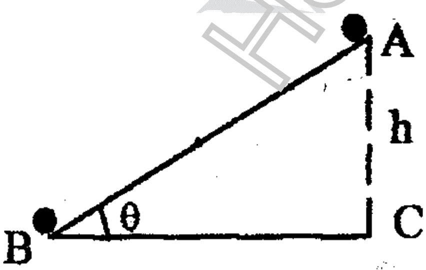
$$\Rightarrow V_{cm}^{2} = \frac{2gh}{\left[1 + \frac{K^2}{R^2}\right]}$$

$$\Rightarrow V_{cm} = \sqrt{\frac{2gh}{\left[1 + \frac{K^2}{R^2}\right]}}$$

$$\boxed{V_{cm} = \sqrt{\frac{2gh}{\left[1 + \frac{K^2}{R^2}\right]}}}$$

53. Give an expression for the **acceleration** of a body rolling on an inclined plane.

Ans:



The velocity of the rolling body when it reaches the bottom is given by the equation of motion,

$\mathbf{v^2 = u^2 + 2aS}$. But here u=0

$$\therefore v^2 = 2aS$$

By geometry, $\sin\theta = h/S$

and so $S = h/\sin\theta$

$$\therefore v^2 = \frac{2ah}{\sin\theta}$$

Or Acceleration,

$$a = \frac{v^2 \sin\theta}{2h}$$

Substituting the value

$$V_{cm} = \sqrt{\frac{2gh}{\left[1 + \frac{K^2}{R^2}\right]}}$$

Acceleration,

$$a = \frac{2gh}{\left[1 + \frac{K^2}{R^2}\right]} \frac{\sin\theta}{2h}$$

$$= \frac{g\sin\theta}{\left[1 + \frac{K^2}{R^2}\right]}$$

54P. a) If the acceleration of a rolling body through an inclined plane is given by $a = \left(\frac{g\sin\theta}{1 + \frac{K^2}{R^2}}\right)$, find the acceleration of a sphere of mass M and radius R.

b) Using the above equation find the acceleration of disc of radius **R** and mass **M**. If a student allows the sphere and disc to roll down simultaneously, which will reach down first? Give the reason

Ans:

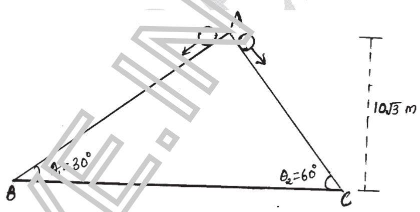
55P. A solid sphere, a ring and a circular disc of identical radii are rolling down an inclined plane without slipping, from the same height, starting from rest.

(a) Which will reach the bottom first?

(b) Which will reach the bottom last?

Ans:

56P. Two inclined frictionless tracks, one gradual and other steeper meet at A as shown in figure. Two spheres of identical mass are allowed to fall down from rest along each track. ($g=10 m/s^2$)



a) Which sphere will reach at the bottom earlier? Explain

b) What is the time taken by this sphere to reach the bottom?

c) Find the ratio of kinetic energies of the two spheres when they reach at the bottom.

Ans:

57P. A ring of radius **2m** weighs **100 kg**. It rolls along a horizontal floor so that its centre of mass has a speed of **20 cm/s**. What is the total kinetic energy?

Ans:

58P. The translational kinetic energy of a rolling body is E. Then the total kinetic energy of the body is