CHAPTER 15

WAVES

Waves are the method of transfer of energy from one part of a medium to another without the actual flow of matter as a whole.

There are three types of waves:

1. Mechanical waves
2. Electromagnetic waves
3. Matter waves

Mechanical waves:-

They are the waves which necessarily need a medium for their propagation.

Eg :- *Waves on a stretched string, water waves, sound waves etc.*

Mechanical waves cannot propagate through vacuum.

Electromagnetic waves :- They are non-mechanical waves which do not require a medium for their propagation. They can travel through vacuum.

There are seven electromagnetic waves:

RMI LUX G

R → Radio waves

M →Microwaves

I→ Infrared Radiations

L →Visible light

U →Ultraviolet Radiations

X →X-Rays

G →Gamma rays

In vacuum all the electromagnetic waves have the same speed of **$c=3x10^8$ m/s**.

Matter waves (or De-Broglie waves):-

They are the waves associated with material particles.

Electrons, protons, neutrons, atoms and molecules have wave like motion. Wave nature is negligible for massive bodies like a cricket ball.

MECHANICAL WAVES

Mechanical waves can be divided into two:

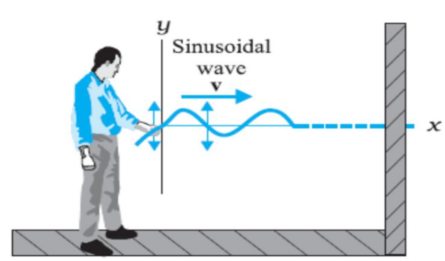
1. **Transverse waves** and
2. **Longitudinal waves.**

The motion of mechanical waves is by the oscillations (or vibrations) of particles of the medium.

Transverse waves :-

If the direction of vibration of the particles of the medium and direction of propagation of the wave are perpendicular, then the wave is called a transverse wave.

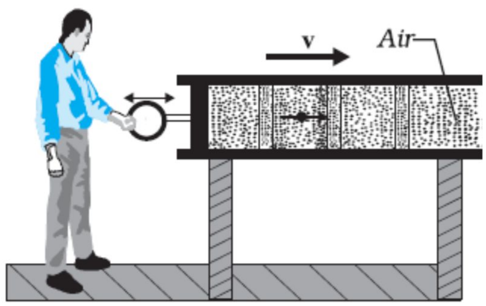
Eg:- Harmonic wave travelling along a stretched string.



Longitudinal waves:-

If the direction of the vibration of the particle and direction of the propagation of the wave are parallel, then the wave is called a longitudinal wave.

Eg:- Propagation of sound through a a medium is longitudinal in nature.



Progressive waves:-

The waves travelling from one part of the medium to another are called travelling or progressive waves.

Displacement Relation of a progressive wave:-

To represent a travelling wave, we need a function of position '**x**' and time'**t**'.

A transverse wave travelling in the **+X** direction can be represented as:

$$\boxed{y(x,t) = a \sin (kx - \omega t + \varphi)}$$

A transverse wave travelling in the **-X** direction can be represented as:

$$\boxed{y(x,t) = a \sin (kx + \omega t + \varphi)}$$

y(x,t) → Displacement as a function of position '**x**' and time '**t**'

a → Amplitude of a wave.

ω → Angular frequency of the wave.

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi\nu}$$

k → Angular wave number $\boxed{k = \frac{2\pi}{\lambda}}$

kx-ωt+ φ→ phase angle.

For a **longitudinal wave**, the displacement of an element of the medium is parallel to the direction of propagation of the wave. **The displacement relation for a longitudinal wave travelling in the +X direction can be written as**

$$\boxed{S(x,t) = a \sin (kx - \omega t + \varphi)}$$

Amplitude (a)

Amplitude is the maximum displacement of the particles of the medium from their equilibrium position.
Displacement (**y**) may be positive or negative but 'a' is always positive.

Phase

The quantity kx-ωt+φ appearing as the argument of the sine function is called the phase of the wave. φ is the phase at x=0 and t=0. Hence φ is called the initial phase angle.

Wavelength (λ)

Wavelength is the distance between two consecutive crests or troughs in a wave.

The Speed of a Travelling Wave (Wave equation)

Speed, $\mathbf{v} = \upsilon\lambda = \frac{\lambda}{T}$

The speed of the wave depends on the medium.

The equation $\boxed{\mathbf{v} = \upsilon\lambda}$

is called the **wave equation.**

Speed of a transverse wave on a stretched string

Speed of a transverse wave on a stretched string $\boxed{v = \sqrt{\frac{T}{\mu}}}$

T is the tension in the string

μ is the linear mass density

$$\mu = \frac{\textit{mass of the string}}{\textit{length of the string}}$$

$\mu = \frac{m}{l}$ → S.I unit is kg/m

Dimension is ML^{-1}

Speed of a longitudinal wave

(speed of sound)

The general formula for velocity of longitudinal waves in a medium is

$$\boxed{v = \frac{B}{\rho}}$$

B → Bulk modulus,

ρ→ density of the medium.4

The speed (v) of longitudinal waves in a solid bar is $v = \sqrt{\frac{Y}{\rho}}$

Y is the Young's modulus of the medium.

Note:- Bulk moduli of solids and liquids are greater than those of gases. Therefore, speed of sound is greater in solids and liquids than that in gases.

Medium	Speed (m/s)
Air (20^0c)	343
Water (20^0c)	1482
Aluminium	6420
Steel	5941

Speed of sound in a gas

According to Newton the propagation of sound through a gas is an **isothermal process**.

For a gas **B = P** (For an isothermal Process)

P = Pressure

Equation for an isothermal process is **PV= constant**

$\therefore\ P\Delta V + V\Delta P = 0$

$$P\Delta V = -V\Delta P$$

$$P = -\frac{V\Delta P}{\Delta V}$$

$$P = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = B, \text{Bulk modulus}$$

∴ The speed of sound (longitudinal wave) in an ideal gas is $v = \sqrt{\frac{P}{\rho}}$, this formula was first given by Newton and is known as **Newton's formula**.

When we calculate the velocity of sound in air at STP using the above formula, the obtained value is **280m/s,** which is about **15%** smaller as compared to the experimental value.

But Laplace pointed out that the propagation of sound through a gas is not isothermal but it is an **adiabatic process** (because the variation of pressure in the propagation of sound is very fast).

For an adiabatic process

$$\mathbf{PV^{\gamma} = constant}$$

$$\Delta\left(PV^{\gamma}\right) = 0$$

$$P\gamma V^{\gamma-1}\Delta V + V^{\gamma}\Delta P = 0$$

$$P\gamma V^{\gamma-1}\Delta V = -V^{\gamma}\Delta P$$

$$\gamma P\frac{V^{\gamma-1}}{V^{\gamma}} = -\frac{\Delta P}{\Delta V}$$

$$\gamma P = -\frac{\Delta P}{\Delta V}V$$

$$\gamma P = B, \text{Bulk modulus}$$

∴ Velocity of sound is an ideal gas

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}}$$

. This formula is known as **Newton-Laplace formula**.

γ is the ratio of specific heat capacities.

$$\gamma = \frac{C_p}{C_v}$$

Using the above formula, the velocity of sound in air at STP is obtained as **331.3m/s** which is very close to the experimental value.

<u>Principle of Superposition of waves</u>

Superposition principle states that "**when a no. of waves meet at a particular point in a medium, each wave produces its own displacement independent of the other and the total displacement is the algebraic sum of displacement due to individual waves**".

$$\mathbf{y(x,t) = y_1(x,t) + y_2(x,t) + \text{-------}}$$

<u>Explanation:-</u>

Consider two harmonic travelling waves on a stretched string both having the same frequency and same amplitude but with different initial phase. Let the waves are travelling along the positive direction of x-axis.

$\mathbf{y_1(x,t) = a\sin(kx-\omega t)}$ and

$\mathbf{y_2(x,t) = a\sin(kx-\omega t+\phi)}$

The net displacement is given by the principle of superposition,

$y(x,t)=y_1(x,t)+y_2(x,t)$

$= a\sin(kx-\omega t) + a\sin(kx-\omega t+\phi)$

$= a[\sin(kx-\omega t) + \sin(kx-\omega t+\phi)]$

$$= a\left[2\sin\left(kx-\omega t+\frac{\phi}{2}\right)\cos(-\phi/2)\right]$$

$= 2a\sin(kx-\omega t+\phi/2)\cos(\phi/2)$

$$\boxed{y(x,t) = 2a\cos(\phi/2)\sin(kx-\omega t+\phi/2)}$$

This also represents a travelling wave in the positive direction of x-axis, with the same frequency and wavelength. But the initial phase angle is $\phi/2$. The amplitude of the resultant wave is

$$\boxed{A(\phi)=2a\cos\phi/2}$$

Special cases

(i) **when $\phi=0$**, the waves are in phase.

$\therefore$ $y(x,t) = 2a\sin(kx-\omega t)$

Amplitude = 2a, which is the largest possible value for A.

This is known as **constructive interference**, in which amplitudes are added.

(ii) For $\phi=\pi$ the waves are completely out of phase.

$\therefore$ $y(x,t) = 0$, the resultant wave has zero displacement everywhere at all times.

This is known as **destructive interference**, in which amplitudes are subtracted.

Reflection of waves

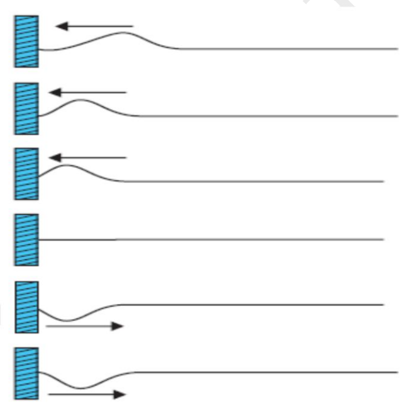
(i) Reflection at a rigid boundary

When a wave is reflected at a rigid boundary, there will be a phase difference of π between the incident and reflected waves.

If $y_i(x,t) = a\sin(kx-\omega t)$ is the incident wave then, the reflected wave,

$$y_r(x,t) = a\sin(kx+\omega t+\pi)$$

$$= -a\sin(kx+\omega t)$$



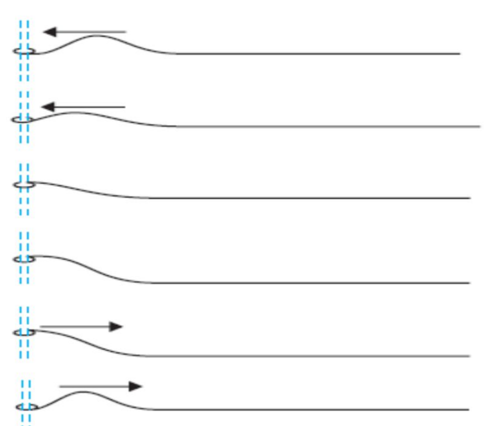
(ii) Reflection at an open boundary(free boundary)

For reflection at an open boundary, the reflected wave is represented as

$$y_r(x,t) = a\sin(kx+\omega t+0)$$

$$= a\sin(kx+\omega t)$$

Here there is no phase difference between the incident wave and reflected wave.



STANDING WAVES AND NORMAL MODES

Consider a system which is bounded at both the ends such as a stretched string fixed at the ends or an air column of finite length. In such a system suppose that we send a continuous sinusoidal wave of a certain frequency toward the right. When the wave reaches the right end, it gets reflected and begins to travel back. The left going wave then overlaps the wave, travelling to the right. When the left going wave reaches the left end, it gets reflected again and the newly reflected wave begins to travel to the right, overlapping the left going wave. This process will continue and, therefore, very soon we have many overlapping waves.

In such a system, at any point x and at any time t, there are always two waves, one moving to the left and other moving to the right.

$\therefore$ we have,

$y_1(x,t) = a \sin(kx - \omega t)$ and

$y_2(x,t) = a \sin(kx + \omega t)$

By the principle of super position, we have the combined wave

$y(x,t) = y_1(x,t) + y_2(x,t)$

$= a \sin(kx - \omega t) + a \sin(kx + \omega t)$

$= a[\sin(kx-\omega t) + \sin(kx + \omega t)]$

$$= a\left[2\sin\left(\frac{kx-\omega t+kx+\omega t}{2}\right)\cos\left(\frac{kx-\omega t-kx-\omega t}{2}\right)\right]$$

$= 2a \sin kx \cos(-\omega t)$

$$\boxed{y(x,t) = 2a \sin kx \cos \omega t}$$

But **y(x,t)** does not represent a travelling wave since it does not contain the terms **kx-ωt** or **kx+ωt**. It represents a standing wave, a wave in which the wave form does not move.

Here the amplitude of the wave is **2a sin kx**, which is a function of **x**. That is the amplitudes are different at different points.

Amplitude is zero for

$\sin kx = 0$

$\Rightarrow kx = n\pi$, for n= 0,1,2,3,......

substituting $k = \frac{2\pi}{\lambda}$

$$\frac{2\pi}{\lambda}x = n\pi \Rightarrow \boxed{x = \frac{n\lambda}{2}}, \text{ for } n = 0,1,2,3,.....$$

The positions of zero amplitude are called nodes. The distance between two consecutive nodes is $\frac{\lambda}{2}$.

Amplitude has maximum value of **2a** for

$|\sin kx| = 1$

$$\Rightarrow kx = (2n+1)\frac{\pi}{2}, \text{ for } n = 0,1,2,3,...$$

$$\frac{2\pi x}{\lambda} = (2n+1)\frac{\pi}{2}, \text{ for } n = 0,1,2,3,...$$

$$\boxed{x = (2n+1)\frac{\lambda}{4}}$$

The positions of maximum amplitude are called antinodes. The distance between two consecutive antinodes is $\frac{\lambda}{2}$.

(i) Standing waves on a stretched string

Consider a string stretched between two rigid supports. When string is excited stationary waves are produced. But string can vibrate only in some definite patterns. There patterns are called **normal modes.**

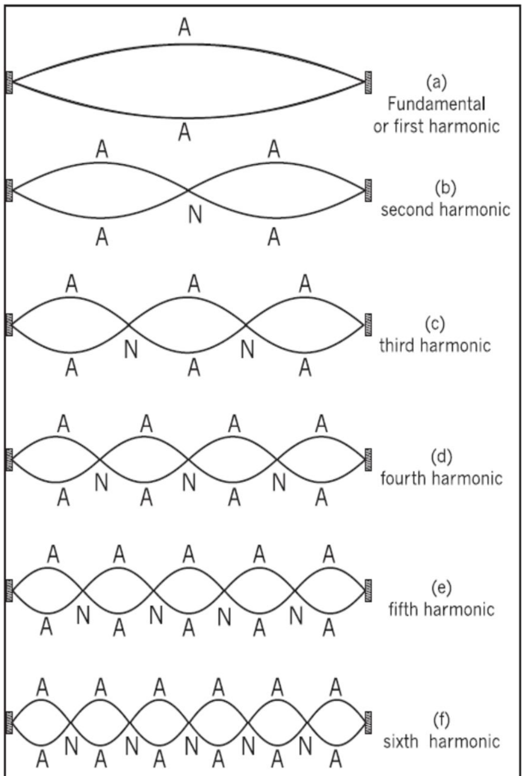


Fig. 15.13 Stationary waves in a stretched string fixed at both ends. Various modes of vibration are shown.

First mode of vibration

Here $L = \frac{\lambda_1}{2}$

$\Rightarrow \lambda_1 = 2L$

Frequency of vibration,

$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$, this frequency is called **fundamental frequency** or **first harmonic**.

Speed of transverse waves in a stretched string is $v = \sqrt{\frac{T}{\mu}}$

$$\therefore \upsilon_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Second mode of vibration

$L = \lambda_2$

$$\upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$= 2\left(\frac{v}{2L}\right) = 2\upsilon_1$, This frequency is called **second harmonic** or **first overtone**.

Third Mode of vibration

Here, $L = \frac{3\lambda_3}{2}$

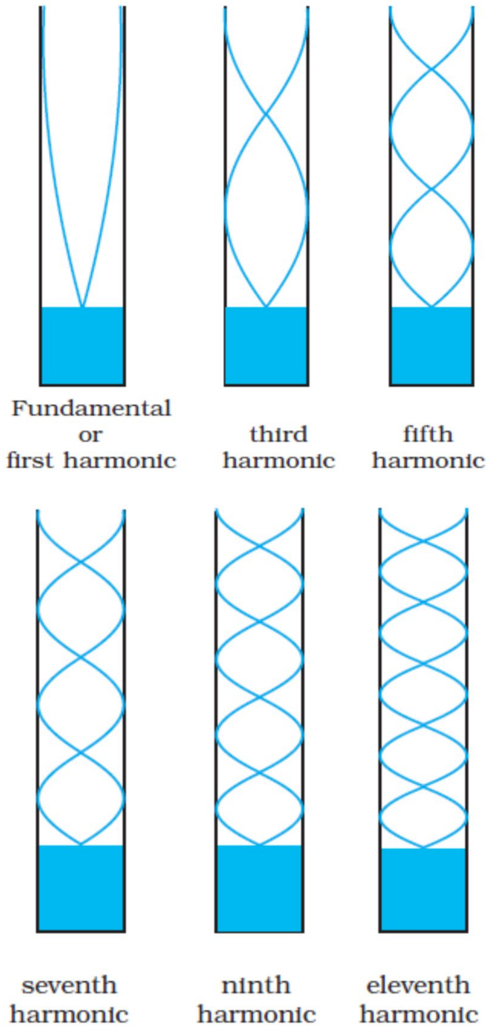
$$\Rightarrow \lambda_3 = \frac{2}{3}L$$

$$\upsilon_3 = \frac{v}{\lambda_3} = 3\left(\frac{v}{2L}\right) = 3\upsilon_1$$

This frequency $\upsilon_3 = 3\upsilon_1$ is called **3rd harmonic** or **second overtone**. Thus for an open pipe all harmonics are present.

ie, $\upsilon_3 : \upsilon_3 : \upsilon_3 : ---- = 1:2:3: ----$

(ii) Standing waves in a closed pipe



First mode of vibration

Here, Length of the pipe L= $\frac{\lambda_1}{4}$

$\Rightarrow \lambda_1 = 4L$

Frequency of vibration,

$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$, this frequency is called **fundamental frequency** or **first harmonic**.

Speed of sound waves in a gas is

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\therefore \upsilon_1 = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}}$$

Second mode of vibration

$L=3\lambda_2/4 \quad \Rightarrow \lambda_2 = \frac{4}{3}L$

$$\upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{\frac{4}{3}L} = 3\frac{v}{4L} = 3\upsilon_1$$

This frequency is called **third harmonic** or **first overtone**.

Third Mode of vibration

Here, $L = \frac{5\lambda_3}{4}$

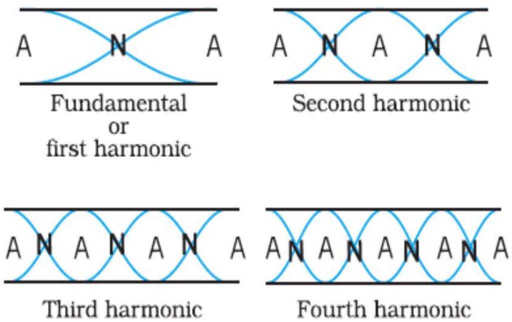
$\Rightarrow \lambda_3 = \frac{4}{5}L$

$$\upsilon_3 = \frac{v}{\lambda_3} = 5\left(\frac{v}{4L}\right) = 5\upsilon_1$$

This frequency $\upsilon_3 = 5\upsilon_1$ is called **5th harmonic** or **second overtone**. Thus for a closed pipe only **odd harmonics** are present.

ie, $\upsilon_3 : \upsilon_3 : \upsilon_3 : ---- = 1:3:5: ----$

(iii) Standing waves in an open pipe



First mode of vibration

Here, Length of the pipe L= $\frac{\lambda_1}{2}$

$$\Rightarrow \lambda_1 = 2L$$

Frequency of vibration,

$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$, this frequency is called **fundamental frequency** or **first harmonic**.

Speed of sound waves in a gas is $v = \sqrt{\frac{\gamma P}{\rho}}$

$$\therefore \upsilon_1 = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}}$$

Second mode of vibration

$L=\lambda_2$

$$\upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$= 2\left(\frac{v}{2L}\right) = 2\upsilon_1$, This frequency is called **second harmonic** or **first overtone**.

Third Mode of vibration

$$\text{Here, } L = \frac{3\lambda_3}{2}$$

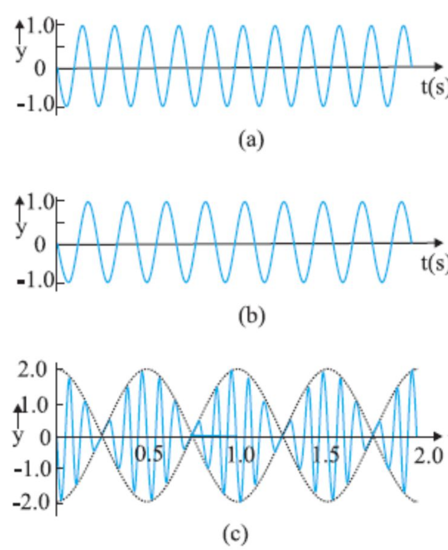
$$\Rightarrow \lambda_3 = \frac{2}{3}L$$

$$\upsilon_3 = \frac{v}{\lambda_3} = 3\left(\frac{v}{2L}\right) = 3\upsilon_1$$

This frequency $\upsilon_3 = 3\upsilon_1$ is called **3rd harmonic** or **second overtone**. Thus for an open pipe all harmonics are present.

ie, $\upsilon_3 : \upsilon_3 : \upsilon_3 : ----= 1:2:3: ----$

Beats



When two sound waves of nearly same amplitudes and slightly different frequencies, travelling in the same direction are

superimposed on each other alternate increase and decrease in the sound intensity are heard. This phenomenon is called beats.

→ Increase in sound intensity is called **waxing**.

→ Decrease in sound intensity **waning**.

→ **1 beat = 1 waxing + 1 waning**.

Derivation of expression for beat Frequency:-

Let us consider two sound waves having slightly different frequencies $S_1 = a\cos\omega_1 t$ and $S_2 = a\cos\omega_2 t$, where $\omega_1 > \omega_2$ travelling in the same directions are superimposed on each other.

According to superposition principle, the resultant displacement is

$S = S_1 + S_2 = a\cos\omega_1 t + a\cos\omega_2 t$

$= a[\cos\omega_1 t + \cos\omega_2 t]$

$$= a\left[2\cos\frac{(\omega_1+\omega_2)t}{2}\cos\frac{(\omega_1-\omega_2)t}{2}\right]$$

$$= 2a\cos\frac{(\omega_1+\omega_2)t}{2}\cos\frac{(\omega_1-\omega_2)t}{2}$$

If $|\omega_1 - \omega_2| << \omega_1$ and ω_2, then the variations in the cosine term with time is very small. So it can be taken along with the amplitude.

$$S = \left(2a\cos\frac{(\omega_1-\omega_2)t}{2}\right)\cos\frac{(\omega_1+\omega_2)t}{2}$$

$S = a'\cos\frac{(\omega_1+\omega_2)t}{2}$, where a' is the amplitude.

$$a' = 2a\cos\frac{(\omega_1-\omega_2)t}{2}$$

Amplitude **a′** is maximum when,

$$\cos\frac{(\omega_1-\omega_2)t}{2} = \pm 1$$

$$\Rightarrow \frac{(\omega_1-\omega_2)t}{2} = 2n\pi$$

$$\Rightarrow t = \frac{2n\pi}{\omega_1-\omega_2}$$

$$\Rightarrow t = \frac{2n\pi}{(2\pi\nu_1 - 2\pi\nu_2)}$$

$$\Rightarrow t = \frac{n}{(\nu_1-\nu_2)}, n = 0,1,2,3,---$$

$$\Rightarrow t = 0, \frac{1}{(\nu_1-\nu_2)}, \frac{2}{(\nu_1-\nu_2)}, ---$$

∴ Time period of beat,

$$T = \frac{1}{(\nu_1-\nu_2)} - 0$$

$$= \frac{1}{(\nu_1-\nu_2)}$$

∴ Beat frequency, $\nu_{beat} = \frac{1}{T} = \nu_1 - \nu_2$

$$\boxed{\nu_{beat} = |\nu_1 - \nu_2|}$$

Doppler Effect in Sound

The apparent change in the frequency (or pitch) of the sound when the source and the listener move relative to each other is called Doppler effect.

When the source and the listener approach (move towards each other), the apparent frequency will be greater than the actual frequency.

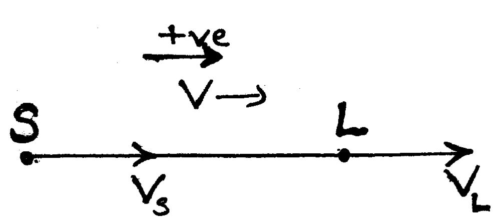
When they recede (move away from each other), the apparent frequency will be less than the actual frequency.

Expression for apparent frequency

Consider a source of frequency ‘ν’. Let V be the velocity of sound in the medium and λ the wave length of sound, when the source and the listener are at rest.

Then the frequency of sound heard by the listener is $\nu = \frac{V}{\lambda}$

Let the source and listener move with velocities V_S and V_L in the direction of propagation of sound from source to listener. The direction S to L is taken as positive.



The relative velocity of sound wave with respect to the source

$$= V - V_S$$

Apparent wave length of sound is

$$\lambda' = \frac{V - V_S}{\nu}$$

Since the listener is moving with a velocity V_L , the relative velocity of the sound with respect to the listener is $V' = V - V_L$

Apparent frequency of sound as heard by the listener is given by,

$$\nu' = \frac{\text{Relative velocity of sound w.r.t. the listener}}{\text{Apparent wavelength}}$$

$$= \frac{V'}{\lambda'} = \frac{V - V_L}{\left(\frac{V - V_S}{\nu}\right)} = \nu\left(\frac{V - V_L}{V - V_S}\right)$$

$$\boxed{\nu' = \nu\left(\frac{V - V_L}{V - V_S}\right)}$$

Case I : Listener at rest and source in linear motion

(a) When the source moves towards the stationary listener, V_S is positive and $V_L=0$

$$\nu' = \nu\left(\frac{V}{V - V_S}\right)$$

That is apparent frequency greater than the actual frequency.

(b) When the source moves away from the stationary listener, V_S is negative and $V_L=0$

$$\nu' = \nu\left(\frac{V}{V+V_S}\right)$$

That is apparent frequency less than the actual frequency.

Case II : Listener in motion and source at rest.

(a) When the listener moves towards the stationary source, V_L is negative and V_S =0

$$\nu' = \nu\left(\frac{V+V_L}{V}\right)$$

That is apparent frequency greater than the actual frequency.

(b) When the listener moves away from the stationary source, V_L is positive and V_S =0

$$\nu' = \nu\left(\frac{V-V_L}{V}\right)$$

That is apparent frequency less than the actual frequency.

Case III : When both the source and listener are in motion

(a) When the source and listener move towards each other, V_S is positive and V_L is negative.

$$\nu' = \nu\left(\frac{V+V_L}{V-V_S}\right)$$

That is apparent frequency is greater than the actual frequency.

(b) When the source and listener move away from each other, V_S is negative and V_L is positive.

$$\nu' = \nu\left(\frac{V-V_L}{V+V_S}\right)$$

That is apparent frequency is less than the actual frequency.

(c) When the source moves away from the listener and the listener moves towards the source, V_S is negative and V_L negative.

$$\nu' = \nu\left(\frac{V+V_L}{V+V_S}\right)$$

(d) When the listener moves away from the source and the source moves towards the listener, V_S is positive and V_L positive.

$$\nu' = \nu\left(\frac{V-V_L}{V-V_S}\right)$$

Effect of motion of the medium

When a wind is blowing in the direction of propagation of sound, the resultant velocity of sound will be V+W.

$$\nu' = \nu\left(\frac{V+W-V_L}{V+W-V_S}\right)$$

Applications of Doppler Effect

1. To estimate the speed of a submarine.

2. To estimate the speed of aeroplane, automobile etc.

3. To track artificial satellites.

4. To estimate the velocity and rotation of sun.

Problems

1. A transverse harmonic wave on a string is described by

$$y(x,t)=3.0\sin(36t+0.018x+\frac{\pi}{4})$$

where x and y are in centimetres and t is in seconds.

(a) Is it a travelling or stationary wave?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) If it is a travelling wave, what are the speed and direction of its propagation?

Ans:

2. Doctors use an ultrasonic scanner to diagnose tumour tissues. If the frequency of the scanner is 4.2 MHz and the speed of sound wave in the tissue is 1.7 km/s, find the wave length of sound wave.

Ans:

3. Open pipes are preferred to closed ones in musical instruments. Why?

Ans: In open pipes all harmonics are present but in closed pipes only odd harmonics are present.

4. a) A pipe of 30 cm long is open at both ends. Which harmonic mode of the pipe is resonantly exerted by a 1.1 kHz source? (Take speed of sound in air to be 330 m/s)

b) Will resonance with the same source be observed if one end of the pipe is closed?

Ans:

5. A violin string resonates in its fundamental frequency of 196 Hz. Where along the string must you place your finger so that the fundamental frequency becomes 440 Hz, if the length of the violin string is 40 cm?

Ans:

6. We can have transverse and longitudinal waves in solids. But we can have only longitudinal waves in fluids. Why?

Ans: Transverse waves can propagate only through such media, which can produce shearing stress. Fluids cannot produce shearing stress.

7. State whether sound is propagated as longitudinal or transverse wave in a string.

Ans: In a string sound is propagated as transverse waves.

8. Name the type of wave produced in an air column.

Ans: Longitudinal stationary wave.

9. If the apparent frequency of the whistle of an engine changes in the ratio 5:4 as the engine passes a man at rest in the railway station, find the velocity of the train.(speed of sound is 340m/s)

Ans:

10. A source of sound of frequency 256 Hz is in between a listener and a wall. If the source is moving towards the wall with a velocity of 5 m/s, how many beats per second will be heard if the sound travels with a speed of 330m/s.

Ans: