Sample Questions

Question :

Write the set in roster form :

A = {x : x is an integer and $-5 < x < 8$}.

Solution :

Roster form :

A = {$-4, -3 - 2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8$}

Question :

Write the set in the set – builder form :

{5, 25, 125, 625}

Solution :

It can be seen that

$5 = 5^1$, $25 = 5^2$, $125 = 5^3$, and $625 = 5^4$.

$\therefore$ {5, 25, 125, 625} = {x : x $= 5^n$, n $\in$ N and 1 $\leq$ n $\leq$ 4}

Question :

List all the elements of the set :

A = {x : x is a month of a year not having 31 days}

Solution :

A = {x : x is a month of a year not having 31 days}

A = {February, April, June, September, November}

Question :

Which of the following are examples of the null set:

(1) Set of odd natural numbers divisible by 2

(2) Set of even prime numbers

(3) {x : x is a natural numbers, $x < 5$ and $x > 7$}

(4) {y : y is a point common to any two parallel lines}

Solution :

(1) a null set

(2) not a null set

(3) a null set

(4) a null set

Question :

State whether each of the following set is finite or infinite:

(1) The set of lines which are parallel to the x – axis.

(2) The set of letters in the English alphabet.

(3) The set of numbers which are multiple of 5.

(4) The set of animals living on the earth.

(5) The set of circles passing through the origin $(0, 0)$

Solution :

(1)   an infinite set

(2)   a finite set

(3)   an infinite set

(4)   a finite set

(5)   an infinite set

Question :

Are the pair of sets equal? Give reasons.

$A = \{5, 2\}$;

$B = \{x : x \text{ is solution of } x^2 + 7x + 10 = 0\}$

Solution :

The equation $x^2 + 7x + 10 = 0$ can be solved as :

$$x^2 + 5x + 2x + 10 = 0$$

$$x(x+5) + 2(x+5) = 0$$

$$(x+5)(x+2) = 0$$

$$x = -5 \quad \text{or} \quad x = -2$$

$\therefore \qquad A = \{5, 2\}$;

$$B = \{-5, -2\}$$

$\therefore \qquad A \neq B$

Question :

Write down all the subsets of the following sets :

(1) $\{x\}$ (2) $\{x, y\}$

(3) $\{7, 8, 9\}$ (4) Φ

Solution :

(1) The subsets of $\{x\}$ are Φ and $\{x\}$.

(2) The subsets of $\{x,y\}$ are $\Phi, \{x\}, \{y\},$ and $\{x,y\}$.

(3) The subsets of $\{7, 8, 9\}$ are $\Phi, \{7\}, \{8\}, \{9\}, \{7, 8\}, \{8, 9\},$
$$\{7, 9\}, \text{and} \{7, 8, 9\}$$

(4) The only subset of Φ is Φ.

Question :

If $A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$, $C = \{5,6,7,8\}$ and $D = \{7,8,9,10\}$;
find

(1) $A \cup B$ (2) $A \cup C$ (3) $B \cup C$ (4) $B \cup D$

(5) $A \cup B \cup C$ (6) $A \cup B \cup D$ (7) $B \cup C \cup D$

Solution :

$A = \{1,2,3,4], B = \{3,4,5,6\}, C = \{5,6,7,8\}$ and $D = \{7,8,9,10\}$

(1) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(2) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(3) $B \cup C = \{3, 4, 5, 6, 7, 8\}$

(4) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

(5) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(6) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(7) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

Question :

If $A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}$ and $D = \{15,17\}$;
find

(1) $A \cap B$ (2) $B \cap C$ (3) $A \cap C \cap D$ (4) $A \cap C$

(5) $B \cap D$ (6) $A \cap (B \cup C)$ (7) $A \cap D$ (8) $A \cap (B \cup D)$

(9) $(A \cap B) \cap (B \cup C)$ (10) $(A \cup D) \cap (B \cup C)$

Solution :

(1) $A \cap B = \{7,9,11\}$

(2) $B \cap C = \{11,13\}$

(3) $A \cap C \cap D = \{A \cap C\} \cap D = \{11\} \cap \{15,17\} = \Phi$

(4) $A \cap C = \{11\}$

(5) $B \cap D = \Phi$

(6) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \{7,9,11\} \cup \{11\} = \{7,9,11\}$

(7) $A \cap D = \Phi$

(8) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D) = \{7,9,11\} \cup \Phi = \{7,9,11\}$

(9) $(A \cap B) \cap (B \cup C) = \{7,9,11\} \cap \{7,9,11,13,15\} = \{7,9,11\}$

(10) $(A \cup D) \cap (B \cup C)$

$= \{3,5,7,9,11, 15,17) \cap \{7,9,11,13,15\} = \{7, 9, 11, 15\}$

Question :

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7, 9\}, B = \{1, 5, 6, 8\}, C = \{1, 4, 6, 7\}$

Verify the following,

(1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(2) $(A \cup B)' = A' \cup B'$

(3) $A - B = A \cap B'$

Solution :

(1) Here $B \cap C = \{1, 6\}$

$A \cup (B \cap C) = \{1, 3, 5, 6, 7, 9\}$

Now, $A \cup B = \{1, 3, 5, 6, 7, 8, 9\}$

$A \cup C = \{1, 3, 4, 5, 6, 7, 9\}$

$(A \cup B) \cap (A \cup C) = \{1, 3, 5, 6, 7, 9\}$

Thus, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(2) $A \cup B = \{1, 3, 5, 6, 7, 8, 9\}$

$\therefore$ $(A \cup B)' = \{2, 4, 10\}$

Now $A' = \{2, 4, 6, 8, 10\}$

$B' = \{2, 3, 4, 7, 9, 10\}$

$\therefore$ $A' \cap B' = \{2, 4, 10\}$

Thus, $(A \cup B)' = A' \cap B'$

(3) $A - B = \{3, 7, 9\}$

$B' = \{2, 3, 4, 7, 9, 10\}$

$A \cap B' = \{3, 7, 9\}$

Hence $A - B = A \cap B'$

Question :

Prove that $\quad A - B \;=\; A - (A \cap B)$

Solution :

We have, by definition,

$$A - B \;=\; A \cup B'$$
$$A - (A \cap B) \;=\; A \cap (A \cap B)'$$
$$= A \cap (A' \cup B')$$
$$= (A \cap A') \cup (A \cap B')$$
$$= 0 \cup (A \cap B')$$
$$= A \cap B'$$
$$= A - B$$

Question :

In a troop of 20 dancers performing Bharatnatyam or Kuchipudi, 12 dancers perform Bharatnatyam and 4 perform both Bharatnatyam and Kuchipudi. Find the number of dancers performing Kuchipudi.

Solution :

Let

A = Set of dancers performing Bharatnatyam

B = Set of dancers performing Kuchipudi

Then given that,

$$n(A) \;=\; 12, \; n(A \cap B) \;=\; 4, \; n(A \cup B) \;=\; 20$$

Now, $\quad n(A \cup B) \;=\; n(A) + n(B) - n(A \cap B)$

$\therefore \qquad\quad 20 \;=\; 12 + n(B) - 4$

$\qquad\qquad\quad 20 \;=\; n(B) + 8$

$\therefore \qquad\quad n(B) \;=\; 12$

Thus number of dancers performing Kuchipudi is 12.

$\therefore$ Number of Dancers performing only Kuchipudi

$$= n(B) - n(A \cap B) \;=\; 12 - 4 \;=\; 8$$

Question :

In a group of people, 28 like Malayalam movies, 30 like Hindi movies, 42 like English movies; 5 like both Malayalam and Hindi

movies, 8 like Hindi and English movies, 8 like Malayalam and English movies and 3like Malayalam, Hindi and English movies. What is the least number of people in the group ?

Solution :

$n(M) = 28, n(H) = 30, n(E) = 42, n(M \cap H) = 5,$

$n(E \cap H) = 8, \quad n(M \cap E) = 8, \quad n(M \cap E \cap H) = 3$

Now,

$n(M \cup E \cup H)$

$\quad = n(M)+n(H)+n(E)-n(M \cap H)-n(E \cap H)-n(M \cap E)+n(M \cap E \cap H)$

$\quad = 28 + 30 + 42 - 5 - 8 - 8 + 3$

$\quad = 103 - 21$

$\quad = 82$

Question :

In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Solution :

Let H be the set of people who speak Hindi, and E be the set of people who speak English.

$\therefore n(H \cup E) = 400, n(H) = 250, n(E) = 200, \quad n(H \cap E) = ?$

We know that :

$\quad n(H \cup E) = n(H)+n(E) - n(H \cap E)$

$\quad \therefore 400 = 250 + 200 - n(H \cap E)$

$\quad\quad 400 = 450 - n(H \cap E) \Rightarrow n(H \cap E)$

$\therefore n(H \cap E) = 450 - 400 = 50$

Thus, 50 people can speak both Hindi and English.

Question :

In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution :

Let C denote the set of people who like coffee, and T denote the set of people who like tea.

$$n(C \cup T) = 70, n(C) = 37, n(T) = 52$$

We know that :

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

$$\therefore \ 70 = 37 + 52 - n(C \cap T)$$

$$70 = 89 - n(C \cap T)$$

$$n(C \cap T) = 89 - 70 = 19$$

Thus, 19 people like both coffee and tea.

Question :

In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages ?

Solution :

Let F be the set of people in the committee who speak French, and S be the set of people in the committee who speak Spanish

$$\therefore n(F) = 50, n(S) = 20, n(S \cap F) = 10$$

$$n(S \cup F) = n(S) + n(F) - n(S \cap F) = 20 + 50 - 10 = 70 - 10 = 60$$

Thus, 60 people in the committee speak at least one of the two languages.

Question :

Which of the following pairs of sets are disjoint

(i) {1, 2, 3, 4} and {x : x is a natural number and $4 \leq x \leq 6$}

(ii) {a, e, i, o, u} and {c, d, e, f}

Solution :

(i) {1, 2, 3, 4} ∩ {4, 5, 6} = {4}

Therefore, this pair of sets is not disjoint.

(ii) {a, e, i, o, u} ∩ (c, d, e, f} = {e}

∴ {a, e, i, o, u} and (c, d, e, f} are not disjoint.

Question :

If A = {3, 6, 9, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20},

C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20};

find (i) A − B (ii) A − C (iii) A − D (iv) B − A

<u>Solution :</u>

(i) A – B = {3, 6, 9, 15, 18, 21}

(ii) A – C = {3, 9, 15, 18, 21}

(iii) A – D = {3, 6, 9, 12, 18, 21}

(iv) B – A = {4, 8, 16, 20}

EXERCISE

1.

a) If A is a subset of the set B, then A∩B =..........

b) Represent the above set A∩B by Venn diagram

c) In a school, there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics, 12 teach physics, how many teach both the subjects.

(March 2016)

Hint or Answer:

a) A

b) Figure

c) n (M ∩ P)= n(M) + n(P) – n(M∪P) = 4

2.

a) A = {x: x is a prime number, $x \leq 6$}

  i. Represent A in roster form

  ii. Write the power set of A.

b) Out of 25 members in an office 17 like to take tea, 16 like to take coffee. Assume that each takes at least one of the two drinks. How many like:

  i. Both coffee and tea?

  ii. Only tea and not coffee?

(Imp 2015)

Hint or Answer:

a) i. A = {2,3,5}

  ii. P(A) = { Φ,{2},{3},{5},{2,3},{2,5},{3,5},{2,3,5}}

b) i. 8

  ii. 17-8=9

3.

Let A = {x: x is a prime number < 5} , and U = {x: x is an integer, $0 \leq x \leq 6$}

  a) Write A, B in roster form

b) Find (A-B) ∪ (B-A)

c) Verify that (A ∪ B)′ = A′ ∩ B′

(March 2015)

Hint or Answer:

a) A = {0, 1, 2, 3, 4} ; B = {0, 1, 2, 3, 4, 5, 6}

b) {0, 1, 4}

c) {5, 6}

4.

a) If two sets A and B are disjoint, which one among the following is true?

(A∪B)=A ; (A∪B)=B ; (A∩B)=B ; (A∩B)= Φ

b) Find the solution set of the equation $x^2 + x - 2$ in roster form.

c) In a group of students, 100 students know Hindi, 50 know English and 33 know both. Each of the students knows either Hindi or English. How many students are there in the group?

(Imp 2014)

Hint or Answer:

a) Φ

b) {1,-2}

c) 117 students

5. Consider the sets

A = {2, 3, 5, 7}, B = {1, 2, 3, 4, 6, 12}

a) Find (A ∩ B)

b) Find (A - B), (B - A) and hence show that

(A ∩ B) ∪ (A - B) ∪(B - A)= (A ∪ B)

c) Write the power set of (A ∩ B)

(March 2014)

Hint or Answer:

a) {2,3}

b) {1,2,3,4,5,6,7,12}

c) {Φ ,{2},{3},{2,3}}

6. A and B are two sets such that A ⊂ B,

a) A ∪ B is ....

b) Draw the Venn Diagram of B – A

c) In a committee, 60 people speak English, 30 speak Hindi and 15 speak both English and Hindi. How many speak at least one of these two language?

(Imp 2013)

Hint or Answer:
a)   B
b)   fig.
c)   75

7.

Let U = {1, 2, 3, 4, 6, 7, 8, 9}, A = {1, 2, 4, 7},
B = {1, 3, 5, 7}.

a)   Find (A ∪ B)
b)   Find A′ , B′ and hence show that
       (A ∪ B)′ = A′ ∩ B′
c)   The power set (A ∩ B) has......elements.

(March 2013)

Hint or Answer:
a)   {1,2,3,4,5,7}
b)   {6,8,9}
c)   64

8.

a)   How many elements has P(A), if A={1, 2, 3}?
b)   Let U = {1, 2, 3, 4, 5, 6, 7}, A = {1, 4, 6, 7},
       B = {1, 2, 3}, Find A′ , B′,(A ∪ B) and hence show that
       (A ∪ B)′ = A′ ∩ B′
c)   If A and B are two sets such that A ⊂ B then what is (A ∩ B).

(Imp 2012)

Hint or Answer:
a)   8
b)   {5}
c)   A

9.   Let A = {x: x is an integer, $\frac{1}{2} < x < \frac{7}{2}$}

a)   Write A in roster form
b)   Find the power set of A ∪ B
c)   Verify that (A-B) ∪ (A ∪ B) = A

(March 2012)

Hint or Answer:
a)   {1,2,3}
b)   ...
c)   A