Sample Questions

Question :

Express the given complex number in the form $a+ib$:

$$(5i)\left(-\frac{1}{6}i\right)$$

Solution :

$$(5i)\left(-\frac{1}{6}i\right) = 5\times-\left(\frac{1}{6}\right)\times i\times i$$

$$= \left(-\frac{5}{6}\right)i^2$$

$$= \left(-\frac{5}{6}\right)(-1)$$

$$= \frac{5}{6}$$

$$= \frac{5}{6} + 0i$$

Question :

Express the given complex number in the form $a+ib$:

$$\frac{5+2i}{3-4i}$$

Solution :

$$\frac{5+2i}{3-4i} = \frac{5+2i}{3-4i}\times\frac{3+4i}{3+4i}$$

$$= \frac{(5+2i)(3+4i)}{(3-4i)(3+4i)}$$

$$= \frac{(15+20i+6i+8i^2)}{(3)^2-(4i)^2}$$

$$= \frac{15+20i+6i+8(-1)}{9-16i^2}$$

$$= \frac{15+20i+6i-8}{9-16(-1)}$$

$$= \frac{7+26i}{9+16}$$
$$= \frac{7+26i}{25}$$
$$= \frac{7}{25} + \frac{26i}{25}$$

Question :

Express the given complex number in the form $a+ib$:

$i^9 - i^{25}$

Solution :

$$i^9 - i^{25} = \left(i^8 \times i\right) - \left(i^{24} \times i\right)$$
$$= (1 \times i) - (1 \times i)$$
$$= i - i$$
$$= 0$$
$$= 0 + 0i$$

Question :(**Kerala Imp 2015**)

What is i^{-35} ?

Solution :

$$i^{-35} = \frac{1}{i^{35}}$$
$$= \frac{1}{i^{32} \times i^3}$$
$$= \frac{1}{1 \times i^3}$$
$$= \frac{1}{-i}$$
$$= \frac{1 \times i}{-i \times i}$$
$$= 0 + i$$

Question :(**Kerala Imp 2010**)

Find sum of $i^1+i^2+i^3+i^4+i^5 \; \ldots \; +i^{99}$ $i^1+i^2+i^3+i^4+i^5 \; \ldots \; +i^{99}$

Solution :

$$=(i^1+i^2+i^3+i^4)+(i^5+i^6+i^7+i^8)+\ldots+(i^{97}+i^{98}+i^{99})$$
$$=(i^1+i^2+i^3+i^4)+i^4(i^1+i^2+i^3+i^4)+\ldots+i^{96}(i^1+i^2+i^3)$$
$$=(i-1-i+1)+i^4(i-1-i+1)+\ldots+(i-1-i)$$
$$=0+i^4\times 0+\ldots+(\cancel{i}-1-\cancel{i})$$
$$=\underline{\underline{-1}}$$

Question :

Express the given complex number in the form $a+ib$:

$$(7i+6)+\left(5-\frac{1}{6}i\right)+(8-17i)$$

Solution :

$$(7i+6)+\left(5-\frac{1}{6}i\right)+(8-17i)$$

$$=(6+5+8)+(7-\frac{1}{6}-17)i$$
$$=19+\left(\frac{7\times 6}{6}-\frac{1}{6}-\frac{17\times 6}{6}\right)i$$
$$=19+\left(\frac{42}{6}-\frac{1}{6}-\frac{102}{6}\right)i$$
$$=\underline{\underline{19+\frac{61}{6}i}}$$

Question :

Express the given complex number in the form $a+ib$:

$$(5i-4)^2+(6-3i)^2$$

Solution :

$$(5i-4)^2+(6-3i)^2$$

$$= \left[(5i)^2 - 2(5i)(4) + 4^2\right] + \left[(6)^2 - 2(6)(3i) + (3i)^2\right]$$
$$= (25i^2 - 40i + 16) + (36 - 36i + 9i^2)$$
$$= 25(-1) - 40i + 16 + 36 - 36i + 9(-1)$$
$$= -25 - 40i + 16 + 36 - 36i - 9$$
$$= (-25+16+36)+(-40i - 36i)$$
$$= \underline{\underline{27 - 76i}}$$

Question :(**Kerala Imp 2013**)

Express $\frac{1+i}{1-i}$ in the form a + i b.

Solution :

$$\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$
$$= \frac{(1+i)^2}{1^2 - i^2}$$
$$= \frac{1 + 2i + i^2}{1+1}$$
$$= \frac{\cancel{1} + 2i - \cancel{1}}{2}$$
$$= \frac{\cancel{2}i}{\cancel{2}}$$
$$= \underline{\underline{i}}$$

Question :

Express $\frac{2-i}{2+i}$ in the form $a + ib$.

Solution :

$$\frac{2-i}{2+i} = \frac{2-i}{2+i} \times \frac{2-i}{2-i}$$
$$= \frac{(2-i)^2}{2^2 - i^2}$$

$$= \frac{4-4i+i^2}{4+1}$$
$$= \frac{4-4i-1}{5}$$
$$= \frac{3-4i}{5}$$
$$= \frac{3}{5} - \frac{4i}{5}$$

Question :(**Kerala March 2011**)

Express $\frac{2+i}{(1+i)(1-2i)}$ in the form $a+ib$.

Solution :

$$\frac{2+i}{(1+i)(1-2i)}$$

$$= \frac{2+i}{1-2i+i-2i^2}$$
$$= \frac{2+i}{1-i+2}$$
$$= \frac{2+i}{3-i} \times \frac{3+i}{3+i}$$
$$= \frac{2+i}{3-i} \times \frac{3+i}{3+i}$$
$$= \frac{6+2i+3i+i^2}{9-i^2}$$
$$= \frac{6+2i+3i-1}{9+1}$$
$$= \frac{5+5i}{10}$$

Question :

Find the conjugate of the complex number $z = 4+7i$

Solution :

Let $z = 4+7i$

Conjugate of the complex number, $\bar{z} = 4 - 7i$

Question :

Find the conjugate of the complex number $z = (2+3i)^2$

Solution :

$$z = (2+3i)^2$$
$$= 4 + 12i + (3i)^2$$
$$= 4 + 12i + 9i^2$$
$$= 4 + 12i - 9$$
$$= -5 + 12i$$

Conjugate of the complex number, $\bar{z} = -5 - 12i$

Question :

Find the multiplicative inverse of the complex number $z = 4 - 2i$

Solution :

Let $z = 4 - 2i$

Then, $\bar{z} = 4 + 2i$

$$|z|^2 = 4^2 + (-2)^2 = 20$$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$
$$= \frac{4+2i}{20}$$
$$= \frac{4}{20} + \frac{2}{20}i$$

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$
$$= \frac{8-6i}{100}$$
$$= \frac{8}{100} - \frac{6}{100}i$$

Question :

Find the multiplicative inverse of the complex number
$z = 8 + 6i$

Solution :

Let $\quad z = 8 + 6i$

Then, $\bar{z} = 8 - 6i$ and $|z|^2 = 8^2 + (6)^2 = 100$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

Question :

Find the multiplicative inverse of the complex number
$z = 8 + 6i$

Solution :

Let $z = -12 - 5i$

Then, $\bar{z} = -12 + 5i$ and $|z|^2 = (-12)^2 + (-5)^2 = 144 + 25 = 169$

Therefore, the multiplicative inverse of $-12 - 5i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

$$= \frac{-12 - 5i}{169}$$

$$= -\frac{12}{169} - \frac{5}{169}i$$

Question :(**Kerala March 2015**)

Find the square root of the complex number : $-7 - 24i$

Solution :

Let $\sqrt{-7 - 24i} = a + ib$

Squaringboth sides

$$\left(\sqrt{-7 - 24i}\right)^2 = (a + ib)^2$$

$$-7 - 24i = a^2 + 2iab + (ib)^2$$

$$-7 - 24i = a^2 + i^2b^2 + 2iab$$

$$-7 - 24i = a^2 - b^2 + 2iab$$

On equating real and imaginary parts, we get

$$a^2 - b^2 = -7 \qquad \dots (1)$$

$$2ab = -24 \qquad \dots (2)$$

We have

$$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$$

$$(a^2 + b^2)^2 = (-7)^2 + (-24)^2 = 49 + 576 = 625$$

$$(a^2 + b^2) = 25 \qquad \dots (3)$$

Add (1) and (3), we get

$$2a^2 = 18$$

$$a^2 = 9$$

$$a = \pm 3$$

and $\quad b = \pm 4 \qquad$ [Using (2)]

Therefore

$$a = 3, b = -4$$

$$a = -3, b = 4$$

Hence

$$\sqrt{-7 - 24i} = \pm(3 - 4i)$$

Question :

Find the square root of the complex number : $3 + 4i$

Solution :

Let $\sqrt{3 + 4i} = a + ib$

Squaring both sides

$$\left(\sqrt{3 + 4i}\right)^2 = (a + ib)^2$$

$$3 + 4i = a^2 + 2iab + (ib)^2$$

$$3+4i = a^2 + i^2b^2 + 2iab$$
$$3+4i = a^2 - b^2 + 2iab$$

On equating real and imaginary parts, we get

$$a^2 - b^2 = 3 \qquad \ldots (1)$$

$$2ab = 4 \qquad \ldots (2)$$

We have

$$(a^2+b^2)^2 = (a^2-b^2)^2 + (2ab)^2$$

$$(a^2+b^2)^2 = (3)^2 + (4)^2 = 9+16 = 25$$

$$a^2 + b^2 = 5 \qquad \ldots (3)$$

Add (1) and (3), we get

$$2a^2 = 8$$
$$a^2 = 4$$
$$a = \pm 2$$

And $\qquad b = \pm 1 \qquad$ [Using (2)]

Therefore $a = 2, b = 1; a = -2, b = -1$

Hence $\sqrt{3+4i} = \pm(2+i)$

Question :

Find the modulus and the argument of the complex number $\sqrt{3} - i$

Solution :

$z = \sqrt{3} - i$

$$r\cos\theta = \sqrt{3}, r\sin\theta = -1,$$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + (-1)^2 = 4$$

$$r^2(\cos^2\theta + \sin^2\theta) = 4$$

$$r^2 \times 1 = 4$$

$$r = 2$$

$\therefore$ Modulus $= \underline{\underline{2}}$

$\therefore$ $2\cos\theta = \sqrt{3}, 2\sin\theta = -1$

$\therefore$ $\cos\theta = \frac{\sqrt{3}}{2}, \ \sin\theta = \frac{-1}{2}$

As θ lies in the III quadrant, $\theta = \theta - \pi$

$$\theta = \frac{\pi}{6} - \pi$$

$$= \underline{\underline{\frac{-5\pi}{6}}}$$

Thus, the modulus and argument of the complex number $\sqrt{3} - i$ are 2 and $\frac{-5\pi}{6}$ respectively.

Question :

Find the modulus and the argument of the complex number $3 + 4i$

Solution :

$z = 3 + 4i$

$r\cos\theta = 3, r\sin\theta = 4,$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = 3^2 + 4^2 = 25$

$r^2(\cos^2\theta + \sin^2\theta) = 25$

$r^2 \times 1 = 25$

$r = 5$

$\therefore \quad \text{Modulus} = 5$

$\therefore \quad 5\cos\theta = 3, 5\sin\theta = 4$

$\therefore \quad \cos\theta = \frac{3}{5}, \sin\theta = \frac{4}{5}$

$\therefore \quad \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{5} \times \frac{5}{3} = \frac{4}{3}$

Argument $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

Thus, the modulus and argument of the complex number $3+4i$ are 5 and $\tan^{-1}\left(\frac{4}{3}\right)$ respectively.

Question :(**March2014, 2015, Imp2012, Imp2009**)

Represent the complex number in the polar from $1+\sqrt{3}i$

Solution :

$z = 1+\sqrt{3}i$

$$r\cos\theta = 1, r\sin\theta = \sqrt{3},$$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (1)^2 + \left(\sqrt{3}\right)^2 = 4$$
$$r^2(\cos^2\theta + \sin^2\theta) = 4$$
$$r^2 \times 1 = 4$$
$$r = 2$$

$\therefore \quad \text{Modulus} = 2$

$\therefore \quad 2\cos\theta = 1, 2\sin\theta = \sqrt{3}$

$\therefore \qquad \cos\theta = \frac{1}{2}, \sin\theta = \frac{\sqrt{3}}{2}$

As θ lies in the I quadrant, $\theta = \frac{\pi}{3}$

Polar form is $\quad 1+\sqrt{3}i = 3(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

Question :

Represent $\left(\frac{1-i}{1+i}\right)$ in polar form.

Solution :

$$\begin{aligned}
&\frac{1-i}{1+i} \\
&= \frac{1-i}{1+i} \times \frac{1-i}{1-i} \\
&= \frac{(1-i)^2}{1^2 - i^2} \\
&= \frac{1-2i+i^2}{1+1} \\
&= \frac{\cancel{1} - 2i - \cancel{1}}{2} \\
&= \frac{-\cancel{2}i}{\cancel{2}} \\
&= -i \\
&= 0 - i
\end{aligned}$$

$$z = 0 - i$$

$$r\cos\theta = 0, r\sin\theta = -1$$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (0)^2 + (-1)^2 = 1$$
$$r^2(\cos^2\theta + \sin^2\theta) = 1$$
$$r^2 \times 1 = 1$$

$$r = 1$$

$\therefore$ $$\text{Modulus} = 1$$

$\therefore$ $$\cos\theta = 0, \sin\theta = -1$$

$$\theta = \frac{3\pi}{2}$$

Polar form is $$-i = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$$

Question :

Solve the equation $x^2+5=0$

Solution :

On comparing with ax^2+bx+c , we obtain

$$a = 1, b = 0, \text{ and } c = 5$$

Therefore, the discriminant is

$$D = b^2 - 4ac = 0^2 - 4 \times 1 \times 5 = -20$$

Therefore, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-0 \pm \sqrt{-20}}{2 \times 1}$$

$$= \frac{\pm i\sqrt{20}}{2}$$

$$= \frac{\pm i\sqrt{4 \times 5}}{2}$$

$$= \frac{\pm \cancel{2} i\sqrt{5}}{\cancel{2}}$$

$$= \pm i\sqrt{5}$$

Question :(**Kerala Imp 2015**)

Solve the equation $\sqrt{5}x^2+x+\sqrt{5}=0$

Solution :

On comparing with ax^2+bx+c , we obtain

$$a=\sqrt{5}, b=1, \text{ and } c=\sqrt{5}$$

Therefore, the discriminant is

$$D=b^2-4ac=1^2-4\times\sqrt{5}\times\sqrt{5}=-19$$

Therefore, the required solutions are

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$x=\frac{-1\pm\sqrt{-19}}{2\times\sqrt{5}}=\frac{-1\pm i\sqrt{19}}{2\sqrt{5}}$$

Question :(**Kerala Imp 2014**)

Solve the equation $-x^2+x-2=0$

Solution :

On comparing with ax^2+bx+c , we obtain

$$a=-1, b=1, \text{ and } c=-2$$

Therefore, the discriminant is

$$D=b^2-4ac=(1)^2-(4\times-1\times-2)=-7$$

Therefore, the required solutions are

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$x = \frac{-1 \pm \sqrt{-7}}{2 \times -1} = \frac{-1 \pm i\sqrt{7}}{-2}$$

Question :

Solve the equation $2x^2 + \frac{x}{2} + 2 = 0$

Solution :

$$2x^2 + \frac{x}{2} + 2 = 0$$

$$4x^2 + x + 4 = 0$$

On comparing with ax^2+bx+c , we obtain

$$a = 4, b = 1, \text{ and } c = 4$$

Therefore, the discriminant is

$$D = b^2 - 4ac = (1)^2 - (4 \times 4 \times 4) = -63$$

Therefore, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-1 \pm \sqrt{-63}}{2 \times 4}$$

$$= \frac{-1 \pm i\sqrt{63}}{8}$$

EXERCISE

1. Express $\frac{5+i}{2+3i}$ in the form $a+ib$.

(Kerala March 2010)

Hint or Answer: 1 - i

2. Express $\frac{2+i}{2-i}$ in the form a+ib

(Kerala Imp 2012)

Hint or Answer: $\frac{3}{5}+\frac{4i}{5}$

3. Express $\frac{2+i}{(1+i)(1-2i)}$ in a+ib form.

(Kerala March 2011)

Hint or Answer: $\frac{1}{2}+\frac{1}{2}i$

4. Express $\frac{5-\sqrt{3}i}{4+2\sqrt{3}i}$ in the form a+ib

(Kerala March 2012)

Hint or Answer: $\frac{1}{2}-\frac{\sqrt{3}i}{2}$

5. Express $\frac{1-i}{\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}}$ in the form a+ib.

(Kerala Imp 2010)

Hint or Answer: $-\sqrt{2}i$

6. Represent $\frac{1+i}{1-i}$ in the form a+ib.

(Kerala Imp 2013)

Hint or Answer: i

7. Express $\frac{2+i}{(1+i)(1-2i)}$ in polar form.

(Kerala March 2011)

Hint or Answer: $\frac{1}{\sqrt{2}}\left[\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right]$

8. Express $\frac{1-i}{\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}}$ in polar form.

(Kerala Imp 2010)

Hint or Answer: $\sqrt{2}\left[\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right]$