Sample Questions

Question:

In a class there are 15 boys and 20 girls. The teacher wants to select a boy and a girl to represent the class in a function. In how many ways can the teacher make this selection?

Solution:

Here the teacher is to perform two jobs :

(i) Selecting a boy among 15boys,and (ii) Selecting a girl among 20 girls.

The first of these can be performed in 15 ways and the second in 20 ways.

$\therefore$ By the fundamental principle of multiplication,

The required number of ways is $= 15 \times 20 = \underline{\underline{300}}$.

Question:

In a class there are 20boys and 10 girls. The teacher wants to select either a boy or a girl to represent the class in a function. In how many ways can the teacher make this selection?

Solution:

Here the eacher is to perform either of the following two jobs:

(i) selecting a boy among 20 boys. or

(ii) Selecting a girl among 10 girls

The first of these can be performed in 20 ways and the second in 10 ways.

$\therefore$ By fundamental principle of addition either of the two jobs can be performed in $(20 + 10) = \underline{\underline{30 \text{ ways}}}$.

Question:

A room has10 doors. In how many ways can a man enter the room through one door and come out through a different door?

Solution:

Clearly, a person can enter the room through any one of the ten doors. So, there are ten ways of entering into the room. After entering into the room, the man can come out through any one

of the remaining 9 doors. So, he can come out through a different door in 9 ways.
Hence, the number of ways in which a man can enter a room through one door and come out through a different door
$= 10 \times 9 = \underline{\underline{90}}$.

Question:
How many words (with or without meaning) of three distinct letters of the English alphabets are there?

Solution:
Here we have to fill up three places by distinct letters of the English alphabets.

Since there are 26letters of the English alphabet, the first place can be filled by any of these letters. So, there are 26 ways of filling up the first place. Now,the second place can be filled up by any of the remaining 25 letters.
So, there are 25 ways of filling up the second place.
After filling up the first two places only 24 letters are left to fill up the third place. So, the third place can be filled in 24 ways.
Hence,the required number of words $= 26 \times 25 \times 24 = \underline{\underline{15600}}$

Question:
How many three digit numbers can be formed by using the digits 1, 2, 3, 4, 5.

Solution:

We have to determine the total number of three digit numbers formed by using the digits 1, 2, 3, 4, 5. Clearly, the repetition of digits is allowed. A three digit number has three places, units, tens and hundreds. Units place can be filled by any of the digits 1, 2, 3, 4, 5. So units place can be filled in 5 ways. Similarly, each one of the tens and hundreds place can be filled in 5 ways.
$\therefore$ Total number of required numbers $= 5 \times 5 \times 5 = \underline{\underline{125}}$

Question:

How many three digit odd numbers can be formed by using the digits 4, 5, 6, 7, 8, 9 if :

(i) the repetition of digits is not allowed?

(ii) the repetition of digits is allowed?

Solution:

For a number to be odd, we must have 5,7 or 9 at the units place. So,there are 3ways of filling the units place

(i) Since the repetition of digits is not allowed, the tens place can be filled with any of the remaining 5digits in 5ways. Now, four digits are left. So,hundreds place can be filled in 4 ways.

So, required number of numbers

$= 3 \times 5 \times 4 = \underline{\underline{60}}$

(ii) Since the repetition of digits is allowed, so each of the tens and hundreds place can be filled in 6 ways.

Hence required number of numbers

$= 3 \times 6 \times 6 = \underline{\underline{108}}$

Question :

How many 3 digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

Solution:

The required number of three digit even numbers

$= 3 \times 6 \times 6 = \underline{\underline{108}}$

Question:

Evaluate (i) $7!$ (ii) $5! - 3!$

Solution:

(i) $7! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$

(ii) $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$

$3! = 1 \times 2 \times 3 = 6$

$\therefore 5! - 3! = 120 - 6$

$= 114$

Question:

If $\frac{1}{5!} + \frac{1}{6!} = \frac{x}{7!}$, find x

Solution:

$$\frac{1}{5!}+\frac{1}{6!}=\frac{x}{7!}$$

$$\Rightarrow \quad \frac{1}{5!}+\frac{1}{6\times 5!}=\frac{x}{7\times 6\times 5!}$$

$$\Rightarrow \quad 1+\frac{1}{6}=\frac{x}{7\times 6}$$

$$\Rightarrow \quad \frac{7}{6}=\frac{x}{42}$$

$$\Rightarrow \quad x=\frac{7}{6}\times 42=49$$

Question:

Evaluate ${}^{8}P_{3}$

Solution:

$${}^{8}P_{3}=\frac{8!}{(8-3)!}=\frac{8!}{5!}=\frac{(8\times 7\times 6)\times 5!}{5!}$$

$$= 8\times 7\times 6 = 336$$

Question: **(Imp 2012)**

If ${}^{n}P_{4} = 360$, find the value of n.

Solution:

$${}^{n}P_{4} = 360$$

$$\frac{n!}{(n-4)!} = 6\times 5\times 4\times 3$$

$$\frac{n!}{(n-4)!} = \frac{6\times 5\times 4\times 3\times 2!}{2!} = \frac{6!}{2!}$$

$$n! = 6! = 6$$

Question: **(Imp 2012)**

If ${}^{9}P_{r} = 3024$, find r.

Solution:

$${}^{9}P_{r} = 3024$$

$$= 9\times 8\times 7\times 6 = {}^{9}P_{4}$$

$$r = 4$$

Question:
How many permutations of the letters of the word 'APPLE' are there?

Solution
Here there are 5 letters, two of which are of the same kind.
The others are each of its own kind.
$\therefore$ Required number of permutations is

$$= \frac{5!}{2!\ 1!\ 1!\ 1!}$$
$$= \frac{5!}{2!} = \frac{120}{2} = \underline{\underline{60}}$$

Question:
How many numbers can be formed with the digits 1,2,3,4,3,2,1 so that the odd digits always occupy the odd places?

Solution
There are 4 odd digits 1,1,3,3 and 4 odd places. So odd digits can be arranged in odd places in $\frac{4!}{2!\ 2!}$ ways.
The remaining 3 even digits 2, 2, 4 can be arranged in 3even places in $\frac{3!}{2!}$ ways.
Hence, the required number of numbers

$$= \frac{4!}{2!\ 2!} \times \frac{3!}{2!} = 6 \times 3$$
$$= \underline{\underline{18}}$$

Question:
How many arrangements can be made with the letters of the word MATHEMATICS?
Solution
There are 11 letters in the word MATHEMATICS of which two are M's, two are A's, two are T's and all other are distinct.
$\therefore$ required number of arrangements

$$= \frac{11!}{2! \times 2! \times 2!}$$
$$= \underline{\underline{4989600}}$$

Question:

In how many ways can 5 different balls be distributed among 3 boxes?

Solution

There are 5 balls and each ball can be placed in 3 ways.

So the total number of ways = 3^5= 243

Question:

In how many ways can 3 prizes be distributed among 4boys, when

(i) no boy gets more than one prize?

(ii) a boy may get any number of prizes?

(iii) no boy gets all the prizes?

Solution

(i) The total number of ways is the number of arrangements of 4 taken 3 at a time.

So, the required number of ways $= {}^4P_3 = 4!$

$= 24$

(ii) The first prize can be given away in 4 ways as it may be given to anyone of the 4boys.The second prize can also be given away in 4 ways, since it may be obtained by the boy who has already received a prize.

Similarly, third prize can be given away in 4 ways.

Hence, the number of ways in which all the prizes can be given away $= 4 \times 4 \times 4 = 4^3 = 64$

(iii) Since any one of the 4 boys may get all the prizes.

So, the number of ways in which a boy get all the 3prizes = 4.

So,the number of ways in which a boy does not get all the prizes $= 64 - 4 = 60$

Question:

In how many ways 10 persons may be arranged in a

(i) line

(ii) circle?

Solution

(i) The number of ways in which 10 persons can be arranged in a line $= {}^{10}P_{10} = 10!$

(ii) The number of ways in which 10 persons can be arranged in a circle $= (10-1)! = 9!$

Question:

In how many ways can 5 gentlemen and 5 ladies sit together at around table, so that no two ladies may be together?

Solution

The number of ways in which 5 gentlemen may be arranged is $(5-1)! = 4!$.

Then the ladies may be arranged among themselves in 5!ways.

Thus the total number of ways $= 4! \times 5!$

$= 24 \times 120 = \underline{\underline{2880}}$

Question:

Find the number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated.

Solution

Considering 4 particular flowers as one flower,we have five flowers, which can be strung to form a garland in 4! ways.

But 4 particular flowers can be arranged in 4! ways.

$\therefore$ Required number of ways $= 4! \times 4! = \underline{\underline{576}}$

Question:

Evaluate ${}^{6}C_3$

Solution

${}^{6}C_3 = {}^{6}P_3$

$3! = 6 \times 5 \times 4$

$1 \times 2 \times 3 = 20$

Question:

If ${}^{n}C_4 = {}^{n}C_6$, find ${}^{12}C_n$

Solution

$$^{n}C_4 = {}^{n}C_6$$

$$n = 4 + 6 = 10$$

Now

$$^{12}C_n = {}^{12}C_{10}$$

$$= {}^{12}C_{(12\text{-}10)}$$

$$= {}^{12}C_2$$

$$= \frac{12 \times 11}{1 \times 2}$$

$$= 66$$

Question:

If $^{15}C_r : {}^{15}C_{(r-1)} = 11 : 5$, find r

Solution

$$^{15}C_r : {}^{15}C_{(r-1)} = 11 : 5$$

$$\frac{^{15}C_r}{^{15}C_{(r-1)}} = \frac{11}{5}$$

$$\frac{\left(\frac{15!}{r!(15-r)!}\right)}{\left(\frac{15!}{(r-1)!\,(15-r+1)!}\right)} = \frac{11}{5}$$

$$\frac{15!}{r!(15-r)!} \times \frac{(r-1)!\,(16-r)!}{15!} = \frac{11}{5}$$

$$\frac{(r-1)!\,(16-r)\,\{(15-r)!\}}{r(r-1)!\,(15-r)!} = \frac{11}{5}$$

$$\frac{16-r}{r} = \frac{11}{5}$$

$$5(16-r) = 11r$$

$$80 = 16r$$

$$r = 5$$

In how many ways players for a cricket team can be selected from a group of 25 players containing 10 batsmen, 8 bowlers,5 all-rounders and 2 wicket keepers? Assume that the team requires 5 batsmen, 3 all-rounder, 2 bowlers and 1 wicket keeper.

Solution

The selection of team is divided into 4 phases

(i) Selection of 5batsmen out of 10. This can be done in ${}^{10}C_5$ ways.

(ii) Selection of 3 all-rounders out of 5. This can be done in ${}^{5}C_3$ ways.

(iii) Selection of 2 bowlers out of 8. This can be done in ${}^{8}C_2$ ways.

(iv) Selection of one wicket keeper out of 2. This can be done in ${}^{2}C_1$ ways.

$\therefore$ The team can be selected in ${}^{10}C_5 \times {}^{5}C_3 \times {}^{8}C_2 \times {}^{2}C_1$ ways

$= 252 \times 10 \times 28 \times 2$ ways

$=$ 141120 ways

EXERCISE

1. a) Write the value of ,${}^{7}C_5$

b) Find the value of n if

$$3.\ {}^{n}P_4 = 5.\ {}^{(n-1)}P_4$$

c) What is the number of ways of choosing four cards from a pack of 52 cards, provided all four cards belong to four different suits?

(March 2016)

OR

a) Write the value of, ${}^{29}C_{29}$.

b) Find the value of n if

12. ${}^{(n-1)}P_3 = 5.\,{}^{(n+1)}P_3$

c) A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has at least one boy and one girl?

(March 2016)

2. a) ${}^7P_7$ = ...

(i) 7 (ii) 7! (iii) 1 (iv) 7^7

b) Find the number of words that can be formed from the letters of the word MALAYALAM". How many of these start with 'Y'?

c) If ${}^{2n}C_3 = 11\,{}^nC_3$. Find n.

OR

a) ${}^nC_r + {}^nC_{r-1}$ = ...

i) ${}^nC_{r+1}C$ ii) ${}^{n+1}C_rC$ iii) ${}^{n-1}C_r$ iv) ${}^{n-1}C_{r-1}$

b) Prove that

${}^{61}C_{57} - {}^{60}C_{56} = {}^{60}C_3$

c) In how many ways can the letters of the word 'ARRANGE' be arranged such that two A's do not occur together?

(IMP 2015)

3. a) $\frac{0!}{1!}$ = ...

i) 0 ii) 1 iii) 2 iv) 3

b) Find r, if $5 \times {}^4P_r = 6 \times {}^5P_{r-1}$

c) Find the number of 8-letters arrangements that can be made from the letters of the word DAUGHTER so that all Vowels do not occur together.

OR

a) ${}^nC_{n-1}$ = ...

i) n-1 ii) n iii) 0 iv) 1

b) If ${}^{n}C_9 = {}^{n}C_8$ Find ${}^{n}C_2$

c) How many ways can a team of 5 persons be selected out of a group of 4 mean and 7 women, if the team has at least one man and one women?

(March 2015)

4. a) Find the number of permutations of the letters of the word ALLAHABAD.

b) Find r, if ${}^{5}P_r = 2 \times {}^{6}P_{r-1}$

OR

a) If, ${}^{n}C_9 = {}^{n}C_8$ find 'n' and ${}^{n}C_{17}$.

b) How many chords can be drawn through 23 points on a circle?

(IMP 2014)

5. a) In how many ways can the letters of the word, PERMUTATIONS be arranged if:

i) the words start with P and end with S?

ii) there are always 4 letters between P and S?

b) In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together.

c) How many chords can be drawn through 21 points? OR

a) What is the minimum numbers of ways of choosing 4 cards from a pack of 52 paying cards? In how many of these:

i) are 4 cards of the same suit?

ii) do 4 cards belong to 4 different suits?

b) Find the number of permutations of the letters of the word ALLAHABAD.

c) How many 5digit telephone numbers can be constructed using the digits 0 t0 9 if each number starts with 67 and no digit appears more than once?

(MARCH 2014)

6. a) How many three digit numbers can be formed form the digits 1,2,3,4,5 assuming that repetition of the digits is not allowed is ....

b) If $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$, find x

c) How many words with or without meaning can be formed using all the letters of the word FRIDAY, using each letter exactly once? How many of them have the first letter is a vowel?

OR

a) If ${}^{n}C_{7} = {}^{n}C_{5}$, then find n

b) A bag contains 5 blue and 6 white balls. Determine the number of ways in which 3 blue and 4 white balls can be selected.

c) What number of choosing 3 cards from a pack of 52 playing cards? In how many of these 3 cards of the same colour?

(IMP 2013)

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