CHAPTER 11

CONIC SECTION

1. **Locus of a point**: When a point moves in such a way that its path traced out is called its locus. E.g.: sector of a circle, circle, conic, etc.

2. A conic is a curve obtained by slicing a cone with a plane which does not pass through the vertex. It is also known as conic section.

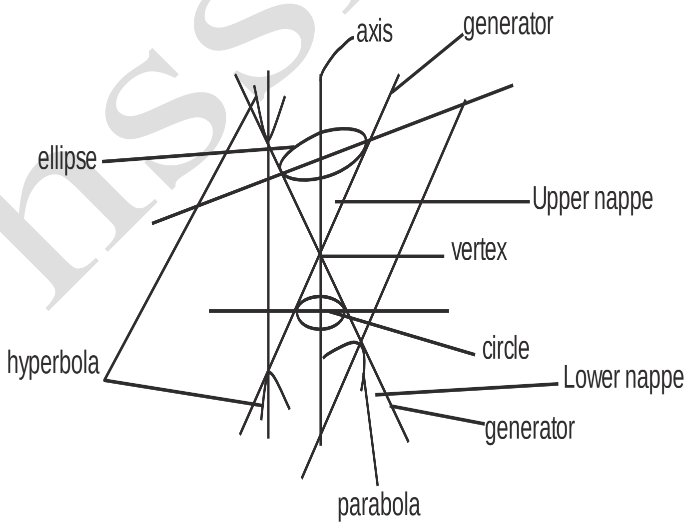
3. **Types of conics**:

   i. **Parabola** : Parabola is a conic obtained by slicing a cone with a plane, which does not pass through the vertex and parallel to any generators.

   ii. **Ellipse** : Ellipse is a conic obtained by slicing a cone with a plane, which does not pass through the vertex not parallel to any generators and cuts only one nappe.

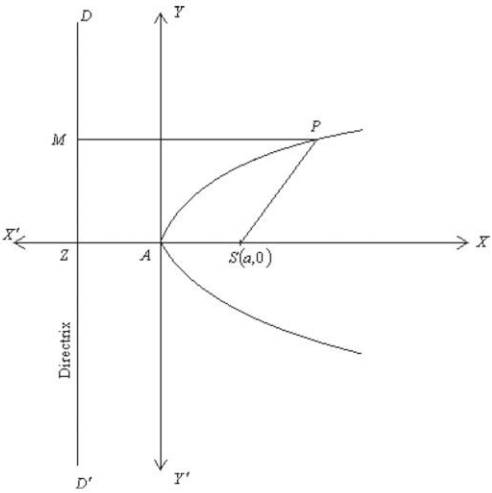
   iii. **Hyperbola** : Hyperbola is a conic obtained by slicing a cone with a plane, which does not pass through the vertex not parallel to any generators and cuts two nappes.

   iv. **Circle** : Circle is also a conic obtained by slicing a cone with a plane, which does not pass through the vertex, parallel to its base.



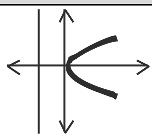
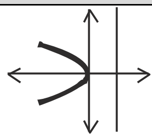
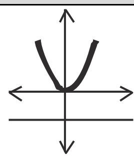
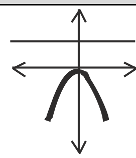
4. **Conic section as a locus**: The conic section is the locus of a point which moves in a plane so that its distance from a fixed point bears a constant ratio to its distance from a fixed line.
5. The fixed point is called focus (S) and the fixed line is called directrix and the constant ratio is called eccentricity of the conic, which is denoted by e.
6. If e=1 the conic is known as a parabola, if $e<1$, the conic is known as an ellipse and if $e>1$, the conic is known as a hyperbola and if $e=0$, the conic is known as a circle.

PARABOLA



Standard Equation

Four types of parabolas:

Name of the parabola	Right handed Parabola	Left handed Parabola	Upward Parabola	Downward Parabola
Graph				
Standard form	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Vertex	$A(0,0)$	$A(0,0)$	$A(0,0)$	$A(0,0)$
Focus	$S(a,0)$	$S(-a,0)$	$S(0,a)$	$S(0,-a)$
Equation of the	$x=-a$	$x=a$	$y=-a$	$y=a$

directrix				
Axis	x axis	x axis	y axis	y axis
Symmetry	x axis	x axis	y axis	y axis
Equation of the axis	y = 0	y = 0	x = 0	x = 0
Equation of the tangent at the vertex	x = 0	x = 0	y = 0	y = 0
Length of the latus rectum	4a	4a	4a	4a
Equation of the latus rectum	$x = a$	$x = -a$	$y = a$	$y = -a$

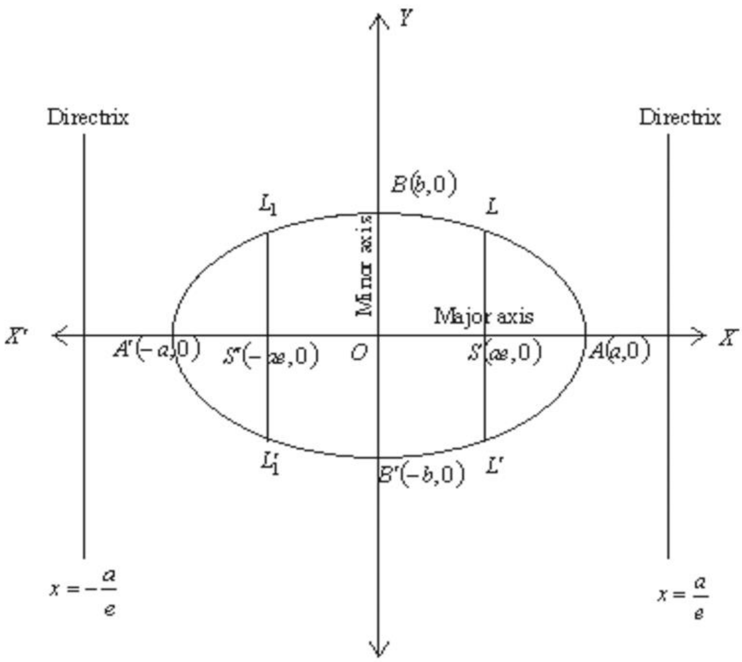
ELLIPSE

Definition2: An ellipse is the set of all points in the plane whose distances from a fixed point in the plane bears a constant ratio, less than one, to their distances from a fixed line in the plane.

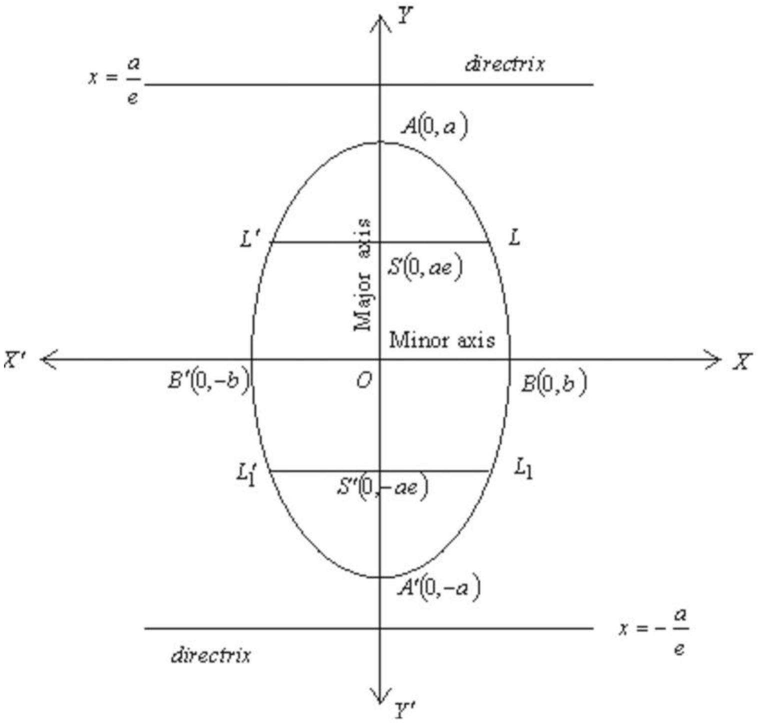
Definition3: An ellipse the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

Definition4: If e<1, then the conic is known as an ellipse.

MAJOR AXIS IS ALONG THE X-AXIS:



MAJOR AXIS IS ALONG THE Y-AXIS



PROPERTIES

Name of the Ellipse	Major axis is along the x axis	Major axis is along the y axis
Standard form	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ ,a^2>b^2$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1,a^2>b^2$
Centre	$C(0,0)$	$C(0,0)$
Foci	$S(ae,0),S'(-ae,0)$	$S(0,ae),S'(0,-ae)$
Vertices	$A(a,0),A'(-a,0)$	$A(0,a),A'(0,-a)$
Equation of the directrix	$x=\pm\frac{a}{e}$	$y=\pm\frac{a}{e}$
Equation of the major axis	$y=0$	$x=0$
Equation of the minor axis	$x=0$	$y=0$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Equation of the latus	$x=\pm ae$	$y=\pm ae$

rectum		
Length of major axis	2a	2a
Length of minor axis	2b	2b

Note:

1. $b^2 = a^2\left(1-e^2\right)$
2. Centre of the ellipse is the point of intersection of the major and minor axis.
3. Foci are the point of intersection of major axis and the latus recta.
4. The ratio of the distances from the centre of the ellipse to one of the foci and to one of the vertices of the ellipse is known as eccentricity.

   The eccentricity, $e = \dfrac{c}{a}$, *where* $c = \sqrt{a^2 - b^2}$

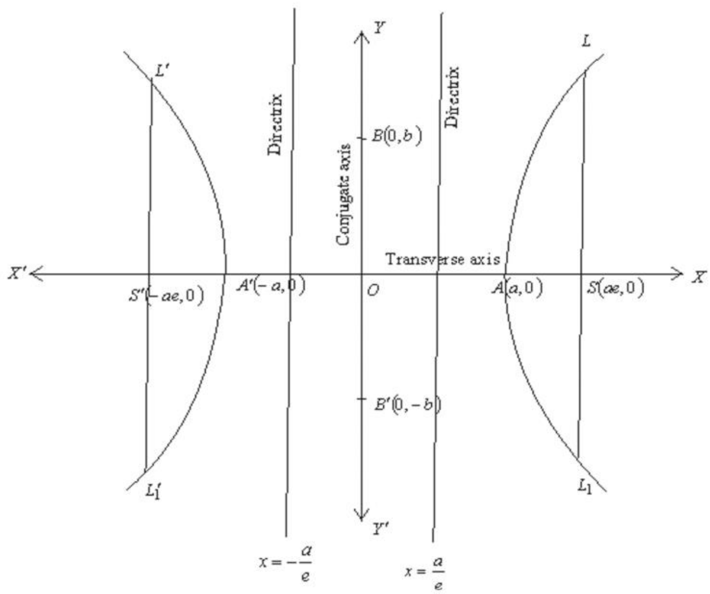
HYPERBOLA

Definition2: A hyperbola is the set of all points in the plane whose distance from a fixed point in the plane bears a constant ratio, greater than one, to their distances from a fixed line in the plane.

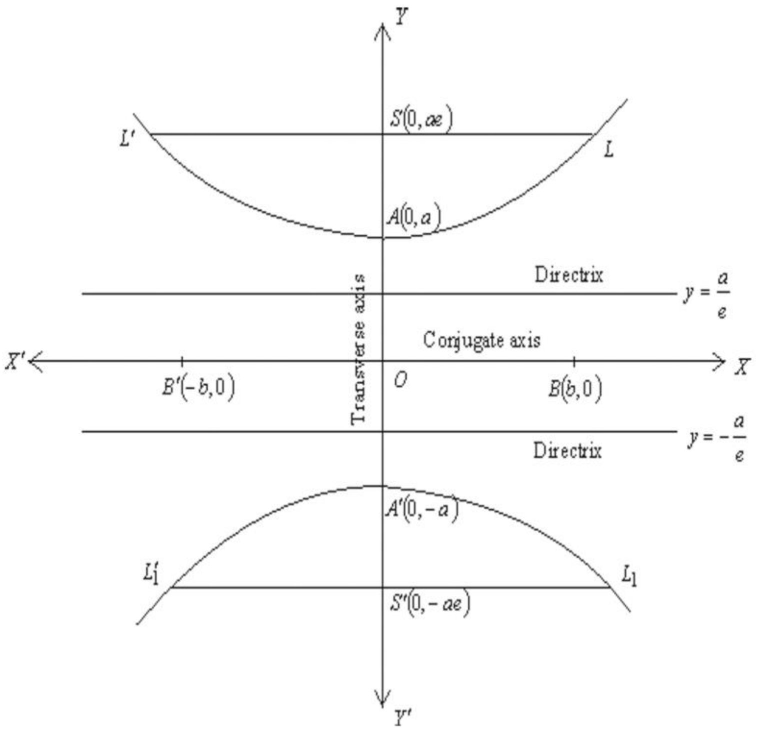
Definition3: A hyperbola the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

Definition4: If e>1, then the conic is known as a hyperbola.

TRANSVERSE AXIS IS ALONG THE X AXIS



TRANSVERSE AXIS IS ALONG THE Y AXIS



PROPERTIES

Name of the hyperbola	Transverse axis is along the x axis	Transverse axis is along the y axis
Standard form	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{a^2}=1$
Centre	$C(0,0)$	$C(0,0)$
Foci	$S(ae,0), S'(-ae,0)$	$S(0,ae), S'(0,-ae)$
Vertices	$A(a,0), A'(-a,0)$	$A(0,a), A'(0,-a)$
Equation of the directrix	$x=\pm\frac{a}{e}$	$y=\pm\frac{a}{e}$
Equation of the transverse axis	$y=0$	$x=0$
Equation of the conjugate axis	$x=0$	$y=0$

Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Equation of the latus rectum	$x = \pm ae$	$y = \pm ae$
Length of transverse axis	2a	2a
Length of conjugate axis	2b	2b
Equation of the transverse axis	$y = 0$	$x = 0$
Equation of the conjugate axis	$x = 0$	$y = 0$

Note:

1. $b^2 = a^2\left(e^2 - 1\right)$
2. Centre of the hyperbola is the point of intersection of the transverse axis and the conjugate axis.
3. Foci are the point of intersection of transverse axis and the latus recta.
4. The ratio of the distances from the centre of the hyperbola to one of the foci and to one of the vertices of
5. the hyperbola is known as eccentricity.
6. The eccentricity, $e = \frac{c}{a}$, where $c = \sqrt{a^2 + b^2}$
7. Hyperbola in which a=b is called equilateral hyperbola.