Conic Sections

A conic is a curve obtained by slicing a cone with a plane which does not pass through the vertex. There are four types of conic.

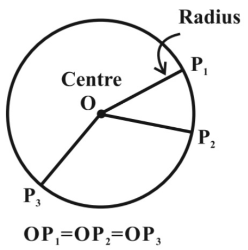
1. Circle
2. Parabola
3. Ellipse
4. Hyperbola

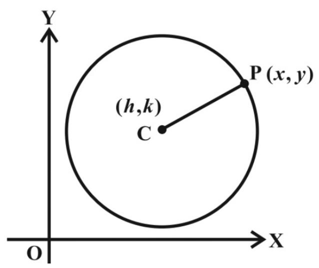
Circle

- A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

Terms related to Circle

- **Centre:** The fixed point is called the **centre** of the circle, denoted by (h, k)
- **Radius:** The distance from the centre to a point on the circle is called the **radius** of the circle,denoted by r





Standard Equation of the Circle

- Let (h, k) be centre and r be the radius of the circle. Then the equation of the circle is
$$(x-h)^2+(y-k)^2=r^2$$
- Let origin $(0, 0)$ be centre and r be the radius of the circle. Then the equation of the circle is
$$x^2+y^2=r^2$$

- The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$, then

$$Centre = (-g, -f)$$

$$Radius = \sqrt{g^2 + f^2 - c}$$

Problems

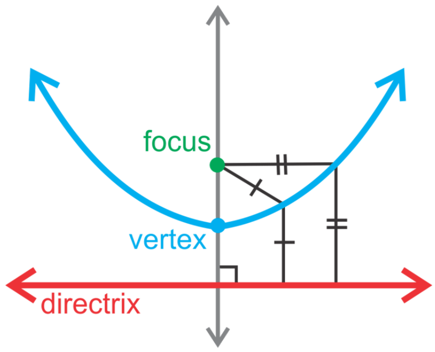
1. Find the equation of the circle with
   (a) Centre $(-3, 2)$ and radius 4.
   (b) Centre $(0, 2)$ and radius 2
   (c) Centre $(-2, 3)$ and radius 4
   (d) Centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{2}$.
   (e) Centre $(1, 1)$ and radius $\sqrt{2}$
   (f) Centre $(-a, -b)$ and radius $\sqrt{a^2 - b^2}$
2. Find the centre and radius of the following circle.
   (a) $x^2 + y^2 + 8x + 10y - 8 = 0$
   (b) $x^2 + y^2 - 4x - 8y - 45 = 0$
   (c) $x^2 + y^2 - 8x + 10y - 12 = 0$ (Improvement 2013)
   (d) $2x^2 + 2y^2 - x = 0$ (March 2009)
   (e) $(x + 5)^2 + (y - 3)^2 = 36$
3. Find the equation of the circle passing through the points $(4, 1)$ and $(6, 5)$ and whose centre is on the line $4x + y = 16$.
4. Find the equation of the circle passing through the points $(2, 3)$ and $(-1, 1)$ and whose centre is on the line $x - 3y - 11 = 0$.
5. Find the equation of the circle passing through the points $(2, -2)$ and $(3, 4)$ and whose centre is on the line $x + y = 2$.
6. Find the equation of the circle with radius 5 whose whose centre lies on x- axis and passes through the point $(2, 3)$.
7. Find the equation of a circle with centre $(2, 2)$ and passes through the point $(4, 5)$. (March 2011)
8. Does the point $(-2.5, 3.5)$ lie inside, outside or on the circle $x^2 + y^2 = 25$

Parabola

- A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point(not on the line) in the plane.

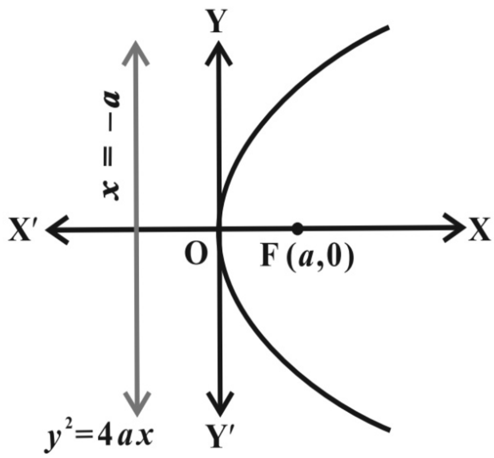
Terms related to Parabola

- **Directrix:** The fixed line is called the **directrix** of the parabola.
- **Focus:** The fixed point is called the **focus** of the parabola.
- **Axis:** A line through the focus and perpendicular to the directrix is called the **axis** of the parabola.
- **Vertex:** The point of intersection of parabola with the axis is called the **vertex** of the parabola.
- **Eccentricity:** The ratio between the distance between a focus and a point on the figure and the distance between the point and the directrix is called **eccentricity**(e) of the parabola. The eccentricity of a parabola is always $1 (e = 1)$.
- **Latus rectum:** The line segment through the focus of a parabola, perpendicular to the axis, which has both endpoints on the parabola is called **lactus rectum** of the parabola.



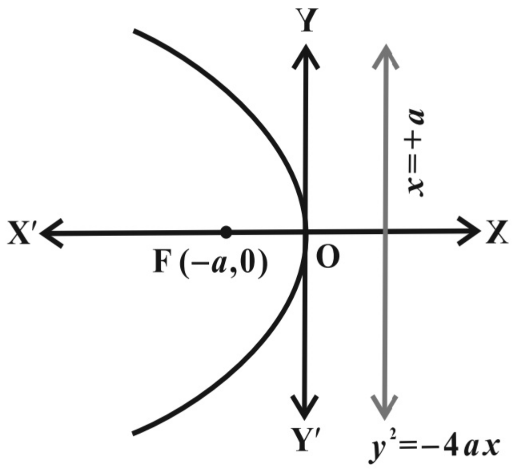
Standard Equations of Parabola

1. **Right Handed Parabola**($y^2 = 4ax$)



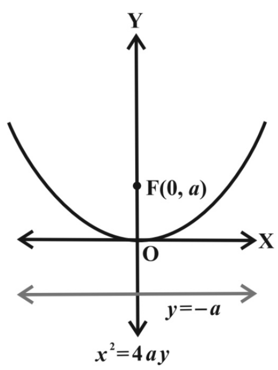
Standard form	$y^2 = 4ax$
Vertex	$O(0, 0)$
Focus	$F(a, 0)$
Equation of the directrix	$x = -a$
Axis	x-axis
Symmetry	$+x$- axis
Equation of the axis	$y = 0$
Length of the latus rectum	$4a$
Equation of the latus rectum	$x = a$

2. **Left Handed Parabola**$(y^2 = -4ax)$



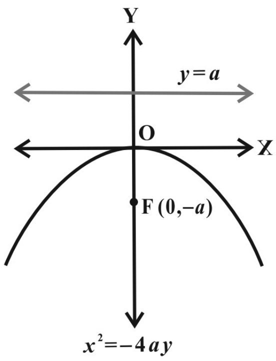
Standard form	$y^2 = -4ax$
Vertex	$O(0, 0)$
Focus	$F(-a, 0)$
Equation of the directrix	$x = a$
Axis	x-axis
Symmetry	$-x$- axis
Equation of the axis	$y = 0$
Length of the latus rectum	$4a$
Equation of the latus rectum	$x = -a$

3. **Upward Parabola**$(x^2 = 4ay)$



Standard form	$x^2 = 4ay$
Vertex	$O(0, 0)$
Focus	$F(0, a)$
Equation of the directrix	$y = -a$
Axis	y-axis
Symmetry	$+y$- axis
Equation of the axis	$x = 0$
Length of the latus rectum	$4a$
Equation of the latus rectum	$y = a$

4. **Downward Parabola**($x^2 = -4ay$)



Standard form	$x^2 = -4ay$
Vertex	$O(0, 0)$
Focus	$F(0, -a)$
Equation of the directrix	$y = a$
Axis	y-axis
Symmetry	$-y$- axis
Equation of the axis	$x = 0$
Length of the latus rectum	$4a$
Equation of the latus rectum	$y = -a$

Problems

1. Find the coordinates of the focus, axis, the equation of the directrix and the length of the latus rectum of the following parabolas:

   (a) $y^2 = 8x$

   (b) $y^2 = -8x$ (Improvement 2017)

   (c) $x^2 = 6y$

   (d) $x^2 = -16y$ (Improvement 2011)

   (e) $y^2 = 12x$

   (f) $y^2 = 10x$

   (g) $x^2 = -9y$

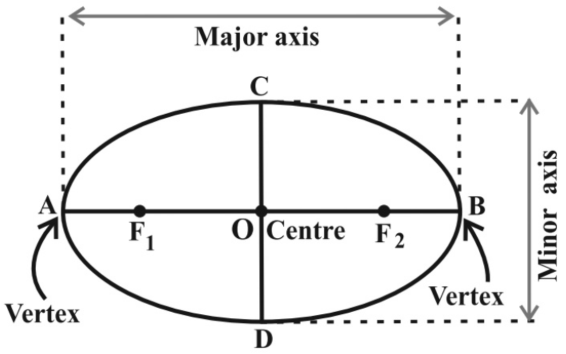
2. Find the equation of the parabola that satisfies the given conditions:
   (a) Focus $(6,0)$; directrix $x = -6$ (March 2017, March 2009)
   (b) Focus $(0,-3)$; directrix $y = 3$
   (c) Focus $(2,0)$; directrix $x = -2$
   (d) Vertex$(0,0)$; Focus $(3,0)$
   (e) Vertex$(0,0)$; Focus $(0,2)$ (Improvement 2009)
   (f) Vertex$(0,0)$; Focus $(-2,0)$
   (g) Symmetric about the $y-$ axis and passes through the point $(2,-3)$
   (h) Vertex $(0,0)$ passing through $(2,3)$ and axis is along $x-$ axis.
   (i) Vertex $(0,0)$ passing through $(5,2)$ and axis is along $y-$ axis.

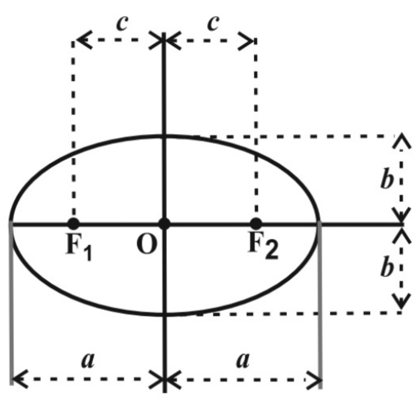
Ellipse

- An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

Terms related to Ellipse

- **Foci:** The two fixed points are called **foci** of the ellipse.
- **Centre:** The mid point of the line segment joining the foci is called the **centre** of the ellipse.
- **Major Axis:** The line segment through the foci of the ellipse is called the **major axis**.
- **Minor Axis:** The line segment through the centre and perpendicular to the major axis is called the **minor axis**.
- **Vertices:** The end points of the major axis are called the **vertices** of the ellipse.
- **Eccentricity:** The eccentricity(e) of an ellipse is the ratio of the distances from the centre of the ellipse to one of the foci and to one of the vertices of the ellipse. The eccentricity of an ellipse is always $< 1 (e < 1)$.
- **Latus Rectum:** Latus rectum of an ellipse is a line segment perpendicular to the major axis through any of the foci and whose end points lie on the ellipse.





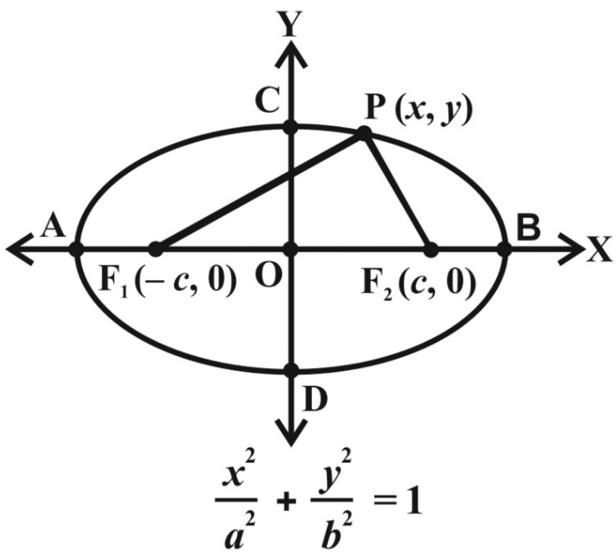
Notation:

1. Length of the semi major axis $= a$, so length of major axis $= 2a$
2. Length of the semi minor axis $= b$, so the length of minor axis $= 2b$
3. Distance between centre and focus $= c$, so the distance between the foci $= 2c$

$$\boxed{\text{Relationship between } a, b \text{ and } c \text{ is } a^2 = b^2 + c^2}$$

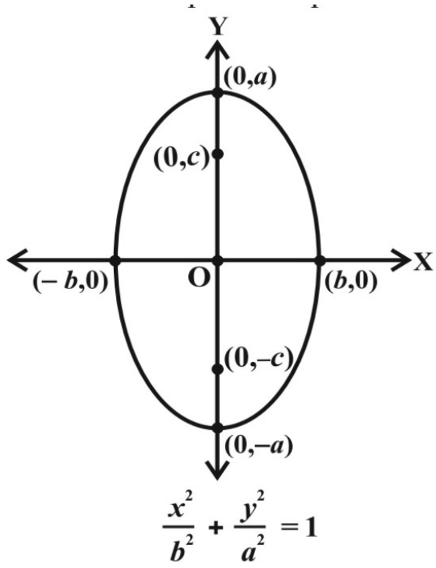
Standard Equations of Ellipse

1. **Major axis along the x- axis** $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right)$



Standard form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a^2 > b^2$
Centre	$(0, 0)$
Foci	$(\pm c, 0)$
Vertices	$(\pm a, 0)$
Equation of the major axis	$y = 0$
Equation of the minor axis	$x = 0$
Length of major axis	$2a$
Length of minor axis	$2b$
Length of latus rectum	$\frac{2b^2}{a}$
Eccentricity(e)	$\frac{c}{a}$

2. **Major axis along the y- axis** $\left(\frac{x^2}{b^2}+\frac{y^2}{a^2}=1\right)$



Standard form	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1,\ a^2>b^2$
Centre	$(0,0)$
Foci	$(0,\pm c)$
Vertices	$(0,\pm a)$
Equation of the major axis	$x=0$
Equation of the minor axis	$y=0$
Length of major axis	$2a$
Length of minor axis	$2b$
Length of latus rectum	$\frac{2b^2}{a}$
Eccentricity(e)	$\frac{c}{a}$

Problems

1. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

   (a) $\frac{x^2}{25}+\frac{y^2}{9}=1$ (Improvement 2017, March 2016, March 2014, March 2013)

   (b) $\frac{x^2}{4}+\frac{y^2}{9}=1$

   (c) $4x^2+9y^2=36$ (March 2011)

   (d) $36x^2+4y^2=144$

   (e) $\frac{x^2}{36}+\frac{y^2}{16}=1$

   (f) $\frac{x^2}{4}+\frac{y^2}{25}=1$

   (g) $\frac{x^2}{25}+\frac{y^2}{100}=1$

(h) $\dfrac{x^2}{49} + \dfrac{y^2}{36} = 1$

(i) $\dfrac{x^2}{100} + \dfrac{y^2}{400} = 1$

(j) $16x^2 + y^2 = 16$

(k) $100x^2 + 25y^2 = 2500$ (Improvement 2015)

2. Find the equation for the ellipse that satisfies the given conditions:

(a) Vertices $(\pm 13, 0)$, foci $(\pm 5, 0)$. (Improvement 2014)

(b) Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$.

(c) Vertices $(0, \pm 13)$, foci $(0, \pm 5)$.

(d) Vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$.

(e) Length og major axis 20, foci $(0, \pm 5)$ (March 2015)

(f) Length of major axis 26, foci $(\pm 5, 0)$

(g) Length of minor axis 16, foci $(0, \pm 6)$

(h) Ends of major axis $(0, \pm\sqrt{5})$, ends of minor axis $(\pm 1, 0)$

(i) Foci $(\pm 3, 0)$, $a = 4$

(j) $b = 3, c = 4$, centre at the origin; foci on the x-axis.

(k) Major axis along the $x-$ axis and passing through the points $(4, 3)$ and $(-1, 4)$ (March 2010)

(l) Major axis on the x- axis and passes through the points $(4, 3)$ and $(6, 2)$.

(m) Centre $(0, 0)$, major axis on the y-axis and passes through the points $(3, 2)$ and $(1, 6)$.

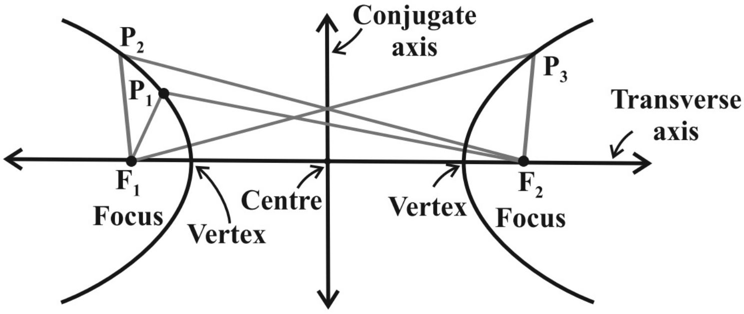
Hyperbola

- A hyperbola is the set of all points in a plane, the difference of whose distances from fixed points in the plane is a constant.

Terms related to Hyperbola

- **Foci:** The two fixed points are called **foci** of the hyperbola.
- **Centre:** The mid point of the line segment joining the foci is called the **centre** of the hyperbola.
- **Transverse Axis:** The line segment through the foci of the hyperbola is called the **transverse axis**.
- **Conjugate Axis:** The line segment through the centre and perpendicular to the transverse axis is called the **transverse axis**.
- **Vertices:** The points at which the hyperbola intersects the transverse axis are called the **vertices** of the hyperbola

- **Eccentricity:** The eccentricity(e) of a hyperbola is the ratio of the distances from the centre of the hyperbola to one of the foci and to one of the vertices of the hyperbola. The eccentricity of a hyperbola is always $> 1 (e > 1)$.
- **Latus Rectum:** Latus rectum of a hyperbola is a line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola.

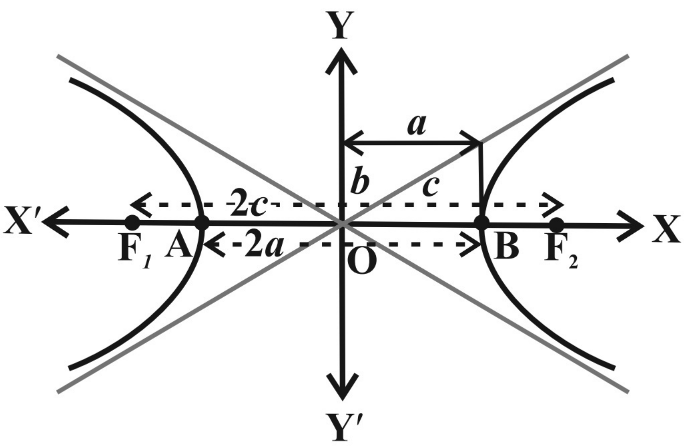


$$P_1F_2 - P_1F_1 = P_2F_2 - P_2F_1 = P_3F_1 - P_3F_2$$

Notation:

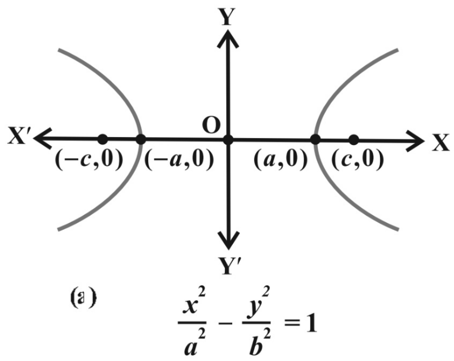
1. Length of the transverse axis $= 2a$
2. Length of the conjugate axis $= 2b$.
3. Distance between centre and focus $= c$, so the distance between the foci $= 2c$

$$\boxed{\text{Relationship between } a, b \text{ and } c \text{ is } c^2 = a^2 + b^2}$$



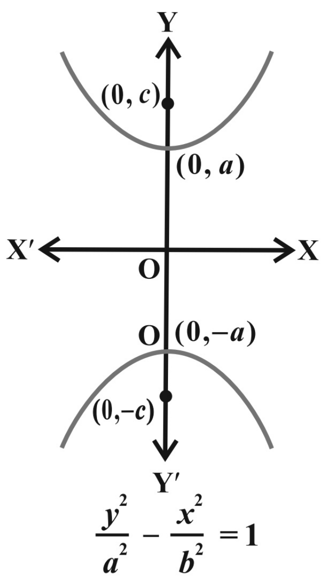
Standard Equations of Hyperbola

1. **Transverse axis along the x- axis** $\left(\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1\right)$



Standard form	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Centre	$(0, 0)$
Foci	$(\pm c, 0)$
Vertices	$(\pm a, 0)$
Equation of the transverse axis	$y = 0$
Equation of the conjugate axis	$x = 0$
Length of transverse axis	$2a$
Length of conjugate axis	$2b$
Length of latus rectum	$\frac{2b^2}{a}$
Eccentricity(e)	$\frac{c}{a}$

2. **Transverse axis along the y- axis** $\left(\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1\right)$



Standard form	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Centre	$(0, 0)$
Foci	$(0, \pm c)$
Vertices	$(0, \pm a)$
Equation of the transverse axis	$x = 0$
Equation of the conjugate axis	$y = 0$
Length of transverse axis	$2a$
Length of conjugate axis	$2b$
Length of latus rectum	$\frac{2b^2}{a}$
Eccentricity(e)	$\frac{c}{a}$

Problems

1. Find the coordinates of the foci, the vertices, the length of transverse axis, the conjugate axis, the eccentricity and the length of the latus rectum of the hyperbola.

   (a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

   (b) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

   (c) $\frac{y^2}{9} - \frac{x^2}{27} = 1$

   (d) $9y^2 - 16x^2 = 36$‘

   (e) $y^2 - 16x^2 = 16$

   (f) $16x^2 - 9y^2 = 576$

   (g) $5y^2 - 9x^2 = 36$

   (h) $49y^2 - 16x^2 = 784$

2. Find the equation for the hyperbola that satisfies the given conditions:

   (a) Foci $(0, \pm 3)$, vertices $\left(0, \pm\frac{\sqrt{11}}{2}\right)$

   (b) Foci $(0, \pm 12)$, length of latus rectum 36.

   (c) Vertices $(0, \pm 5)$, foci $(0, \pm 8)$

   (d) Vertices $(\pm 2, 0)$, foci $(\pm 3, 0)$

   (e) Vertices $(0, \pm 3)$, foci $(0, \pm 5)$

   (f) Foci $(\pm 5, 0)$, transverse axis of length 8.

   (g) Foci $(0, \pm 13)$, conjugate axis of length 24.

   (h) Foci $(\pm 3\sqrt{5}, 0)$, length of latus rectum 8.

   (i) Foci $(\pm 4, 0)$, length of latus rectum 12.

   (j) Vertices $(\pm 7, 0)$, $e = \frac{4}{3}$

   (k) Foci $(0, \pm\sqrt{10})$, passing through $(2, 3)$