Sample Questions

Question 1:

Find $\lim\limits_{x\to 1}\frac{x^2-1}{x-1}$

Solution:

$$\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{x\to 1}\frac{(x-1)(x+1)}{x-1}$$

$$=\lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$$

$$=\lim_{x\to 1}(x+1)=1+1=\underline{\underline{2}}$$

Question 2:

Find $\lim\limits_{x\to -3}\frac{x^2+7x+12}{x^2-9}$

Solution:

$$\lim_{x\to -3}\frac{x^2+7x+12}{x^2-9}=\lim_{x\to -3}\frac{(x+4)(x+3)}{(x+3)(x-3)}$$

$$=\lim_{x\to -3}\frac{(x+4)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$$

$$=\lim_{x\to -3}\frac{(x+4)}{(x-3)}=\frac{-3+4}{-3-3}=\underline{\underline{-\frac{1}{6}}}$$

Question 3:

Evaluate $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-1}{x}$

Solution:

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\cdot\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}+1\right)}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x\to 0}\frac{1+x-1}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x\to 0}\frac{x}{x\left(\sqrt{1+x}+1\right)}$$

$$= \lim_{x\to 0}\frac{1}{\left(\sqrt{1+x}+1\right)} = \frac{1}{\left(\sqrt{1+0}+1\right)} = \frac{1}{2}$$

Question 4 :

Find $\lim_{x\to 1}\frac{x^5-1}{x-1}$

Solution :

$$\lim_{x\to 1}\frac{x^5-1}{x-1} = 5(1)^{5-1} = 5(1)^4 = 5$$

Question 5 :

Find $\lim_{x\to 0}\frac{\sin 6x}{6x}$

Solution :

$$\lim_{x\to 0}\frac{\sin 6x}{6x} = \lim_{6x\to 0}\frac{\sin 6x}{6x}$$
$$= 1$$

Question 6 :

Find $\lim_{\theta\to 0}\frac{\tan\theta-\sin\theta}{\sin^3\theta}$

Solution :

$$\lim_{\theta\to 0}\frac{\tan\theta-\sin\theta}{\sin^3\theta} = \lim_{\theta\to 0}\frac{\frac{\sin\theta}{\cos\theta}-\sin\theta}{\sin^3\theta}$$
$$= \lim_{\theta\to 0}\frac{\sin\theta-\sin\theta\cos\theta}{\sin^3\theta.\cos\theta}$$
$$= \lim_{\theta\to 0}\frac{\sin\theta(1-\cos\theta)}{\sin^3\theta.\cos\theta}$$

$$= \lim_{\theta \to 0} \frac{(1-\cos\theta)}{\sin^2\theta . \cos\theta}$$

$$= \lim_{\theta \to 0} \frac{(1-\cos\theta)}{\left(1-\cos^2\theta\right).\cos\theta}$$

$$= \lim_{\theta \to 0} \frac{(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)\cos\theta}$$

$$= \lim_{\theta \to 0} \frac{1}{(1+\cos\theta)\cos\theta} = \frac{1}{(1+1)1} = \underline{\underline{\frac{1}{2}}}$$

Question 7 :

Find $\lim_{x \to 0} \frac{\sin 4x}{x}$

Solution :

$$\lim_{x \to 0} \frac{\sin 4x}{x} = \lim_{x \to 0} \frac{\sin 4x}{x} \times \frac{4}{4}$$

$$= \lim_{x \to 0} \frac{\sin 4x}{4x} \times 4$$

$$= 4 \lim_{x \to 0} \frac{\sin 4x}{4x}$$

$$= 4 \times 1 = \underline{\underline{4}}$$

Question 8 :

Find $\lim_{x \to 0} \frac{\sin 3x}{\sin 8x}$

Solution :

$$\lim_{x \to 0} \frac{\sin 3x}{\sin 8x} = \lim_{x \to 0} \frac{\frac{\sin 3x}{x}}{\frac{\sin 8x}{x}}$$

$$= \frac{3}{8} \lim_{x \to 0} \frac{\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8x}}$$

$$= \underline{\underline{\frac{3}{8}}}$$

Question 9 :

Find $\lim_{\theta\to 0}\frac{e^{\theta}-\sin\theta-1}{\theta}$

Solution :

$$\lim_{\theta\to 0}\frac{e^{\theta}-\sin\theta-1}{\theta}=\lim_{\theta\to 0}\frac{e^{\theta}-1-\sin\theta}{\theta}$$

$$=\lim_{\theta\to 0}\frac{\left(e^{\theta}-1\right)-\sin\theta}{\theta}$$

$$=\lim_{\theta\to 0}\frac{\left(e^{\theta}-1\right)}{\theta}-\frac{\sin\theta}{\theta}$$

$$=\lim_{\theta\to 0}\frac{\left(e^{\theta}-1\right)}{\theta}-\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1-1=\underline{\underline{0}}$$

Question 9 :

Find the derivative of $\frac{x+1}{x-1}$ from first principle.

Solution :

Let $f(x)=\frac{x+1}{x-1}, f(x+h)=\frac{(x+h)+1}{(x+h)-1}=\frac{(x+h+1)}{(x+h-1)}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{(x+h+1)}{(x+h-1)}-\frac{x+1}{x-1}}{h}$$

$$=\lim_{h\to 0}\frac{(x-1)(x+h+1)-(x+1)(x+h-1)}{h(x+h-1)(x-1)}$$

$$=\lim_{h\to 0}\frac{x^2+xh+x-x-h-1-x^2-xh+x-x-h+1}{h(x+h-1)(x-1)}$$

$$=\lim_{h\to 0}\frac{-h-h}{h(x+h-1)(x-1)}$$

$$=\lim_{h\to 0}\frac{-2h}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{-2}{(x+h-1)(x-1)}$$

$$= \frac{-2}{(x-1)(x-1)}$$

$$= \frac{-2}{(x-1)^2}$$

Question 9 :

Find the derivative of the following

(a) $5\sec x + 3\tan x$

(b) sinx cosx

(c) $\frac{\text{sinx}}{\text{cosx}}$

Solution :

(a) $5\sec x + 3\tan x$

$$\frac{d}{dx}(5\sec x + 3\tan x) = \frac{d}{dx}(5\sec x) + \frac{d}{dx}(3\tan x)$$

$$= 5\frac{d}{dx}(\sec x) + 3\frac{d}{dx}(\tan x)$$

$$= 5\sec x.\tan x + 3\sec^2 x$$

(b) sinx cos2x

By Leibnitz product rule

$$\frac{d}{dx}(\text{sinx cosx}) = \text{sinx}\frac{d}{dx}(\text{cosx}) + \text{cosx}\frac{d}{dx}(\sin x)$$

$$= \text{sinx}(-\text{sinx}) + \cos x \cos x$$

$$= -\sin^2 x + \cos^2 x$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x$$

(c) $\frac{\text{sinx}}{\text{cosx}}$

$$\frac{d}{dx}\left(\frac{\text{sinx}}{\text{cosx}}\right) = \frac{\text{cosx}\frac{d}{dx}(\text{sinx}) - \text{sinx}\frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

EXERCISE

1.

a) $\frac{d}{dx}\left(\frac{x^n}{n}\right) = \ldots\ldots\ldots$

b) Differentiate $\frac{\sin x}{x+1}$ w.r.t x

c) Using first principle, find the derivative of cosx

(March 2016)

Hint or Answer:

a) x^{n-1} b) $\frac{x\cos x + \cos x - \sin x}{(x+1)^2}$ c) $-\sin x$

OR

2.

a) $\frac{d}{dx}(-\sin x) = \ldots\ldots\ldots$

b) Find $\frac{dy}{dx}$, if $y = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$;a, b are constants

c) Using first principle, find the derivative of sinx

(March 2016)

Hint or Answer:

a) $-\cos x$ b) $\frac{-4a}{x^5} + \frac{2b}{x^3} - \sin x$ c) $\cos x$

3.

a) $\lim\limits_{x \to 0} \frac{\tan x}{x} = \ldots\ldots\ldots$

b) Find $\lim\limits_{x \to 0} \frac{\sin 4x}{\sin 2x}$

c) Find the derivative of cosx using first principle.

(Imp 2015)

Hint or Answer:

a) 1

b) 1

c) $-\sin x$

4.

a) Derivative of $x^2 - 2$ at x = 10 is .....

b) $f(x) = \begin{cases} 2x+3 & x \le 0 \\ 3(x+1) & x > 0 \end{cases}$. Find $\lim\limits_{x \to 0} f(x)$ and $\lim\limits_{x \to 1} f(x)$

c) If $xy = c^2$, prove that $x^2 \frac{dy}{dx} + c^2 = 0$

(Imp 2015)

Hint or Answer:

a) 20

b) 3,6

c)

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