

CHAPTER 13

LIMITS AND DERIVATIVES

IMPROVEMENT 2018

1. Consider the real valued function

$$f(x) = \frac{x-3}{x^2-x-6}$$

- a) Find the domain of $f(x)$ (2)
 b) Evaluate $\lim_{x \rightarrow 3} f(x)$ (1)

2. a) Find the derivative of $f(x) = x \sin x$ with respect to x . (1)
 b) Find the derivative of the function $y = \sqrt{x}$ with respect to x by using first principles. (3)

MARCH 2018

3. a) $\lim_{x \rightarrow 2} [x] = \dots\dots\dots$ (1)
 i) 2 ii) 3
 iii) 0 iv) does not exist
 b) Evaluate: $\lim_{x \rightarrow 2} \frac{x^3 - 4x^2 + 4x}{x^2 - 4}$ (3)
 4. a) Find the derivative of $y = \sin x$ from the first principle. (3)
 b) Find $\frac{dy}{dx}$, if $y = \frac{x^5 - \cos x}{\sin x}$ (3)

IMPROVEMENT 2017

5. a) Find $\lim_{x \rightarrow 2} (x^2 - 2)$ (1)
 b) Find the derivative of $\frac{1}{x}$ from the first principles. (3)
 c) Find the derivative of $x \sin x$ (2)

OR

- a) Find $\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$ (2)
 b) Find the derivative of $f(x) = \cos x$ from the first principles. (3)
 c) $\frac{d}{dx}(x^n) = \dots\dots\dots$ (1)

MARCH 2017

6. a) $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x} = \dots\dots\dots$
 i) 0 ii) 1
 iii) 2 iv) 3 (1)
 b) Find $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$ (2)
 c) Find the derivative of $f(x) = \sin x$ by using the first principle. (3)

IMPROVEMENT 2016

7. a) $\frac{d}{dx}(\tan x) = \dots\dots\dots$ (1)
 b) Compute: $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$. (2)
 c) Using the first principle, find the derivative of $\cos x$ (3)

OR

- a) $\frac{d}{dx}(9 + \sin x) = \dots\dots\dots$ (1)
 b) Evaluate: $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$ (2)
 c) Find $\frac{dy}{dx}$, if $y = \frac{4 + 5 \sin x}{3 + 7 \cos x}$ (3)

MARCH 2016

8. a) $\frac{d}{dx}\left(\frac{x^n}{n}\right) = \dots\dots\dots$ (1)

b) Differentiate $y = \frac{\sin x}{x+1}$ with respect to x (2)

c) Using first principles, find the derivative of $\cos x$. (3)

OR

a) $\frac{d}{dx}(-\sin x) = \dots\dots\dots$ (1)

b) Find $\frac{dy}{dx}$ if $y = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$, where a, b are constants. (2)

c) Using first principles, find the derivative of $\sin x$ (3)

IMPROVEMENT 2015

9. a) $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \dots\dots\dots$

- i) 1 ii) -1
iii) 0 iv) 2 (1)

b) Find $\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 2x}$ (2)

c) Find the derivative of $\cos x$ using first principle. (3)

OR

a) Derivative of $x^2 - 2$ at $x = 10$ is $\dots\dots\dots$ (1)

- i) 10 ii) 20
iii) -10 iv) -20

b) If $f(x) = \begin{cases} 2x+3, & x \leq 0 \\ 3(x+1), & x > 0 \end{cases}$

Find $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 1} f(x)$. (2)

c) If $xy = c^2$, prove that $x^2 \frac{dy}{dx} + c^2 = 0$ (3)

MARCH 2015

10. a) Match the following: (3)

i) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$	1) 1
ii) $\lim_{x \rightarrow 0} \cos 5x$	2) 0
iii) $\lim_{x \rightarrow 0} \frac{\sin 8x}{x}$	3) 2
	4) 8

b) Find the derivative of $\tan x$ using the first principle. (3)

OR

a) Match the following: (3)

i) $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$	1) 0
ii) $\lim_{x \rightarrow 0} \frac{\sin \pi x}{\cos 2x}$	2) 32
iii) $\lim_{r \rightarrow 1} \pi r^2$	3) 3.0
	4) π

b. If $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots\dots\dots + \frac{x^2}{2} + x + 1$,
prove that $f'(1) = 100 f'(0)$. (3)

IMPROVEMENT 2014

11. a) Evaluate $\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$. (3)

b) Using first principles, find the derivative of $\cos x$ (3)

OR

Find the derivative of $\frac{\cos x}{2x+3}$ w.r.t. x .

MARCH 2014

12. a) Find the derivative of $\sin x$, using first principle. (3)

OR

Find the derivative of $\frac{x^5 - \cos x}{\sin x}$, using quotient rule. (3)

13. a) The value of $\lim_{x \rightarrow 0} \frac{\sin 5x}{5x}$ is (1)
- b) Evaluate $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$, $ab \neq 0$ (2)

IMPROVEMENT 2013

14. If $f(x) = \begin{cases} a+bx & ; x < 1 \\ 4 & ; x = 1 \\ b-ax & ; x > 1 \end{cases}$ and $\lim_{x \rightarrow 1} f(x) = f(1)$, then find the values of a and b . (3)

15. Find the derivative of $y = \operatorname{cosec} x$, using first principle. (3)

OR

Find the derivative of $\frac{x+1}{x-1}$, using first principle. (3)

MARCH 2013

16. a) The value of $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$ is (1)
- b) Evaluate $\lim_{x \rightarrow 1} \frac{x^{15} - 1}{x^{10} - 1}$. (2)

17. Find the derivative of $y = \sin x$, using first principle. (3)

OR

Find the derivative of $\frac{x + \cos x}{x - 1}$ w.r.t. x . (3)

IMPROVEMENT 2012

18. Evaluate $\lim_{x \rightarrow 0} \frac{(x+5)^2 - 25}{x}$ (2)

19. Find the derivative of the following:

i) $f(x) = \frac{2x+3}{x-1}$ (2)

ii) $f(x) = \cos(x-5)$ (2)

OR

Compute the derivative of $\sec x \sec x$ with respect to x from first principle. (4)

MARCH 2012

20. a) Evaluate $\lim_{x \rightarrow 3} \left(\frac{x^3 - 27}{x^2 - 9} \right)$ (1)

b) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{\sin^3 x} \right)$ (2)

21. Prove that $\frac{d}{dx} \left(\frac{\cos x}{1 + \sin x} \right) = \frac{-1}{1 + \sin x}$ (3)

OR

Find the derivative of $\cot x$, using first principle. (3)

IMPROVEMENT 2011

22. a) Evaluate $\lim_{x \rightarrow -1} \frac{x^5 + 1}{x + 1}$ (1)

- b) Find the derivative of $\lim_{x \rightarrow -1} x^2 + x + 1$ from first principle. (2)

23. a) Differentiate $\frac{x^2 \tan x}{1+x}$ (3)

OR

b) Differentiate $\frac{x + 2 \cos x}{3x + 4 \sin x}$. (3)

MARCH 2011

24. i) Evaluate $\lim_{x \rightarrow -1} \frac{x^2 - 5x + 6}{x - 1}$ (3)

ii) Evaluate $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$ (3)

25. Using first principle, find the derivative of

$f(x) = \frac{1}{x}$ (3)

OR

Using quotient rule, find the derivative of

$f(x) = \cot x$. (3)

IMPROVEMENT 2010

26. Choose the most appropriate answer from those given in the bracket, choose either (a), (b), (c) or (d), (e), (f).

a) If $\lim_{x \rightarrow 2} \frac{x^p - 2^p}{x - 2} = 192$, then

$P = [2, 4, 6, 10]$ (1)

b) $\lim_{x \rightarrow 0} \frac{\sqrt{1-x} - 1}{x} = \dots$
 $\left[0, 1, \frac{1}{2}, \text{not defined}\right]$ (1)

c) $\lim_{x \rightarrow 0} \frac{\sin ax}{x \cos bx} = \dots$
 $[0, a, b, \text{not defined}]$ (1)

OR

d) $\lim_{x \rightarrow 0} \frac{\sqrt{z} - 1}{1 - z} = \dots$ (1)
 $\left[0, -\frac{1}{2}, \frac{1}{2}, 1\right]$

e) $\lim_{x \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} - x\right)}{\left(\frac{\pi}{4} - x\right)}$ is (1)

$\left[0, 1, \frac{\pi}{4}, \text{not defined}\right]$

f) If $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = k \neq 0$, find $\lim_{x \rightarrow 0} f(x) =$
 $[0, 1, k, \text{not defined}]$ (1)

MARCH 2010

27. i) Evaluate $\lim_{x \rightarrow 1} \frac{x^7 - 1}{x^4 - 1}$ (1)

ii) Evaluate $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$ (2)

28. If $xy = c^2$, prove that $x^2 \frac{dy}{dx} + c^2 = 0$ (3)

OR

Find the derivative of $\tan x$ from first principle.

IMPROVEMENT 2009

29. i) Evaluate $\lim_{x \rightarrow a} \frac{\sqrt{x} + \sqrt{a}}{x + a}$. (1)

ii) Evaluate $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$ (2)

iii) Evaluate $\lim_{x \rightarrow 0} (\cos ex - \cot x)$. (3)

MARCH 2009

30. a) Choose the correct value of

$\lim_{x \rightarrow -1} \left[\frac{x^{10} + x^5 + 1}{x - 1} \right]$ from the bracket. (1)
 $\left[\frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, 0 \right]$

b) Evaluate $\lim_{x \rightarrow 1} \left[\frac{\sqrt{1+x}-1}{x} \right]$ (2)

c) Evaluate $\lim_{x \rightarrow 0} \left[\frac{\sqrt{1-\cos 4x}}{x^2} \right]$ (3)

