Chapter 14:Mathematical Reasoning

Sample Questions

Question 1 :

Determine which of the following are statements and give reasons for your answer.

(1) Tomorrow is a holiday.

(2) For real number x, [x] is an integer.

(3) He died young.

(4) Galois was a mathematician. He died young.

Solution :

(1) It is not a statement. It involves variable time.

(2) Though x is a variable, it is a true statement for every x.So it is a statement.

(3) Pronoun is used. It is not a statement. Whom does 'he' refer to ?

(4) Here 'he' refers to Galois. This is a statement.

Question 2 :

Give negations of following statements.

(1) Christmas is celebrated on 25th of December.

(2) Diwali marks the end of current Hindu year.

Solution :

(1) Christmas is not celebrated on 25th of December.

(2) Diwali does not mark the end of current Hindu year.

Question 3 :

Identify component statements in the following compound statements and determine the truth value of the compound statement.

(1) Delhi is the capital of Gujarat and 7 X 5 = 75.

(2) Ahmedabad and Vadodara are cities of Gujarat.

Solution :

(2) Let p : Delhi is the capital of Gujarat.

q : 7 X 5 = 75

p and q are both false and hence given statement $p \wedge q$ is also false.

(3) Let p : Ahmedabad is a city of Gujarat.

q : Vadodara is a city of Gujarat.

p and q both are true and hence $p \wedge q$ is also true.

Question 4 :

Determine where 'or' used in following example is in inclusive sense or exclusive sense.

'Two distinct coplanar lines intersect in a point or are parallel.'

Solution :

Let p : Two distinct coplanar lines intersect in a point.

q : Two distinct coplanar lines are parallel.

The given disjunction is $p \wedge q$ and it is in exclusive sense as lines intersecting in a point and to be parallel are exclusive events.

Question 5 :

Express the following in the implication form.

(1) The square of an even number is even.

(2) The sum of digits of an integer is divisible by 9, if it is divisible by 9.

(3) The roads will be wet only if it rains.

Solution :

(1) Let p : x is an even number.

q : x^2 is even.

Implication $p \Rightarrow q$ is : if x is an even number, x^2 is even.

(2) Let p : An integer is divisible by 9.

q : The sum of its digits is divisible by 9.

The implication $p \Rightarrow q$ is : if an integer is divisible by 9, the sum of its digits is divisible by 9.

Question 6 :

For each of the following statements, identify component statements and state whether the compound statements are true or not.

(1) If a triangle is equiangular, all its sides are congruent.

(2) If a number is a real number, it is a natural number.

Solution :

(1) Let p : A triangle is equiangular.

      q : All the sides of the triangle are congruent.

We have $p \Rightarrow q$. Now it cannot happen that p is true and q is false. If p is true, then q is true.

Therefore, $p \Rightarrow q$ is true.

(2) Let p : A number is a real number.

      q : The number is a natural number.

Here $\sqrt{2} \in R$ and $\sqrt{2} \notin N$.

Thus p is true and q is false for the number $\sqrt{2}$.

Thus there exists a real number for which $p \Rightarrow q$ is false.

$\therefore p \Rightarrow q$ is false.

Question 7 :

Give contrapositive and converse of following.

(1) If it rains, the roads are wet.

(2) If x is prime, then x is odd.

Solution :

(1) Let p : It rains.

      q : The roads are wet.

Contrapositive is, 'if the roads are not wet, it has not rained'.

Converse is, 'if the roads are wet, it has rained'.

(2) p : x is prime.

   q : xis odd.

$\sim q \Rightarrow \sim p$ is 'if x is not odd, then x is not a prime'.

Converse is 'if x is odd, then x is a prime'.

Question 8 :

Using the method of contradiction, prove that sum of an irrational number and a rational number is irrational.

Solution :

Let x be an irrational number and y be a rational number.

Let x + y = z be a rational number, if possible.

Since z and y are both rational, z-y is also rational.

∴ x = z - y is also rational.

But x is irrational.

We come to a contradiction.

∴ z = x + y is irrational.

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EXERCISE

1.

   a)   Write the negation of the statement:

        "Every natural number is greater than zero".

   b)   Verify by the method of contradiction:

        "$\sqrt{13}$ is irrational."

(March 2016)

2.

   a)   Write the negation of the statement:

        " $\sqrt{7}$ is irrational."

   b)   Prove that : " $\sqrt{7}$ is irrational." by the method of contradiction.

(March 2015)

3.

   a)   Write the negation of the statement: " $\sqrt{5}$ is irrational."

b) Verify by the method of contradiction: " $\sqrt{2}$ is irrational."

(March 2014)

4.

a) Write the negation of the statement:

"All triangles are not equilateral triangle".

b) Verify by the method of contradiction:

" $\sqrt{7}$ is irrational."

(March 2013)

5.

Consider the statement:

"If x is an integer and x^2 is even, then x is also even".

a) Write the converse of this statement.

b) Prove the statement by the contra-positive method.

(March 2012)

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