Question 1 :

Expand $(x+4)^5$

Solution :

$(x+4)^5$

$= {}^5C_0(x)^5 + {}^5C_1(x)^4(4) + {}^5C_2(x)^3(4)^2 + {}^5C_3(x)^2(4)^3$
$+ {}^5C_4(x)^1(4)^4 + {}^5C_5(x)^0(4)^5$

$= x^5 + 5(x^4)(4) + 10(x^3)(16) + 10(x^2)(64) + 5(x)(256) + (1024)$

$= x^5 + 20x^4 + 160x^3 + 640x^2 + 1280x + 1024$

Question 2 :

Expand $(2x-3)^6$

Solution :

$(2x-3)^6$

$= {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3$
$+ {}^6C_4(2x)^2(3)^4 - {}^6C_5(2x)(3)^5 + {}^6C_6(3)^6$

$= 64x^6 - 6(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27)$
$+ 15(4x^2)(81) - 6(2x)(243) + 729$

$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

Question 3 :

Expand $\left(2x^2 - \frac{3}{x}\right)^4$

Solution :

$\left(2x^2 - \frac{3}{x}\right)^4$

$= {}^4C_0\,(2x^2)^4\left(-\frac{3}{x}\right)^0 + {}^4C_1\,(2x^2)^3\left(-\frac{3}{x}\right)^1 + {}^4C_2\,(2x^2)^2\left(-\frac{3}{x}\right)^2$
$+ {}^4C_3\,(2x^2)^1\left(-\frac{3}{x}\right)^3 + {}^4C_4\,(2x^2)^0\left(-\frac{3}{x}\right)^4$

$$= (1)16x^8(1) + 4(8x^6)\left(-\frac{3}{x}\right)+6(4x^4)\frac{9}{x^2}+4(2x^2)\left(-\frac{27}{x^3}\right)+(1)(1)\left(\frac{81}{x^4}\right)$$

$$= 16x^8 - 96x^5 + 216x^2 - \frac{216}{x} + \frac{81}{x^4}$$

Question 4 :

Expand $\left(\frac{x}{2}+\frac{1}{x}\right)^5$

Solution :

$$\left(\frac{x}{2}+\frac{1}{x}\right)^5$$

$$= {}^5C_0\left(\frac{x}{2}\right)^5\left(\frac{1}{x}\right)^0 + {}^5C_1\left(\frac{x}{2}\right)^4\left(\frac{1}{x}\right)^1 + {}^5C_2\left(\frac{x}{2}\right)^3\left(\frac{1}{x}\right)^2$$
$$+ {}^5C_3\left(\frac{x}{2}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{2}\right)^1\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{x}{2}\right)^0\left(\frac{1}{x}\right)^5$$

$$= 1\left(\frac{x^5}{32}\right)\left(\frac{1}{x}\right)^0 + 5\left(\frac{x^4}{16}\right)\left(\frac{1}{x}\right)+ 10\left(\frac{x^3}{8}\right)\left(\frac{1}{x^2}\right)$$
$$+10\left(\frac{x^2}{4}\right)\left(\frac{1}{x^3}\right)+ 5\left(\frac{x}{2}\right)\left(\frac{1}{x^4}\right)+ 1\left(\frac{1}{x^5}\right)$$

$$= \frac{x^5}{32}+ \frac{5}{16}x^3+ \frac{5}{4}x + \frac{5}{2x}+ \frac{5}{2x^3}+ \frac{1}{x^5}$$

Question 5 :

$(0.99)^5$

Solution :

$(0.99)^5$

$$= (1-0.01)^5$$

$$= {}^5C_0(1)^5(0.01)^0+{}^5C_1(1)^4(0.01)^1+{}^5C_2(1)^3(0.01)^2+{}^5C_3(1)^2(0.01)^3$$
$$+{}^5C_4(1)^1(0.01)^4+ {}^5C_5\ (1)^0(0.01)^5$$

$$= 1(1)^5(0.01)^0+5(1)^4(0.01)^1+10(1)^3(0.01)^2+10(1)^2(0.01)^3$$
$$+5(1)^1(0.01)^4+ 1(1)^0(0.01)^5$$

$= 1+5(0.01)+10(0.0001)+10(0.000001)^3+5(0.00000001)^4 + (0.0000000001)^5$

$= 0.9509900499$

Question 6 :

Using binomial theorem, find the 7th power of 11.

Solution :

11^7

$= (1 + 10)^7$

$= {}^7C_0(1)^7(10)^0+{}^7C_1(1)^6(10)^1+{}^7C_2(1)^5(10)^2+{}^7C_3(1)^4(10)^3 + {}^7C_4(1)^3(10)^4+ {}^7C_5\ (1)^2(10)^5+ {}^7C_6(1)^1(10)^6+ {}^7C_7\ (1)^0(10)^7$

$= 1 + 70 + 2100 + 35000 + 350000 + 2100000 + 7000000 + 10000000$

$= 19487171$

Question 7 :

Find the coefficient of x^5 in the expansion of $\left(x+\frac{1}{x^3}\right)^{17}$

Solution :

The general term is $T_{r+1} = {}^{17}C_r x^{17-r}\left(\frac{1}{x^3}\right)^r = {}^{17}C_r x^{17-4r}$

Let T_{r+1} be the term containing x^5 then, $17 - 4r = 5,\ r = 3$

Question 8 :

Expand $(2x + 3y)^5$

Solution :

$(2x + 3y)^5$

$= {}^5C_0(2x)^5(3y)^0+ {}^5C_1\ (2x)^4(3y)^1+ {}^5C_2(2x)^3(3y)^2 + {}^5C_3(2x)^2(3y)_3+ {}^5C_4(2x)^1(3y)^4+ {}^5C_5(2x)^0(3y)^5$

$= 1(32)x^5(1)+ 5(16x^4)(3y)+ 10(8x^3)(9y^2)+ 10(4x^2)(27y^3) + 5(2x)\ (81y^4)+ (1)(1)(243y^5)$

$= 32x^5+ 240x^4y + 720x^3y2+ 1080x^2y^3+ 810xy^4+ 243y^5$

Question 9 :

Expand $\left(x^2+\frac{2}{x}\right)^4$

Solution :

$$\left(x^2+\frac{2}{x}\right)^4$$

$$={}^4C_0\left(x^2\right)^4+{}^4C_1\left(x^2\right)^3\left(\frac{2}{x}\right)+{}^4C_2\left(x^2\right)^2\left(\frac{2}{x}\right)^2+{}^4C_3\left(x^2\right)^1\left(\frac{2}{x}\right)^3+{}^4C_3\left(\frac{2}{x}\right)^4$$

$$=1\left(x^8\right)+4\left(x^6\right)\left(\frac{2}{x}\right)+6\left(x^4\right)\left(\frac{4}{x^2}\right)+4\left(x^2\right)\left(\frac{8}{x^3}\right)+1\left(\frac{16}{x^4}\right)$$

$$=x^8+8x^5+24x^2+\frac{32}{x}+\frac{16}{x^4}$$

Question 10 :

Find a if the 17th and 18th terms of the expansion $(2+a)^{50}$ are equal.

Solution :

We have $T_{r+1} = {}^nC_r x^{n-r} y^r$

Therefore,

$T_{17} = T_{16+1} = {}^{50}C_{16}\,(2)^{50-16}\,a^{16} = {}^{50}C_{16}\,(2)^{34}\,a^{16}$

$T_{18} = {}^{50}C_{17}\,(2)^{50-17}\,a^{17} = {}^{50}C_{17}\,(2)^{33}\,a^{17}$

Therefore

$${}^{50}C_{16}\,(2)^{34}\,a^{16} = {}^{50}C_{17}\,(2)^{33}\,a^{17}$$

$$\frac{{}^{50}C_{16}\,(2)^{34}}{{}^{50}C_{17}\,(2)^{33}} = \frac{a^{17}}{a^{16}}$$

$$a = \frac{{}^{50}C_{16}\times 2}{{}^{50}C_{17}\,(2)^{33}}$$

$$a = \frac{{}^{50}C_{16}\times 2}{{}^{50}C_{17}\,(2)^{33}} = 1$$

Question 11:

Which is smaller ? $(1.1)^{100000}$ or 100000

Solution:

$(1.1)^{100000} = (1 + 0.1)^{100000}$

= 1 + 100000(0.1) + Sum of some posttlve terms

= 1 + 10000 + Sum of positive terms

> 10000 $\quad\therefore\quad$ 10000 is smaller

Question 12:

If the middle term in the expansion of $\left(\frac{x}{3}+3\right)^{10}$ is 8064, find x.

Solution:

Here n =10

$\therefore$ n is even, so middle term is $\frac{n+2}{2} = 6^{th}$ term

$$\therefore \quad T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} 3^5$$

$$\therefore\ 8064 = 252x^5$$

$$\therefore \quad x^5 = \frac{8064}{252} = 32 = 2^5$$

$$\therefore \quad x = 2$$

EXERCISE

1. a) The 8th terms in the expansion of $\left(\sqrt{2} + \sqrt{3}\right)^7$

$(27\sqrt{2},\ 27\sqrt{3},\ 72\sqrt{2},\ 72\sqrt{3})$

b) Find the term independent of x in the expansion of $\left(x + \frac{1}{2x}\right)^{18}, x > 0$

(March 2016)

2. a) Number of terms in the expansion of $\left(x + \frac{1}{x}\right)^{20}$ is .....

(19, 20, 21, 22)

b) Consider the expansion of $\left(3x^2 - \frac{1}{3x}\right)^9$ .Find the coefficient of x^6 and the term independent of x.

(Imp 2015)

3. a) The number of terms in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is

i) n +1 ii) n iii) 2n +1 iv) 2n + 2

b) Find if the 17th term and 18th term of the expansion of , a $(2+\alpha)^{50}$ are equal.

(March 2015)

4. a) Write the expansion of $(a+b)^n$ where n is any positive integer.

b) Find the value of α if the 17th term and 18th term in the expansion of $(2+a)^{50}$ are equal.

(Imp2014)

5. a) Write the number of terms in the expansion of $(a-b)^{2n}$

b) Find the general term in the expansion of $\left(x^2 - yx\right)^{12}$

c) Find the coefficient of x^6y^3 in the expansion of $(x+2y)^9$

(March 2014)

6. a) Number of terms in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is .....

b) Find the middle term in the above expansion.

(Imp2013)