Sample Questions

Question 1 :

For an A.P. 3, 8, 13, 18, ... find the 17th and 40th terms.

Solution :

$$a = 3, d = 5$$

$$n^{th} \text{ term } a_n = a + (n - 1)d$$
$$= 3 + (n - 1)5$$
$$= 5n - 2$$
$$a_{17} = 5(17) - 2 = \underline{83}$$
$$a_{40} = 5(40) - 2 = \underline{198}$$

Question 2 :

If the 11th term of an A.P. is zero, then prove that its 31st term is double than the 21st term.

Solution :

$$a_{11} = a + 10d$$
$$0 = a + 10d$$
$$2(a_{21}) = 2(a + 20d)$$
$$= 2(a + 40d)$$
$$= (a + 30d) + (a + 10d)$$
$$= a_{31} + 0$$
$$= a_{31}$$

Question 3 :

The sum of n terms of two APs are in the ratio (3n+6) : (5n-13), Find the ratio of their 11th terms.

Solution :

According to the given condition,

$$\frac{\frac{n}{2}\left[2A + (n-1)D\right]}{\frac{n}{2}\left[2a + (n-1)d\right]} = \frac{3n+6}{5n-13}$$

$$\frac{2A+(n-1)D}{2a+(n-1)d} = \frac{3n+6}{5n-13}$$

$$\frac{T_{11}}{t_{11}} = \frac{A+10D}{a+10d}$$

$$= \frac{2A+20D}{2a+20d}$$

$$= \frac{2A+(21-1)D}{2a+(21-1)d}$$

Substituting $n = 21$

$$\frac{T_{11}}{t_{11}} = \frac{3(21)+6}{5(21)-13} = \frac{3}{4}$$

∴ The ratio of the 11th terms of the two A.P.s is 3 : 4.

Question 4 :

The sum and the product of three consecutive terms of an A.P. are 24 and 312 respectively. Find the three terms.

Solution :

Suppose the three consecutive terms of the A.P. are

$$a - d, a, a + d$$

$$(a - d) + a + (a + d) = 24$$

$$3a = 24$$

$$a = 8$$

and

$$(a - d) \times a \times (a + d) = 312$$

$$(8 - d) \times 8 \times (8 + d) = 312$$

$$64 - d^2 = 39$$

$$d = 5 \text{ or } -5$$

∴ The required terms are 3, 8, 13

Question 5 :

If the third term of a G.P. is 18 and its sixth term is 486, find the 9th term.

Solution :

$$t_3 = ar^2 = 18$$

$$t_6 = ar^5 = 486$$

$$\frac{t_6}{t_3} = \frac{ar^5}{ar^2} = \frac{486}{18}$$

$$r^3 = 27$$

$$r = 3$$

Also $ar^2 = 18$.

So $9a = 18$

$$a = 9$$

$$t_9 = ar^8 = 9(3^8) = 13122$$

Question 6 :

Three numbers are consecutive term of a G.P. Their sum and product are $\frac{31}{5}$ and 1 respectively, find the numbers.

Solution :

Let the three numbers are $\frac{a}{r}$, a, ar

$$\left(\frac{a}{r}\right) \times a \times ar = 1$$

$$a^3 = 1 \quad or \quad a = 1$$

and

$$\frac{a}{r} + a + ar = \frac{31}{5}$$

$$\frac{1}{r} + 1 + r = \frac{31}{5}$$

$$5r^2 - 26r + 5 = 0$$

$$(5r - 1)(r - 5) = 0$$

$$r = \frac{1}{5}, 5$$

$\therefore$ The numbers are 5, 1, $\frac{1}{5}$

Question 7 :

Find the sum of the sequence 5, 55, 555, ... upto first n terms.

Solution :

S_n

$= 5 + 55 + 555 + ... \text{ n terms}$

$= \dfrac{5}{9}[9 + 99 + 999 + ... \text{ n terms}]$

$= \dfrac{5}{9}[(10 - 1) + (10^2 - 1) + (10^3 - 1) + ... \text{ n terms}]$

$= \dfrac{5}{9}[(10 + 10^2 + 10^3 + ... \text{ n terms}) - (1 + 1 + 1 + ... \text{ n terms})]$

$= \dfrac{5}{9}\left[\dfrac{10(10^n - 1)}{10 - 1} - n\right]$

$= \dfrac{5}{9}\left[\dfrac{10}{9}(10^n - 1) - n\right]$

$= \dfrac{50}{81}(10^n) - \dfrac{50}{81} - \dfrac{5n}{9}$

Question 8 :

Find the sum of n terms of the series $1 \times 2 + 2 \times 3 + 3 \times 4 ...$

Solution :

$a_n = n(n+1)$

$S_n = \displaystyle\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k(k+1)$

$= \displaystyle\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k$

$= \dfrac{n(n+1)(2n+1)}{6} + \dfrac{n(n+1)}{2}$

$= \dfrac{n(n+1)}{2}\left(\dfrac{(2n+1)}{3} + 1\right)$

$= \dfrac{n(n+1)}{2}\left(\dfrac{(2n+4)}{3}\right) = \dfrac{n(n+1)(n+2)}{3}$

EXERCISE

1.

a) The 8th term in the expansion of $\left(\sqrt{2} + \sqrt{3}\right)^7$ is...

$$\left(27\sqrt{2}\,;27\sqrt{3}\,;72\sqrt{2}\,;72\sqrt{3}\right)$$

b) Find the term independent of x in the expansion of

$$\left(x + \frac{1}{2x}\right)^{18} ; x > 0$$

(March 2016)

2.

a) The nth term of the G. P. 5, 25, 125 ... is .....

$n^5;\ 5^n;\ (2n)^5;\ 5^{2n}$

b) Find the sum of all natural numbers between 200 and 1000 which are multiples of 10.

c) Calculate the sum of n-terms of the series whose n^{th} term is $a_n = n(n + 3)$

(March 2016)

3.

a) Geometric mean of 16 and 4 is ... (20,4,10,8)

b) Find the sum to terms, 5 + 55 + 555 + ...

c) Find the sum to terms of the A.P. whose k^{th} term is 5k + 1

(Imp 2015)

OR

4.

a) If the first 3 terms of an A.P. are x - 1, x + 1, 2x + 3 then x is.......   (-2, 0, 2, 4)

b) Find the sum to *n* terms of the sequence

$$1 \times 2 + 2 \times 3 + 3 \times 4 \ \ ...$$

c) The nth term of a G.P.

$$5, \frac{-5}{2}, \frac{5}{4}, \frac{-5}{8}, \ldots \text{ is } \frac{5}{1024}. \text{ Find } n$$

(Imp 2015)

5.

a) If the sum of a certain number of terms of A.P. 25, 22, 19, ... Is 116, then find the last term.

b) Find the sum to n- terms of the series

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 \ldots$$

OR

6.

a) A man starts repaying a loan as a first installment of Rs.1.000. If he increases the installment by Rs. 150 every month, What amount will he pay in the 30th installment?

b) Find the sum to n-terms of the sequence:

7, 77, 777, 7777, ...

(Imp 2015)

7.

a) If the sum of a certain number of terms of the A.P 25, 22, 19, ... is 116, then find the last term.

b) Find the sum to n terms of the series

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 \ldots$$

(March 2014)

8.

a) Find the sum of multiples of 7 between 200 and 400.
b) The sum of first 3 terms of a Geometric progression is $\frac{39}{10}$ and their product is 1. Find the terms.

(Imp 2014)