Kerala Engineering Entrance Examination 2007
Mathematics

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1. If the curves $x^2 = 9A(9-y)$ and $x^2 = A(y+1)$ intersect orthogonally, then the value of A is

(A) 3 (B) 4 (C) 5 (D) 7 (E) 9

2. If $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$, then $f(x)$ is

(A) increasing in $(-\infty, -2)$ and in $(0, 1)$

(B) increasing in $(-2, 0)$ and in $(1, \infty)$

(C) decreasing in $(-2, 0)$ and in $(0, 1)$

(D) decreasing in $(-\infty, -2)$ and in $(1, \infty)$

(E) increasing in $(-2, 0)$ and in $(0, 1)$

3. If the distance S covered by a particle in time t is proportional to the cube root of its velocity, then the acceleration is

(A) a constant (B) $\propto S^3$ (C) $\propto \frac{1}{S^3}$

(D) $\propto S^5$ (E) $\propto \frac{1}{S^5}$

4. $\int\left(\sin^6 x + \cos^6 x + 3\sin^2 x \cos^2 x\right) dx$ is equal to

(A) $x+c$ (B) $\frac{3}{2}\sin 2x+c$ (C) $-\frac{3}{2}\cos 2x+c$

(D) $\frac{1}{3}\sin 3x - \cos 3x+c$ (E) $\frac{1}{3}\sin 3x + \cos 3x+c$

5. $\int \frac{4^{x+1} - 7^{x-1}}{28^x} dx$ is equal to

(A) $\frac{1}{7\log_e 4} 4^{-x} - \frac{4}{\log_e 7} 7^{-x} + c$

(B) $\frac{1}{7\log_e 4} 4^{-x} + \frac{4}{\log_e 7} 7^{-x} + c$

(C) $\frac{4^{-x}}{\log_e 7} - \frac{7^{-x}}{\log_e 4} + c$

(D) $\frac{4^{-x}}{\log_e 4} - \frac{7^{-x}}{\log_e 7} + c$

(E) $\frac{1}{28}\log_e 4^{-x} + \frac{1}{7}\log_e 7^{-x} + c$

6. The value of $\int e^{\tan^{-1} x} \frac{(1+x+x^2)}{1+x^2} dx$ is

(A) $\tan^{-1} x + c$

(B) $e^{\tan^{-1} x} + 2x + c$

(C) $e^{\tan^{-1} x} + c$

(D) $e^{\tan^{-1} x} - x + c$

(E) $xe^{\tan^{-1} x} + c$

7. The value of $\int \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{3\log_e x} - e^{2\log_e x}} dx$ is

(A) $x^2 + c$

(B) $\frac{x^2}{2} + c$

(C) $\frac{x^3}{3} + c$

(D) $\frac{x}{2} + c$

(E) e

8. If $f(x)=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$ and $g(x)=e^{\sin^{-1}x}$, then $\int f(x)g(x)\,dx$ is equal to

(A) $e^{\sin^{-1}x}\left(\sin^{-1}x-1\right)+C$ (B) $e^{\sin^{-1}x}+C$

(C) $e^{(\sin^{-1}x)^2}+C$ (D) $e^{2\sin^{-1}x}+C$ (E) $e^{\sin^{-1}x}\sin^{-1}x+C$

9. The value of $\int\dfrac{x^2+1}{x^4-x^2+1}dx$ is

(A) $\tan^{-1}(2x^2-1)+c$ (B) $\tan^{-1}\dfrac{x^2+1}{x}+c$ (C) $\sin^{-1}\left(x-\dfrac{1}{x}\right)+c$

(D) $\tan^{-1}x^2+c$ (E) $\tan^{-1}\left(\dfrac{x^2-1}{x}\right)+c$

10. $\int\cos\left\{2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right\}dx$ is equal to

(A) $\dfrac{1}{8}(x^2-1)+c$ (B) $\dfrac{x^2}{4}+c$ (C) $\dfrac{x}{2}+c$

(D) $\dfrac{x}{4}+c$ (E) $\dfrac{x^2}{2}+c$

11. The area bounded by the parabola $y^2=8x$ and its latus rectum in sq. units is

(A) $\dfrac{16}{3}$ (B) $\dfrac{32}{3}$ (C) $\dfrac{8}{3}$ (D) $\dfrac{64}{3}$ (E) $\dfrac{4}{3}$

12. $\int_{\pi/6}^{\pi/3} \frac{1}{1+\tan^3 x}\, dx$ is

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi}{2}$

13. $\int_{-1}^{1} \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2+1}\, dx$ is

(A) $\frac{4}{5}$ (B) $\frac{5}{4}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$ (E) $\frac{2}{3}$

14. If $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then $\frac{1}{I_3 + I_5}$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) 4 (E) 6

15. If $\int_0^{\frac{\pi}{2}} \sin^6 x\, dx = \frac{5\pi}{32}$, then the value of $\int_{-\pi}^{\pi} \left(\sin^6 x + \cos^6 x\right) dx$ is

(A) $\frac{5\pi}{8}$ (B) $\frac{5\pi}{16}$ (C) $\frac{5\pi}{2}$ (D) $\frac{5\pi}{4}$ (E) $\frac{5\pi}{32}$

16. The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{y}{x} + \dfrac{\Phi\left(\frac{y}{x}\right)}{\Phi'\left(\frac{y}{x}\right)}$ is

(A) $x\Phi\left(\dfrac{y}{x}\right) = k$ (B) $\Phi\left(\dfrac{y}{x}\right) = kx$ (C) $y\Phi\left(\dfrac{y}{x}\right) = k$

(D) $\Phi\left(\dfrac{y}{x}\right) = ky$ (E) $\Phi\left(\dfrac{y}{x}\right) = ke^{\frac{y}{x}}$

17. If the integrating factor of the differential equation $\dfrac{dy}{dx} + P(x)y = Q(x)$ is x, then $P(x)$ is

(A) x (B) $\dfrac{x^2}{2}$ (C) $\dfrac{1}{x}$ (D) $\dfrac{1}{x^2}$ (E) $\dfrac{1}{2x}$

18. If c_1, c_2, c_3, c_4, c_5 and c_6 are constants, then the order of the differential equation whose general solution is given by $y = c_1 \cos(x + c_2) + c_3 \sin(x + c_4) + c_5 e^{x+c_6}$ is

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

19. $y = 2e^{2x} - e^{-x}$ is a solution of the differential equation

(A) $y_2 + y_1 + 2y = 0$ (B) $y_2 - y_1 + 2y = 0$ (C) $y_2 + y_1 = 0$

(D) $y_2 - y_1 - 2y = 0$ (E) $y_2 - 2y_1 - y = 0$

20. On the set N of all natural numbers define the relation R by $a\ R\ b$ if and only if the GCD of a and b is 2. Then R is

(A) Reflexive, but not symmetric

(B) Symmetric only

(C) Reflexive and transitive

(D) Reflexive, symmetric and transitive

(E) Not reflexive, not symmetric and not transitive

21. Let Z denote the set of all integers and $A = \{(a, b) : a^2 + 3b^2 = 28,\ a, b \in Z\}$ and $B = \{(a, b) : a > b,\ a, b \in Z\}$. Then the number of elements in $A \cap B$ is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

22. If $f(x) = 2x^2 + bx + c$ and $f(0) = 3$ and $f(2) = 1$, then $f(1)$ is equal to

(A) 1 (B) 2 (C) 0 (D) $\frac{1}{2}$ (E) -2

23. The domain of the real valued function $f(x) = \sqrt{1-2x} + 2\sin^{-1}\left(\frac{3x-1}{2}\right)$ is

(A) $\left[\frac{-1}{3}, 1\right]$ (B) $\left[\frac{1}{2}, 1\right]$ (C) $\left[\frac{-1}{2}, \frac{1}{3}\right]$

(D) $\left[-1, \frac{1}{3}\right]$ (E) $\left[\frac{-1}{3}, \frac{1}{2}\right]$

24. The period of the function $f(x) = a^{\{\tan(\pi x)\}+x-[x]}$, where $a > 0$, [.] denotes the greatest integer function and x is a real number, is

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) 2π (E) 1

25. If ω is a complex cube root of unity then the value of $\sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{6}\right\}$ is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{1}{\sqrt{2}}$ (D) $-\frac{\sqrt{3}}{2}$ (E) $\frac{1}{2}$

26. Let $z = \frac{11-3i}{1+i}$. If α is a real number such that $z - i\alpha$ is real, then the value of α is

(A) 4 (B) -4 (C) 7 (D) -7 (E) 3

27. Let z_1 and z_2 be the roots of the equation $z^2 + pz + q = 0$ where p, q are real. The points represented by z_1, z_2 and the origin form an equilateral triangle if

(A) $p^2 = 3q$ (B) $p^2 > 3q$ (C) $p^2 < 3q$ (D) $p^2 = 2q$ (E) $p = 3q$

28. If α, β, γ are the cube roots of a negative number p, then for any three real numbers x, y, z the value of $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$ is

(A) $\frac{1-i\sqrt{3}}{2}$ (B) $\frac{-1-i\sqrt{3}}{2}$ (C) $(x+y+z)i$

(D) pi (E) $\frac{x+y+z}{2}pi$

29. The magnitude and amplitude of $\frac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$ are respectively

(A) $2, \frac{3\pi}{4}$ (B) $4, \frac{3\pi}{4}$ (C) $2\sqrt{2}, \frac{\pi}{4}$

(D) $2\sqrt{2}, \frac{\pi}{2}$ (E) $2\sqrt{2}, \frac{3\pi}{4}$

30. If $1+x^2=\sqrt{3}x$, then $\sum_{n=1}^{24}\left(x^n-\frac{1}{x^n}\right)^2$ is equal to

(A) 0 (B) 48 (C) -24 (D) 24 (E) -48

31. If α and β are the roots of the equation $ax^2+bx+c=0$, $\alpha\beta=3$ and a, b, c are in AP, then $\alpha+\beta$ is equal to

(A) -4 (B) -1 (C) 4 (D) -2 (E) 2

32. If one root of the equation $x^2+px+12=0$ is 4, while the equation $x^2+px+q=0$ has equal roots, then the value of q is

(A) 4 (B) 12 (C) 3 (D) $\frac{29}{4}$ (E) $\frac{49}{4}$

33. Given tan A and tan B are the roots of $x^2-ax+b=0$. The value of $\sin^2(A+B)$ is

(A) $\frac{a^2}{a^2+(1-b)^2}$ (B) $\frac{a^2}{a^2+b^2}$ (C) $\frac{a^2}{(a+b)^2}$

(D) $\frac{b^2}{a^2+(1-b)^2}$ (E) $\frac{a^2}{b^2+(1-a)^2}$

34. The number of roots of the equation $|x| = x^2 + x - 4$ is

(A) 4 (B) 3 (C) 1 (D) 0 (E) 2

35. The first term of an infinite G.P is 1 and each term is twice the sum of the succeeding terms, then the sum of the series is

(A) 2 (B) $\frac{5}{2}$ (C) $\frac{7}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{2}$

36. If $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P. then

(A) a, b, c are in A.P.

(B) c, a, b are in A.P.

(C) a^2, b^2, c^2 are in A.P.

(D) a, b, c are in G.P.

(E) c^2, a^2, b^2 are in A.P.

37. In an infinite geometric series the first term is a and common ratio is r. If the sum of the series is 4 and the second term is $\frac{3}{4}$ then (a, r) is

(A) $\left(\frac{4}{7}, \frac{3}{7}\right)$ (B) $\left(2, \frac{3}{8}\right)$ (C) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (D) $\left(3, \frac{1}{4}\right)$ (E) $\left(4, \frac{3}{4}\right)$

38. The sets $S_1, S_2, S_3 \ldots$ are given by

$S_1 = \left\{\frac{2}{1}\right\}, S_2 = \left\{\frac{3}{2}, \frac{5}{2}\right\}, S_3 = \left\{\frac{4}{3}, \frac{7}{3}, \frac{10}{3}\right\}, S_4 = \left\{\frac{5}{4}, \frac{9}{4}, \frac{13}{4}, \frac{17}{4}\right\}, \ldots$. Then the sum of the numbers in the set S_{25} is

(A) 320 (B) 322 (C) 324 (D) 325 (E) 326

39. If H_1, H_2 are two harmonic means between two positive numbers a and b $(a \neq b)$, A and G are the arithmetic and geometric means between a and b then $\frac{H_2 + H_1}{H_2 H_1}$ is

(A) $\frac{A}{G}$ (B) $\frac{2A}{G}$ (C) $\frac{A}{2G^2}$ (D) $\frac{A}{G^2}$ (E) $\frac{2A}{G^2}$

40. If $\log_{\sqrt{3}} 5 = a$ and $\log_{\sqrt{3}} 2 = b$, then $\log_{\sqrt{3}} 300 =$

(A) $2(a + b)$ (B) $2(a + b + 1)$ (C) $2(a + b + 2)$

(D) $a + b + 4$ (E) $a + b + 1$

41. If a, b, c are distinct positive numbers each being different from 1 such that $(\log_b a.\log_c a - \log_a a) + (\log_a b.\log_c b - \log_b b) + (\log_a c.\log_b c - \log_c c) = 0$, then abc is

(A) 0 (B) e (C) 1 (D) 2 (E) 3

42. If α and β are the roots of the equation $x^2 + px + q = 0$ and if the sum $(\alpha+\beta)x - \frac{\alpha^2+\beta^2}{2}x^2 + \frac{\alpha^3+\beta^3}{3}x^3 - \frac{\alpha^4+\beta^4}{4}x^4 + \ldots$ exists, then it is equal to

(A) $\log(x^2+px+q)$ (B) $\log(x^2-px+q)$

(C) $\log(1+px+qx^2)$ (D) $\log(1-px+qx^2)$ (E) $\log(x^2+qx+p)$

43. If $m = {}^nC_2$, then ${}^mC_2$ is equal to

(A) $3\ {}^nC_4$ (B) ${}^{n+1}C_4$ (C) $3\ {}^{n+1}C_4$ (D) $3\ {}^{n+1}C_3$ (E) $3\ {}^{n+1}C_2$

44. The number of permutations of the letters of the word 'CONSEQUENCE' in which all the three Es are together is

(A) $9!\,3!$ (B) $\frac{9!}{2!}$ (C) $\frac{9!}{2!\,2!\,3!}$

(D) $\frac{9!}{2!\,3!}$ (E) $\frac{9!}{2!\,2!}$

45. If $(1+x-3x^2)^{10} = 1+a_1x+a_2x^2+...+a_{20}x^{20}$, then $a_2+a_4+a_6+...+a_{20}$ is equal to

(A) $\frac{3^{10}+1}{2}$ (B) $\frac{3^9+1}{2}$ (C) $\frac{3^{10}-1}{2}$ (D) $\frac{3^9-1}{2}$ (E) $2^{19}-1$

46. The value of $\left(\frac{^{50}C_0}{1}+\frac{^{50}C_2}{3}+\frac{^{50}C_4}{5}+...+\frac{^{50}C_{50}}{51}\right)$ is

(A) $\frac{2^{50}}{51}$ (B) $\frac{2^{50}-1}{51}$ (C) $\frac{2^{50}-1}{50}$ (D) $\frac{2^{51}-1}{51}$ (E) $\frac{2^{51}-1}{50}$

47. In the expansion of $(1+x+x^2+x^3)^6$, the coefficient of x^{14} is

(A) 130 (B) 120 (C) 128 (D) 125 (E) 115

48. If $n = 5$, then $({}^nC_0)^2+({}^nC_1)^2+({}^nC_2)^2+...+({}^nC_5)^2$ is equal to

(A) 250 (B) 254 (C) 245 (D) 252 (E) 258

49. If l, m and n are real numbers such that $l^2+m^2+n^2=0$, then

$$\begin{vmatrix} 1+l^2 & lm & ln \\ lm & 1+m^2 & mn \\ ln & mn & 1+n^2 \end{vmatrix}$$ is equal to

(A) 0 (B) 1 (C) $l+m+n+2$

(D) $2(l+m+n)+3$ (E) $lmn-1$

50. If $f(x) = x^2+4x-5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ then $f(A)$ is equal to

(A) $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$ (E) $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

51. If $s_r = \alpha^r + \beta^r + \gamma^r$, then the value of $\begin{vmatrix} s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{vmatrix}$ is equal to

(A) 0

(B) $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

(C) $(\alpha+\beta+\gamma)^6$

(D) $(\alpha-\beta)^2(\beta-\gamma)^2(\gamma-\alpha)^2$

(E) $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma$

52. The coefficient of x in $f(x) = \begin{vmatrix} x & 1+\sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}$, $-1 < x \le 1$, is

(A) 1 (B) -2 (C) -1 (D) 0 (E) 2

53. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$, then x is equal to

(A) 9 (B) −9 (C) 0 (D) −1 (E) 1

54. If ω be the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to

(A) 0 (B) $-H$ (C) H (D) H^2 (E) I

55. Number of integral solutions of $\dfrac{x+2}{x^2+1} > \dfrac{1}{2}$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

56. If r is a real number such that $|r| < 1$ and if $a = 5(1-r)$, then

(A) $0 < a < 5$ (B) $-5 < a < 5$ (C) $0 < a < 10$

(D) $0 \leq a < 10$ (E) $-10 < a < 10$

57. If p : 4 is an even prime number

q : 6 is a divisor of 12 and

r : the HCF of 4 and 6 is 2, then which one of the following is true

(A) $(p \wedge q)$ (B) $(p \vee q) \wedge \sim r$ (C) $\sim (q \wedge r) \vee p$

(D) $\sim p \vee (q \wedge r)$ (E) $p \leftrightarrow (q \wedge r)$

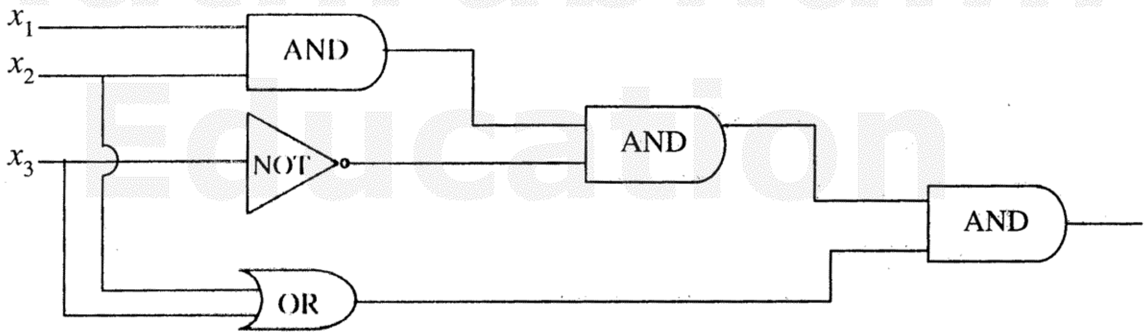
58. Which of the following is not true for any two statements p and q

(A) $\sim[p \vee (\sim q)] \equiv (\sim p) \wedge q$ (B) $(p \vee q) \vee (\sim q)$ is a tautology

(C) $(p \wedge q) \wedge (\sim q)$ is a contradiction (D) $\sim[p \wedge (\sim p)]$ is a tautology

(E) $\sim(p \vee q) \equiv (\sim p) \vee (\sim q)$

59. The output of the circuit is



(A) $(x_2 + x_3) \cdot [(x_1 \cdot x_2) \cdot x_3']$ (B) $(x_2 + x_3') \cdot [(x_1 \cdot x_2) \cdot x_3']$

(C) $(x_2 + x_3) + [(x_1 \cdot x_2) \cdot x_3']$ (D) $(x_2 \cdot x_3) + [(x_1 \cdot x_2) \cdot x_3']$

(E) $(x_1 + x_3) \cdot [(x_1 \cdot x_2) \cdot x_3']$

60. If $A + B + C = \pi$, then

$\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)+\tan\left(\frac{B}{2}\right)\tan\left(\frac{C}{2}\right)+\tan\left(\frac{C}{2}\right)\tan\left(\frac{A}{2}\right)$ is equal to

(A) $\frac{\pi}{6}$ (B) 3 (C) 2 (D) 1 (E) -1

61. If $\sin 4A - \cos 2A = \cos 4A - \sin 2A, \left(0 < A < \frac{\pi}{4}\right)$, then the value of $\tan 4A$ is

(A) 1 (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$

(D) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (E) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

62. $\tan^{-1}\frac{m}{n} - \tan^{-1}\frac{m-n}{m+n}$ is equal to

(A) $\tan^{-1}\frac{n}{m}$ (B) $\tan^{-1}\frac{m+n}{m-n}$ (C) $\frac{\pi}{4}$

(D) $\tan^{-1}\left(\frac{1}{2}\right)$ (E) $\frac{\pi}{2}$

63. In a ΔABC if $(\sqrt{3}-1)a = 2b$, $A = 3B$, then C is

(A) 60° (B) 120° (C) 30° (D) 45° (E) 90°

54. If $\sec^{-1}\sqrt{1+x^2}+\operatorname{cosec}^{-1}\frac{\sqrt{1+y^2}}{y}+\cot^{-1}\frac{1}{z}=\pi$ then $x+y+z$ is equal to

(A) xyz (B) $2xyz$ (C) xyz^2

(D) x^2yz (E) $3xyz$

55. If $\sec\alpha$ and $\operatorname{cosec}\alpha$ are the roots of the equation $x^2-px+q=0$, then

(A) $p^2=p+2q$ (B) $q^2=p+2q$ (C) $p^2=q(q+2)$

(D) $q^2=p(p+2)$ (E) $p^2=q(q-2)$

56. If $x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$ and $x\sin\theta=y\cos\theta$, then x^2+y^2 is

(A) 2 (B) 0 (C) 3 (D) 4 (E) 1

57. In a ΔABC if $\tan\frac{A}{2}=\frac{5}{6}$, $\tan\frac{C}{2}=\frac{2}{5}$ then

(A) a,c,b are in A.P. (B) a,b,c are in G.P. (C) b,a,c are in A.P.

(D) a,b,c are in A.P. (E) a,c,b are in G.P.

58. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is

(A) $\sqrt{3}:2+\sqrt{3}$ (B) $1:6$ (C) $1:2+\sqrt{3}$

(D) $2:3$ (E) $\sqrt{2}:2+\sqrt{3}$

69. In a triangle ABC, $(b+c)(bc)\cos A+(a+c)(ac)\cos B+(a+b)(ab)\cos C$ is

(A) $a^2+b^2+c^2$ (B) $a^3+b^3+c^3$ (C) $(a+b+c)(a^2+b^2+c^2)$
(D) $(a+b+c)(ab+bc+ca)$ (E) abc

70. If in ΔABC, $\sin\frac{A}{2}\sin\frac{C}{2}=\sin\frac{B}{2}$ and $2s$ is the perimeter of the triangle, then s is

(A) $2b$ (B) b (C) $3b$ (D) $4b$ (E) $\frac{3b}{2}$

71. ABC is a right angled triangle with $\angle B=90°$, $a=6$ cm. If the radius of the circumcircle is 5 cm, then the area of ΔABC is

(A) 25 cm^2 (B) 30 cm^2 (C) 36 cm^2 (D) 24 cm^2 (E) 48 cm^2

72. The vertices A, B, C of a triangle are (2, 1), (5, 2) and (3, 4) respectively. Then the circumcentre is

(A) $\left(\frac{13}{4},\frac{-9}{4}\right)$ (B) $\left(\frac{-13}{4},\frac{9}{4}\right)$ (C) $\left(\frac{-13}{4},\frac{-9}{4}\right)$
(D) $\left(\frac{13}{4},\frac{9}{4}\right)$ (E) $\left(\frac{13}{2},\frac{9}{4}\right)$

73. The x-axis, y-axis and a line passing through the point $A(6, 0)$ form a triangle ABC. If $\angle A = 30°$, then the area of the triangle, in sq. units, is

(A) $6\sqrt{3}$ (B) $12\sqrt{3}$ (C) $4\sqrt{3}$ (D) $8\sqrt{3}$ (E) $2\sqrt{3}$

74. The midpoint of the line joining the points (– 10, 8) and (– 6, 12) divides the line joining the points (4, – 2) and (– 2, 4) in the ratio

(A) 1 : 2 internally (B) 1 : 2 externally (C) 2 : 1 internally

(D) 2 : 1 externally (E) 2 : 3 externally

75. The equations of the lines through the point (3,2) which makes an angle of 45° with the line $x - 2y = 3$ are

(A) $3x - y = 7$ and $x + 3y = 9$ (B) $x - 3y = 7$ and $3x + y = 9$

(C) $x - y = 3$ and $x + y = 2$ (D) $2x + y = 7$ and $x - 2y = 9$

(E) $2x - y = 7$ and $x + 2y = 9$

76. The straight lines $3x + 4y - 5 = 0$ and $4x = 3y + 15$ intersect at the point P. On these lines the points Q and R are chosen so that $PQ = PR$. The slopes of the lines QR passing through (1, 2) are

(A) $-7, \frac{1}{7}$ (B) $7, \frac{1}{7}$ (C) $7, -\frac{1}{7}$ (D) $3, -\frac{1}{3}$ (E) $-3, \frac{1}{3}$

77. The equation of the line which is such that the portion of line segment intercepted between the coordinate axes is bisected at (4, –3) is

(A) $3x+4y=24$ (B) $3x-4y=12$ (C) $3x-4y=24$

(D) $4x-3y=24$ (E) $4x-3y=12$

78. The acute angle between the lines joining the origin to the points of intersection of the line $\sqrt{3}\,x+y=2$ and the circle $x^2+y^2=4$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi}{12}$

79. If the circle $x^2+y^2+4x+22y+c=0$ bisects the circumference of the circle $x^2+y^2-2x+8y-d=0$, then $c+d$ is equal to

(A) 30 (B) 50 (C) 40 (D) 56 (E) 52

80. Two diameters of the circle $3x^2+3y^2-6x-18y-7=0$ are along the lines $3x+y=c_1$ and $x-3y=c_2$. Then the value of $c_1\,c_2$ is

(A) – 48 (B) 80 (C) –72 (D) 54 (E) 24

81. The area of an equilateral triangle that can be inscribed in $x^2+y^2-4x-6y-12=0$ is

(A) $\frac{25\sqrt{3}}{4}$ (B) $\frac{35\sqrt{3}}{4}$ (C) $\frac{55\sqrt{3}}{4}$ (D) $\frac{75\sqrt{3}}{4}$ (E) $\frac{25}{4}$

82. Length of the tangents from the point (1, 2) to the circles $x^2+y^2+x+y-4=0$ and $3x^2+3y^2-x-y-k=0$ are in the ratio 4 : 3, then k is equal to

(A) $\frac{21}{2}$ (B) $\frac{4}{21}$ (C) 21 (D) 7 (E) $\frac{21}{4}$

83. The parametric representation of a point on the ellipse whose foci are (3, 0) and (–1, 0) and eccentricity $\frac{2}{3}$ is

(A) $(1+3\cos\theta, \sqrt{3}\sin\theta)$ (B) $(1+3\cos\theta, 5\sin\theta)$

(C) $(1+3\cos\theta, 1+\sqrt{5}\sin\theta)$ (D) $(1+3\cos\theta, 1+5\sin\theta)$

(E) $(1+3\cos\theta, \sqrt{5}\sin\theta)$

84. The eccentricity of the conic $\frac{(x+2)^2}{7}+(y-1)^2=14$ is

(A) $\sqrt{\frac{7}{8}}$ (B) $\sqrt{\frac{6}{17}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{\frac{6}{11}}$ (E) $\sqrt{\frac{6}{7}}$

85. If for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, y-axis is the minor axis and the length of the latus rectum is one half of the length of its minor axis, then its eccentricity is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{3}{4}$ (E) $\frac{3}{5}$

86. If the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{100}-\frac{4y^2}{225}=1$ have the same directrices, then the value of b^2 is

(A) 9 (B) 144 (C) 12 (D) 4 (E) 25

87. The position vectors of the points A and B with respect to O are $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$. The length of the internal bisector of $\angle BOA$ of ΔAOB is

(A) $\frac{\sqrt{136}}{9}$ (B) $\frac{\sqrt{136}}{3}$ (C) $\frac{20}{3}$ (D) $\frac{\sqrt{217}}{9}$ (E) $\frac{25}{3}$

88. Given $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=5\vec{c}+6\vec{d}$, then the value of $\vec{a}\,.\,\vec{b}\times\left(\vec{a}+\vec{c}+2\vec{d}\right)$ is

(A) 7 (B) 16 (C) −1 (D) 4 (E) −17

89. Let $\vec{a},\vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{-1}{4}|\vec{b}||\vec{c}|\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$ then the angle between $\vec{a}$ and $\vec{c}$ is equal to

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{6}$

90. A vector of magnitude 12 units perpendicular to the plane containing the vectors $4\hat{i}+6\hat{j}-\hat{k}$ and $3\hat{i}+8\hat{j}+\hat{k}$ is

(A) $-8\hat{i}+4\hat{j}+8\hat{k}$ (B) $8\hat{i}+4\hat{j}+8\hat{k}$ (C) $8\hat{i}-4\hat{j}+8\hat{k}$

(D) $8\hat{i}-4\hat{j}-8\hat{k}$ (E) $4\hat{i}-8\hat{j}-8\hat{k}$

91. Forces of magnitudes 3 and 4 units acting along $6\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+6\hat{k}$ respectively act on a particle and displace it from (2, 2, −1) to (4, 3, 1). The work done is

(A) $\frac{124}{7}$ (B) $\frac{120}{7}$ (C) $\frac{125}{7}$ (D) $\frac{121}{7}$ (E) $\frac{123}{7}$

92. Let $ABCD$ be a parallelogram and M be the point of intersection of the diagonals. If O is any point, then $\vec{OA}+\vec{OB}+\vec{OC}+\vec{OD}$ is

(A) $3\vec{OM}$ (B) $4\vec{OM}$ (C) $\vec{OM}$ (D) $2\vec{OM}$ (E) $\frac{1}{2}\vec{OM}$

93. If D, E and F are the midpoints of the sides $\overrightarrow{BC}$, $\overrightarrow{CA}$ and $\overrightarrow{AB}$ respectively of the triangle ABC and G is the centroid of the triangle, then $\overrightarrow{GD}+\overrightarrow{GE}+\overrightarrow{GF}$ is

(A) $\vec{0}$ (B) $2\overrightarrow{AB}$ (C) $2\overrightarrow{GA}$ (D) $2\overrightarrow{GC}$ (E) $\overrightarrow{GA}+\overrightarrow{GB}$

94. The shortest distance from the point (1,2,−1) to the surface of the sphere $x^2+y^2+z^2=54$ is

(A) $3\sqrt{6}$ (B) $2\sqrt{6}$ (C) $\sqrt{6}$ (D) 2 (E) 4

95. If from a point $P(a, b, c)$ perpendiculars PA, PB are drawn to YZ and ZX planes, then the equation of the plane OAB is

(A) $bcx + cay + abz = 0$ (B) $bcx + cay - abz = 0$

(C) $bcx - cay + abz = 0$ (D) $-bcx + cay + abz = 0$ (E) $ax + by + cz = 0$

96. If $P(x, y, z)$ is a point on the line segment joining $Q(2,2,4)$ and $R(3,5,6)$ such that projections of $\vec{OP}$ on the axes are $\frac{13}{5}, \frac{19}{5}, \frac{26}{5}$ respectively, then P divides QR in the ratio

(A) 1 : 2 (B) 3 : 2 (C) 2 : 3 (D) 1 : 3 (E) 3 : 1

97. The equation to the plane through the point (2, 3, 1) and (4, –5, 3) parallel to x-axis is

(A) $x+y+4z = 7$ (B) $x+4z = 7$ (C) $y-4z = 7$

(D) $y+4z = -7$ (E) $y + 4z = 7$

98. The angle between $\vec{r} = (1+2\mu)\vec{i} + (2+\mu)\vec{j} + (2\mu - 1)\vec{k}$ and the plane $3x - 2y + 6z = 0$ (where μ is a scalar) is

(A) $\sin^{-1}\left(\frac{15}{21}\right)$ (B) $\cos^{-1}\left(\frac{16}{21}\right)$ (C) $\sin^{-1}\left(\frac{16}{21}\right)$

(D) $\frac{\pi}{2}$ (E) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

99. The length of the shortest distance between the two lines $\vec{r} = (-3\vec{i} + 6\vec{j}) + s(-4\vec{i} + 3\vec{j} + 2\vec{k})$ and $\vec{r} = (-2\vec{i} + 7\vec{k}) + t(-4\vec{i} + \vec{j} + \vec{k})$ is

(A) 7 units (B) 13 units (C) 8 units (D) 9 units (E) 11 units

100. The perpendicular distance of the point (6, 5, 8) from y-axis is

(A) 5 units (B) 6 units (C) 8 units (D) 9 units (E) 10 units

101. The equation of the plane passing through the origin and containing the line $\frac{x-1}{5} = \frac{y-2}{4} = \frac{z-3}{5}$ is

(A) $x+5y-3z=0$ (B) $x-5y+3z=0$ (C) $x-5y-3z=0$

(D) $3x-10y+5z=0$ (E) $x+5y+3z=0$

102. The standard deviation of the numbers 31, 32, 33, ….. , 46, 47 is

(A) $\sqrt{\frac{17}{12}}$ (B) $\sqrt{\frac{47^2-1}{12}}$ (C) $2\sqrt{6}$ (D) $4\sqrt{3}$ (E) $\frac{5}{12}$

103. A random variable X takes values 0, 1, 2, 3, ... with probability $P(X=x) = k(x+1)\left(\frac{1}{5}\right)^x$ where k is constant, then $P(X=0)$ is

(A) $\frac{7}{25}$ (B) $\frac{18}{25}$ (C) $\frac{13}{25}$ (D) $\frac{19}{25}$ (E) $\frac{16}{25}$

104. Out of 15 persons 10 can speak Hindi and 8 can speak English. If two persons are chosen at random, then the probability that one person speaks Hindi only and the other speaks both Hindi and English is

(A) $\frac{3}{5}$ (B) $\frac{7}{12}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$ (E) $\frac{2}{3}$

105. A random variable X has the following probability distribution

$X=x_i$	1	2	3	4
$P(X=x_i)$	0.1	0.2	0.3	0.4

The mean and the standard deviation are respectively

(A) 3 and 2 (B) 3 and 1 (C) 3 and $\sqrt{3}$

(D) 2 and 1 (E) 3 and $\sqrt{2}$

106. If $g(x)=\int_0^x \cos^4 t \, dt$ then $g(x+\pi)$ is equal to

(A) $g(x)+g(\pi)$ (B) $g(x)-g(\pi)$ (C) $g(x)g(\pi)$

(D) $\frac{g(x)}{g(\pi)}$ (E) $\frac{g(\pi)}{g(x)}$

107. If $\lim_{x\to1}\frac{ax^2+bx+c}{(x-1)^2}=2$, then (a,b,c) is

(A) (2, – 4, 2) (B) (2, 4, 2) (C) (2, 4, –2)

(D) (2, – 4, – 2) (E) (– 2, 4, 2)

108. Let $[x]$ denote the greatest integer $\leq x$. If $f(x) = [x]$ and $g(x) = |x|$, then the value of $f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(-\frac{8}{5}\right)\right)$ is

(A) 2 (B) –2 (C) 1 (D) 0 (E) –1

109. If $f(x) = \frac{\log_e(1 + x^2 \tan x)}{\sin x^3}$, $x \neq 0$, is to be continuous at $x = 0$, then $f(0)$ must be defined as

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) –1 (E) 2

110. The derivative of $f(\tan x)$ w. r. t. $g(\sec x)$ at $x = \frac{\pi}{4}$ where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$ is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 1 (D) 2 (E) $\frac{1}{2}$

111. If $\sec\left(\frac{x - y}{x + y}\right) = a$, then $\frac{dy}{dx}$ is

(A) $\frac{y}{x}$ (B) $\frac{-y}{x}$ (C) $\frac{x}{y}$ (D) $\frac{-x}{y}$ (E) $\frac{x - y}{x + y}$

112. If $x = \frac{2at}{1+t^3}$ and $y = \frac{2at^2}{(1+t^3)^2}$ then $\frac{dy}{dx}$ is

(A) ax (B) a^2x^2 (C) $\frac{x}{a}$ (D) $\frac{x}{2a}$ (E) $2a$

113. If $\cos\frac{x}{2}.\cos\frac{x}{2^2}...\cos\frac{x}{2^n} = \frac{\sin x}{2^n \sin\frac{x}{2^n}}$, then $\frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + ... + \frac{1}{2^n}\tan\frac{x}{2^n}$ is

(A) $\cot x - \cot\frac{x}{2^n}$ (B) $\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right) - \cot x$

(C) $\frac{1}{2^n}\tan\left(\frac{x}{2^n}\right) - \tan x$ (D) $\frac{1}{2}\cot x - \frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)$ (E) $\cot\left(\frac{x}{2^n}\right) - \cot x$

114. If $y = \underset{n\to\infty}{\text{Lt}} (1+x)(1+x^2)(1+x^4)\ldots\left(1+x^{2^n}\right)$ and $x^2 < 1$ then y' is equal to

(A) 1 (B) $\frac{1}{1-x}$ (C) $\frac{1}{1+x}$ (D) $\frac{-1}{(1-x)^2}$ (E) $\frac{1}{(1-x)^2}$

115. If $f(x) = \log_{x^3}\left(\log_e x^2\right)$, then $f'(x)$ at $x = e$ is

(A) $\frac{1}{3e}(1-\log_e 2)$ (B) $\frac{1}{3e}(1+\log_e 2)$ (C) $\frac{1}{3e}(-1+\log_e 2)$

(D) $-\frac{1}{3e}(1+\log_e 2)$ (E) $\frac{1}{3e}(\log_e 2)$

116. If $f(x) = (x-2)(x-4)(x-6)\ldots(x-2n)$, then $f'(2)$ is

(A) $(-1)^n 2^{n-1}(n-1)!$ (B) $(-2)^{n-1}(n-1)!$ (C) $(-2)^n n!$

(D) $(-1)^{n-1} 2^n (n-1)!$ (E) $2^{n-1}(n-1)!$

117. If θ is the semi vertical angle of a cone of maximum volume and given slant height, then $\tan\theta$ is equal to

(A) 2 (B) 1 (C) $\sqrt{2}$ (D) $\sqrt{3}$ (E) $\sqrt{3}+\sqrt{2}$

118. A man of 2 m height walks at a uniform speed of 6 km/hr away from a lamp post of 6 m height. The rate at which the length of his shadow increases is

(A) 2 km/hr (B) 1 km/hr (C) 3 km/hr

(D) 6 km/hr (E) $\frac{3}{2}$ km/hr

119. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at (2, 3) then

(A) $p = 2, q = -7$ (B) $p = -2, q = 7$ (C) $p = -2, q = -7$

(D) $p = 2, q = 7$ (E) $p = 0, q = 7$

120. A missile is fired from the ground level rises x meters vertically upwards in t seconds where $x = 100t - \frac{25}{2}t^2$. The maximum height reached is

(A) 200 m (B) 125 m (C) 160 m (D) 190 m (E) 300 m