

TIRUNELVELI

A. BALAJI

Time: 2.30 hours.

COMMON HALF-YEARLY EXAMINATION - 2019

Standard X
MATHEMATICS

Reg.No. :

--	--	--	--	--

Marks: 100

- Instructions :**
- i) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
 - ii) Use blue or black ink to write and pencil to draw diagrams.

Note : This question paper contains four parts.

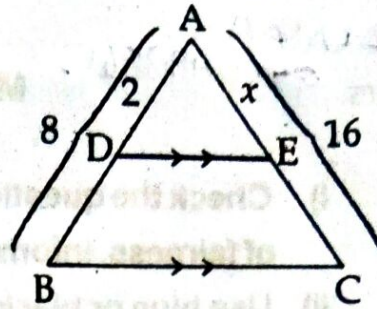
Part - I

- Note :**
- i) Answer all questions. 14 x 1 = 14
 - ii) Choose the correct answer from the four alternatives and write the option code and the corresponding answer

1. The range of the relation $R = \{(x, x^2) / x \text{ is a prime number less than } 13\}$ is
 - a) $\{2, 3, 5, 7\}$
 - b) $\{2, 3, 5, 7, 11\}$
 - c) $\{4, 9, 25, 49, 121\}$
 - d) $\{1, 4, 9, 25, 49, 121\}$
2. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then fog is
 - a) $\frac{3}{2x^2}$
 - b) $\frac{2}{3x^2}$
 - c) $\frac{2}{9x^2}$
 - d) $\frac{1}{6x^2}$
3. If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is
 - a) 4
 - b) 2
 - c) 1
 - d) 3
4. If the sequence t_1, t_2, t_3, \dots are in A.P then the sequence $t_6, t_{12}, t_{18}, \dots$ is
 - a) a geometric progression
 - b) an arithmetic progression
 - c) neither an arithmetic progression nor a geometric progression
 - d) a constant sequence
5. $\frac{x}{x^2-25} - \frac{8}{x^2+6x+5}$ gives
 - a) $\frac{x^2-7x+40}{(x^2-25)(x+1)}$
 - b) $\frac{x^2+7x+40}{(x-5)(x+5)(x+1)}$
 - c) $\frac{x^2-7x+40}{(x^2-25)(x+1)}$
 - d) $\frac{x^2+10}{(x^2-25)(x+1)}$
6. Find the matrix X if $2X + \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 9 & 5 \end{bmatrix}$
 - a) $\begin{bmatrix} -2 & -2 \\ 2 & -1 \end{bmatrix}$
 - b) $\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$
 - c) $\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$
 - d) $\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$

7. In the given figure, the value of x is

- a) 2 b) 8
c) 4 d) 12



8. The area of triangle formed by the points $(-5,0)$, $(0,-5)$ and $(5,0)$ is
a) 0 sq.units b) 25 sq.units c) 5 sq.units d) none of these
9. $(2,1)$ is the point of intersection of two lines
a) $x - y - 3 = 0$; $3x - y - 7 = 0$ b) $x + y = 3$; $3x + y = 7$
c) $3x + y = 3$; $x + y = 7$ d) $x + 3y - 3 = 0$; $x - y - 7 = 0$
10. $\cos 60^\circ \sin 30^\circ + \cos 30^\circ \sin 60^\circ =$
a) 90° b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) 1
11. The height of a right circular cone whose radius is 3 cm and slant height is 5 cm will be
a) 12 cm b) 4 cm c) 13 cm d) 5 cm
12. The total surface area of a hemisphere is how much times the square of its radius?
a) π b) 4π c) 3π d) 2π
13. The standard deviation of a data is 5. If each value is multiplied by 2, then the new variance is
a) 3 b) 100 c) 10 d) 225
14. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is
a) $\frac{3}{10}$ b) $\frac{7}{10}$ c) $\frac{3}{9}$ d) $\frac{7}{9}$

Part - II

Note. Answer any ten Questions. Question Number 28 is Compulsory. $10 \times 2 = 20$

15. Let $A = \{1, 2, 3, 4\}$ and $B = \mathbb{N}$. Let $f: A \rightarrow B$ be defined by $f(x) = x^3$ then,
i) Find the range of f . ii) Identify the type of function.
16. a and b are two positive integers such that $a^b \times b^a = 800$. Find ' a ' and ' b '.
17. Show that the sequence described by $a_n = \frac{1}{3}n + \frac{1}{6}$ is an A.P.
18. Find the sum of $1 + 3 + 5 + \dots + 55$
19. If α and β are the roots of $x^2 + 6x - 4 = 0$, find the value of $(\alpha - \beta)^2$.

20. If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$, then verify $(A^T)^T = A$.

21. The line through the points $(-2, a)$ and $(9, 3)$ has slope $-\frac{1}{2}$. Find the value of a .
22. Find the area of the triangle formed by the points $(1, -1)$, $(-4, 6)$ and $(-3, -5)$.
23. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.
24. The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.
25. The volume of a solid right circular cone is 11088 cm^3 . If its height is 24 cm then find the radius of the cone.
26. An aluminium sphere of radius 15 cm is melted to make a cylinder of radius 10 cm. Find the height of the cylinder.
27. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.
28. A coin is tossed twice. What is the probability of getting exactly one head?

Part - III

Note: Answer any Ten Questions Question Number 42 is Compulsory $10 \times 5 = 50$

29. Let $A = \{x \in \mathbb{N} / 1 < x < 4\}$, $B = \{x \in \mathbb{W} / 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} / x < 3\}$. Then verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
30. Find the value of k , such that $\text{fog} = \text{gof}$ if $f(x) = 3x + 2$, $g(x) = 6x - k$.
31. In a Geometric Progression, the 4th term is 8 and the 8th term is $\frac{128}{625}$. Find the Geometric Progression.
32. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm, ..., 24 cm. How much area can be decorated with these colour papers?
33. Solve: $x + y + z = 5$; $2x - y + z = 9$; $x - 2y + 3z = 16$
34. If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b .
35. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.
36. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$, show that $(A - B)^T = A^T - B^T$.
37. State and prove Pythagoras theorem.
38. $A(-3, 0)$, $B(10, -2)$ and $C(12, 3)$ are the vertices of $\triangle ABC$. Find the equation of the altitude through A.

39. From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are 45° and 60° respectively. Find the height of the tower. ($\sqrt{3} = 1.732$).
40. A doll is made by surmounting a cone on a hemisphere of equal radius. The radius of the hemisphere is 7 cm and slant height of the cone is 11 cm. Find the surface area of the doll.
41. Find the standard deviation for the following data.

x	10	15	18	20	25
f	3	2	5	8	2

42. If the roots of $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$.

Part - IV

Note : Answer All the Questions.

2 x 8 = 16

43. a) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR. (Scaler factor $\frac{3}{5} < 1$)
(or)
- b) Draw a tangent to the circle from the point P having radius 3.6 cm and centre at O. Point P is at a distance 7.2 cm from the centre.
44. a) Graph the quadratic equation $x^2 - 9x + 20 = 0$ and state its nature of solutions.
(or)
- b) Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$.

choose

1. c) {49, 25, 49, 121}
2. c) $\frac{2}{9x^2}$
3. b) 2
4. b) an arithmetic progression
5. c) $\frac{x^2 - 7x + 40}{(x^2 - 25)(x + 1)}$
6. b) $\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$
7. c) 4

8. 25 sq. units
9. b) $x + y = 3$, $3x + y = 7$
10. d) 1
11. b) 4cm
12. c) 3π
13. b) ~~25~~ 100
14. b) $\frac{7}{10}$

Two marks

15.

Range = {1, 8, 27, 64}

Type: One-one function and into function.

20. $A^T = \begin{bmatrix} 5 & -\sqrt{7} & 8 \\ 2 & 0.7 & 3 \\ 2 & 5/2 & 1 \end{bmatrix}$
 $(A^T)^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{7} & 0.7 & 3 \\ 8 & 3 & 1 \end{bmatrix} = A$

16.

$a^b \times b^a = 100 = 2^5 \times 5^2$ or $5^2 \times 2^5$
 $a = 5$ or 2 $b = 2$ or 5 .

21.

$\frac{3\bar{a}}{9+a} = -\frac{1}{2}$
 $6 - 2a = \cancel{9+a} - 11$
 $\boxed{a = \frac{17}{2}} \quad \boxed{a = \frac{17}{2}}$

17.

$a_1 = \frac{3}{b}$ $a_2 = \frac{5}{b}$ $a_3 = \frac{7}{b}$
 $t_2 - t_1 = t_3 - t_2$
 $\therefore 2t_1$ an A.P.

22. Area = $\frac{1}{2} \begin{vmatrix} 1 & -4 & -3 & 1 \\ -1 & 6 & -5 & -1 \end{vmatrix}$
 $= \frac{1}{2} [6 + 20 + 3 - 4 + 18 + 5]$
 $= 24$ sq. units.

18.

$s_n = \left(\frac{n+1}{2}\right)^2 = \left(\frac{28}{2}\right)^2 = 784$.

23.

$\tan \theta = \frac{10\sqrt{3}}{30} = \frac{1}{\sqrt{3}}$
 $\boxed{\theta = 30^\circ}$

19.

$\alpha + \beta = -6$ $\alpha\beta = -4$
 $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
 $= (-6)^2 - 4(-4)$
 $= 36 + 16 = 52$.

$$24. \quad 4\pi r_1^2 : 4\pi r_2^2$$

$$1^3 \times 12^3 = 16^3 \times 16^3$$

$$\boxed{19:16}$$

$$25. \quad \frac{1}{3} \pi r^2 h = 11088$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$$

$$r^2 = \frac{11088 \times 3 \times 7}{22 \times 24}$$

$$\boxed{r = 21 \text{ cm}}$$

$$26. \quad \frac{4}{3} \pi r^3 = \pi r^2 h$$

$$\frac{4}{3} \times 15^3 = 10 \times 10 \times h$$

$$\boxed{h = 45 \text{ cm}}$$

$$27. \quad L - 13.4 = 36.8$$

$$\boxed{L = 50.2}$$

128.

$$n(A) = 4$$

$$A = \{HT, TH\}$$

$$P(A) = \frac{2}{4} = \frac{1}{2}$$

5 Marks

$$29. \quad A = \{2, 3\} \quad B = \{0, 1\}$$

$$C = \{1, 2\}$$

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

$$30. \quad \text{fog} = gof$$

$$3(6x-k) + 2 = 6(3x+2) - k$$

$$18x - 3k + 2 = 18x + 12 - k$$

$$-2k = 10$$

$$\boxed{k = -5}$$

$$31. \quad ar^3 = 8$$

$$ar^7 = \frac{128}{625}$$

$$r^4 = \frac{16}{625} = \frac{2^4}{5^4}$$

$$\boxed{r = \frac{2}{5}}$$

$$a = 8 \times \frac{5^3}{2^3} = 125$$

The G.P is 125, 50, 20, 8, ...

$$32. \quad 10^2 + 11^2 + \dots + 24^2 = (1^2 + \dots + 24^2) - (1^2 + \dots + 9^2)$$

$$= 4900 - 285$$

$$= 4615 \text{ sq. cm.}$$

$$33. \quad x + y + z = 5$$

$$2x - y + z = 9$$

$$x - 2y + 3z = 16$$

Ans: $x = 2, y = -1, z = 4.$

34.

$$\begin{array}{r|rrr}
 & 3 & 2 & 1 \\
 \hline
 3 & 9 & 12 & 28 & a & 5 \\
 & 7 & & & & \\
 \hline
 6 & 2 & & 12 & 24 & \\
 & & & 12 & 4 & \\
 \hline
 6 & 4 & 4 & & 24 & a & 5 \\
 & & & & 24 & 16 & 16 \\
 \hline
 & & & & 0 & & \\
 \hline
 \boxed{a=16} & & & & \boxed{b=16} & &
 \end{array}$$

35.

$$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$B = \{(1,3), (2,2), (3,1)\}$$

$$n(A \cup B) = 8 \quad P(A \cup B) = \frac{8}{36} = \frac{2}{9}$$

$$\boxed{P(A \cup B) = \frac{2}{9}}$$

36.

$$A - B = \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix}$$

$$(A - B)^T = \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix}$$

$$\therefore (A - B)^T = A^T - B^T$$

37. In a right angle triangle the square of the hypotenuse is equal to sum of the squares of other two sides.



$$\triangle ABD \sim \triangle ABC$$

$$AB^2 = BD \times BC \quad \text{--- (1)}$$

$$\triangle ADC \sim \triangle ABC$$

$$AC^2 = DC \times BC$$

$$\therefore AD^2 + AC^2 = BC^2$$

38. slope of BC (m_1) = $\frac{5}{2}$.

$$m_2 = -2/5$$

equation $\Rightarrow y - 0 = -2/5(x + 3)$

(A)

$$5y = -2x - 6$$

$$\boxed{2x + 5y + 6 = 0}$$

39. $\tan 45^\circ = \frac{30}{x}$

$$\boxed{x = 30}$$

$$\tan 60^\circ = \frac{30 + y}{x}$$

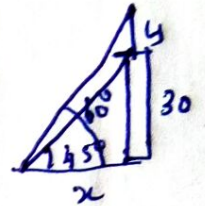
$$\sqrt{3} = \frac{30 + y}{30}$$

$$30\sqrt{3} = 30 + y$$

$$y = 30(0.732)$$

$$= 21.960 = 21.96 \text{ m}$$

$$\boxed{\text{Height of the tower} = 21.96 \text{ m}}$$



40.

$$\text{Surface Area} = 2\pi r^2 + \pi r l$$

$$= \pi r [2r + l]$$

$$= \frac{22}{7} \times 7 [2 \times 7 + 11]$$

$$= 22 [25] = \underline{\underline{550 \text{ cm}^2}}$$



41.

$$\boxed{A = 18}$$

$$2fd^2 = 340, \quad fd = 0$$

$$fd = 20$$

$$d = \sqrt{\frac{340}{22} - (0)^2} = \sqrt{17} = \boxed{4.12}$$

42.

$$(b-c)^2 - 4(a-b)(c-a) = 0$$

$$b^2 - 2bc + c^2 - 4ac + 4a^2 + 4bc - 4ab > 0$$

$$(2a - b - c)^2 = 0$$

$$\boxed{2a = b + c}$$

44) a), $\in (4, 5)$ roots are real and unequal.

g. $\in (-2, 1)$ "