

JAIN COLLEGE

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SUBJECT: MATHEMATICS
II PUC MOCK II 2019-2020

Timings Allowed: 3Hrs 15Mins

Total Marks: 100

PART A

I Answer ALL the following question.

1 X 10 =10

1. Define bijective function.
2. Give an example of a relation which is symmetric and transitive but not reflexive.
3. Construct 2X2 matrix $A=[a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$
4. Find the value of $\begin{vmatrix} 200 & 201 \\ 202 & 203 \end{vmatrix}$ without actual expansion.
5. If $y = \sin(5x + 8)$ find $\frac{dy}{dx}$
6. Check the continuity of the function $f(x) = 2x + 3$ at $x = 1$
7. Evaluate $\int \sec^2(7 - 4x)dx$
8. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$
9. Define feasible region.
10. A fair die is rolled $E = \{1,5\}$ $F = \{1,4\}$ Find $P\left(\frac{E}{F}\right)$.

PART-B

II. answer any ten of the following.

2 X 10 =20

11. A relation R is defined on the set $A = \{1,2,3,4,5,6\}$ by $R = \{(x,y): y \text{ is divisible by } x\}$ verify whether R is symmetric and reflexive or not.
12. Prove that $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $x \in [-1,1]$
13. Simplify $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right]$ if $\frac{a}{b} \tan x > -1$
14. Find the equation of the line joining (3,1) and (9,3) using the determinant.
15. If $2x + 3y = \sin x$ then find $\frac{dy}{dx}$.
16. If $x = at^2$ $y = 2at$ find $\frac{dy}{dx}$
17. Find the approximate change in the surface area of a cube of side x meters caused by increasing the side by 1%.
18. Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{2}$.
19. Integrate $\frac{\cos\sqrt{x}}{\sqrt{x}}$
20. Evaluate $\int \log x dx$.
21. If \vec{a} is a unit vector such that $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$ find $|\vec{x}|$
22. Find the order and degree of the differential equation, $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$
23. Find the distance of the plane $3x - 3y + 4z - 6 = 0$ from the origin
24. An unbiased die is thrown twice. Let the event A be "odd number on the first throw", B the event "odd number on second throw" Are A and B independent?

PART-C

III. Answer any TEN of the following questions.

3M X10 =30

25. Show that the relation R in the set of all Z defined by $R = \{(x,y): 2 \text{ divides } x - y\}$ is an equivalence relation.
26. Prove that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$
27. Express $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.
28. Find the numbers whose sum is 24 and product is maximum.
29. $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ prove that $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$
30. Verify Rolle's theorem for $f(x) = x^2 + 2x - 8$, $x \in [-4,2]$.

31. Evaluate $\int \frac{4x+1}{\sqrt{2x^2+x-3}} dx$
32. Evaluate $\int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$
33. In a cultural test, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours the count reaches 2,00,000 if the rate of growth of the bacteria is proportional to the number present.
34. Find the area of the region bounded by the curve $y^2=4x$ and its latus rectum
35. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ find value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$
36. Prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$
37. Find the cosine of the angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$
38. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{-1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$
39. Find the probability of getting 5 exactly twice in 7 throws of a die

PART-D

IV. Answer any SIX of the following questions.

6 X 5=30

40. Let $f: N \rightarrow R$ be defined by $f(x) = 4x^2 + 12x + 15$ show that $f: N \rightarrow S$ where S is the range of f is invertible. Also find inverse of f .
41. Solve by matrix method $x - y + 2z = 7, 3x + 4y - 5z = -5, 2x - y - 3z = 12$.
42. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ then show that $A^3 - 23A - 40I = 0$
43. If $y = \sin^{-1} x$ show that $(1 - x^2)y_2 - xy_1 = 0$
44. A ladder 5m long is leaning against the wall. The bottom of the wall is pulled along the ground away from the wall at the rate of 2cm/sec. How fast its height on the wall decreasing when the foot of the wall is 4m away from the wall.
45. If $e^y(x + 1) = 1$ show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$
46. Find the integral of $\sqrt{a^2 - x^2}$ with respect to x and hence evaluate $\int \sqrt{3 - 2x - x^2} dx$.
47. Find the integral of $\frac{3x-2}{(x+1)^2(x+3)} dx$
48. Find general solution of differential equation $\frac{dy}{dx} - y = \cos x$

PART-E

V. Answer any ONE of the following.

10 X 1 =10

49. a) Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ and hence evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

b) For what value of 'k' $f(x)$ is continuous, $f(x) = \begin{cases} k(x^2 - 2) & x \leq 0 \\ 4x + 1 & x > 0 \end{cases}$

50. a) Minimize and Maximize $Z = 3x + 9y$ subject to the constraints $x + 3y \leq 60, x + y \geq 10, x \leq y,$

$x, y \geq 0$ Graphical method

b) Prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & xz \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy + yz + zx)$

*****ALL THE BEST*****