SECTION A

1) $kx(x-2)+6=0$

$kx^2-2kx+6=0$

For equal roots $b^2-4ac=0$

$(-2k)^2-4\times k\times 6=0$

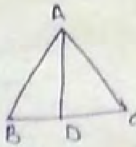
$4k^2-24k=0$

$4k(k-6)=0$

$4k=0,\ k=0$

$k-6=0,\ k=6$, Here $k=6$ since it cannot be 0.

2) From $\triangle ABC$ & $\triangle ADC$



$\angle BAC=\angle ADC$ (given)

$\angle ACB=\angle ACD$ Common.

$\therefore \triangle ABC \sim \triangle DAC$

ie $\frac{BC}{AC}=\frac{AB}{AD}=\frac{AC}{DC}$

$\frac{BC}{AC}=\frac{AC}{DC}$

$AC^2=BC\cdot DC \Rightarrow$ Hence proved

3) $d=3,\ a=12,\ a_n=99.$

$a_n=a+(n-1)d$

$99=12+(n-1)3$

$87=3n-3$

$90=3n$

$n=30$

4)

Height	No of students	cf
150-155	15	15
155-160	13	28
160-165	10	38
165-170	8	46
170-175	9	55
175-180	5	60

$\frac{n}{2}=\frac{60}{2}=30$

$\therefore$ ~~uppe~~ median class : 160-165

upper limit of median class $=165$

5) Two lines are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3}{6} = \frac{-1}{-2} = \frac{-5}{-p}$

$\frac{1}{2} = \frac{5}{p}$

$p = 10$

The lines are parallel when p is any real number except 10.

6) Total area = CSA of cone + CSA of cylinder + base area of cylinder.

$= \pi r l + 2\pi r h + \pi r^2$

7) $(k-1)x^2 - 10x + 3 = 0$

$\alpha \cdot \frac{1}{\alpha} = \frac{c}{a} = 1$

$\frac{3}{k-1} = 1$

$3 = k-1$

$k = 4$

8) $\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{35}{45} = \frac{7}{9}$

$\therefore \frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \left(\frac{7}{9}\right)^2 = 49:81$

9) 2, —, 26, —

$a_3 - a_2 = a_2 - a_1$

$2a_2 = 26 + 2$

$a_2 = \frac{28}{2} = 14.$

$d = a_2 - a_1 = 14 - 2 = 12.$

$a_4 = 26 + 12 = 38.$

$\therefore$ AP is 2, 14, 26, 38

10) $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$

squares are always positive

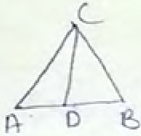
i.e. there are no squares less than 1 other than 0.

No. of square $\leq 1 = 3$.

No of events . 11

$\therefore$ probability $= \frac{3}{11}$.

11) From ΔCAD & ΔABC



$\angle CDA = \angle ACB$ (given)

$\angle CAD = \angle CAB$ (common)

$\therefore \Delta CAD \sim \Delta BAC$ (AA criterion)

$\frac{AB}{AC} = \frac{CB}{CD} = \frac{AC}{AD}$

$\frac{AB}{6} = \frac{6}{3}$

$AB = 12 cm$

12) AP $= b, c, 2b$. To find $\frac{b}{c}$

$c - b = 2b - c$

$2c = 3b$

$\frac{b}{c} = \frac{2}{3}$

13) $x^2 + 2\sqrt{2}kx + 18 = 0$

For equal roots $b^2 - 4ac = 0$

$(2\sqrt{2}k)^2 - 4 \times 18 = 0$

$8k^2 - 72 = 0$

$8(k^2 - 9) = 0$

$k^2 = 9$

$k = \pm 3$.

14) $2x^2 - 4x + 3 = 0$

$b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3$

$= 16 - 24$

$= -8$ i.e. the roots are unreal.

15) $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a} \cdots$

$d = a_3 - a_2$

$= \frac{3-2a}{3a} - \frac{3-a}{3a}$

$= \frac{3-2a-3+a}{3a}$

$= \frac{-a}{3a}$

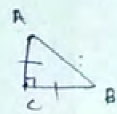
$= -\frac{1}{3}$

16) $AB = \sqrt{AC^2 + CB^2}$

$= \sqrt{4^2 + 4^2}$

$= \sqrt{32}$

$= 4\sqrt{2}$ cm



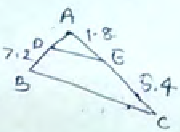
17) Since $DE \parallel BC$

$\frac{AD}{DB} = \frac{AE}{EC}$

$AD = \frac{AE \times DB}{EC}$

$= \frac{1.8 \times 7.2}{5.4}$

$= 2.4$ cm



18) $P(\text{blue}) = \frac{1}{5}$

$P(\text{black}) = \frac{1}{4}$

No. of green marbles = 11

Let x be the total no. of marbles.

$\therefore \frac{1}{5} + \frac{1}{4} + \frac{11}{x} = 1$

$\frac{4x + 5x + 220}{20x} = 1$

$9x + 220 = 20x$

$220 = 11x$

$x = 20$

19) $x + 2y = 5 \qquad 3x + ky + 15 = 0$

For a unique solution.

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}.$$

$$\frac{1}{3} \neq \frac{2}{k}$$

$$k \neq 6.$$

ie for any value other than 6, the eqns will have a unique soln.

20) $x + x + 18 = 180$

$2x + 18 = 180$

$2x = 162$

$x = \frac{162}{2} = \underline{81} \Rightarrow$ smaller angle.

larger angle $= 81 + 18$

$= \underline{\underline{99}}$

21)

	Sunil's age	son's age.
	$3x = y$	x
5 yrs later	$\frac{5}{2}(x+5) = y.$	$x+5$

$y = 3x.$

$2y = 5x + 25$

$6x = 5x + 25$

$x = 25$

$6x + 10 = 5x + 25$

21)

	Sumit's age.	son's age.
present day	$y = 3x.$	$x.$
5 yrs later	$y + 5 = \frac{5}{2}(x+5)$	$x+5$

$3x + 5 = \frac{5}{2}(x+5)$

$6x + 10 = 5x + 25$

$\underline{\underline{x = 15}}$ Sumit's age $\underline{\underline{= 45}}$

22) Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$= 35 + \left[\frac{50 - 34}{100 - 34 - 42}\right] \times 5$

$= 35 + \left[\frac{16}{24}\right] \times 5$

$= 35 + 3.33$

$= \underline{\underline{38.33}}$

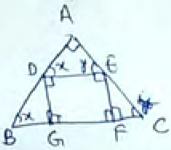
23) $a = 110 \quad a_n = 990 \quad d = 10$

$a_n = a + (n-1)d.$

$990 = 110 + 10n - 10$

$890 = 10n$

$n = \underline{\underline{89}}$



24) Given DEFG is a square.

i.e all angles of the square $= 90°$

$DE = EF = FG = DG$.

From $\triangle DBG$ & $\triangle ADE$.

$\angle ADE = \angle DBG$ (corresponding $\angle s$)

$\angle A = \angle DGB = 90°$

$\triangle ADE \sim \triangle GDB$ (AA criterion)

$\frac{DE}{DB} = \frac{AE}{DG} = \frac{AD}{BG}$.

From $\triangle ADE$ & $\triangle EFC$

$\angle A = \angle EFC = 90°$

$\angle AED = \angle ECF$ (corresponding $\angle s$)

$\triangle ADE \sim \triangle FEC$ (AA criterion)

From $\triangle DBG$ & $\triangle GFC$

$\angle DGB = \angle EFC = 90°$

$\angle DBG = \angle FEC$

$\therefore \triangle DBG \sim \triangle CEF$ (AA criterion)

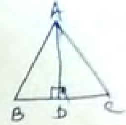
$\frac{DB}{EC} = \frac{BG}{EF} = \frac{DG}{FC}$

Subst. EF & DG with FG

$\frac{BG}{EF} = \frac{DG}{FC}$

$FG^2 = BG \times FC$ $\Rightarrow$ Hence proved

25) Let AB = BC = AC = a.



AD = d.

BD = DC = $\frac{a}{2}$. (altitude & median are the same)

altitude^2 = hypotenuse^2 − base.

$$d^2 = a^2 - \left(\frac{a}{2}\right)^2$$

$$d^2 = a^2\left(1 - \frac{1}{4}\right)$$

$$d^2 = \frac{3a^2}{4}$$

$4d^2 = 3a^2 \Rightarrow$ Hence proved

26) Jayanti $P(E) = \frac{1}{36}$.

Pihu $P(E) = \frac{1}{6}$.

Pihu has a better chance.

27) Total no. of outcomes = (100 − 70) − 1 = 29.

(a) prime number.

No. of favourable outcomes = 71, 73, 79, 83, 89, 97.

$P(Pr) = \frac{6}{29}$

(b) divisible by 7.

No. of favourable outcome = $\frac{4}{29}$

28) h = 10 cm.

r = 3 cm.

Vol. of glass a = Vol. of cyl − vol of hemisphere

$$= \pi r^2 h - \frac{2}{3}\pi r^3$$

$$= \pi r^2\left[h - \frac{2}{3}r\right]$$

$$= \pi \times 3^2\left[10 - \frac{2 \times 3}{3}\right]$$

$$= 72\pi \text{ cm}^3$$

Vol. of glass b = vol. of cylinder − vol. of cone.

$= \pi r^2 h - \frac{1}{3}\pi r^2 h_1$

$= \pi r^2 \left[h - \frac{h_1}{3}\right]$

$= \pi \times 3^2 \left[10 - \frac{1.5}{3}\right]$

$= 9\pi \left[9.5\right]$

$= 85.5\pi \text{ cm}^2$

Glass b would contain more liquid.

29) A.P = 10, 7, 4 . . . . −62.

$d = -3$.

Reversing the AP.

A.P = −62, −59, ... 4, 7, 10

$a_n = a + (n-1)d$.

$= -62 + (11-1)\times 3$

$= -62 + 30$

$= -32$

30) Let d. be the common difference of two APs

$a_1, a_2, a_3 \ldots$ & $A_1, A_2, A_3, \ldots$

$A_{100} - a_{100} = 100$. (given)

$A_1 + 99d - (a_1 + 99d) = 100$

$A_1 + 99d - a_1 - 99d = 100$

$A_1 - a_1 = 100$.

$A_{1000} - a_{1000}$.

$= A_1 + 999d - (a_1 + 999d)$

$= A_1 + 999d - a_1 - 999d$

$= A_1 - a_1$

$= 100$

31) Total no. of cards = 52.

No. of red cards = 26.

No. of queens = 4 (of which two are red.

~~Probabi~~ No. of favourable cards = 52 − (26 + 2) = 24

Probability $= \frac{24}{52} = \frac{6}{13}$

32) Total no. of outcomes = 36.
No. of favourable outcomes = 11.
[(1,5) (2,5) (3,5) (4,5) (6,5) (5,1) (5,2) (5,3) (5,4) (5,5) (5,6)]

Probability $= \frac{11}{36}$

33) $2x + 3y = 7$
$(a-b)x + (a+b)y = 3a+b-2$
For infinitely many solutions $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{2-3a-b}$

or

$\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$

$\frac{2}{a-b} = \frac{3}{a+b}$

$2a + 2b = 3a - 3b$

$a = 5b$

$\frac{3}{a+b} = \frac{7}{3a+b-2}$

$\frac{3}{5b+b} = \frac{7}{15b+b-2}$

$\frac{3}{6b} = \frac{7}{16b-2}$

$16b - 2 = 14b$

$2b = 2$

$b = 1$

$a = 5$

SECTION C

34) $3x^2 - 2\sqrt{6}x + 2 = 0$

$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$(\sqrt{3}x - \sqrt{2})^2 = 0$ $\quad \sqrt{3}x - \sqrt{2} = 0$ $\quad x = \sqrt{2}/\sqrt{3}$

35) Let the speed of the train be x km/h.
Total distance travelled = 480 km.
Time taken to travel 480 km = $\frac{480}{x}$ h.

If speed is 8 km/h less, time = $\frac{480}{x-8}$ h.

ie $\frac{480}{x-8} = \frac{480}{x} + 3$.

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$480\left[\frac{1}{x-8} - \frac{1}{x}\right] = 3$$

$$480\left[\frac{x-x+8}{x(x-8)}\right] = 3$$

$$160 \times 8 = x^2 - 8x.$$

$$x^2 - 8x - 1280 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{8 \pm \sqrt{64+5120}}{2}$$

$$= \frac{4 \pm 72}{2}$$

$$= 2 \pm 36$$

$$= 38, -34.$$

Since speed cannot be negative, $x = 38$ km/h.

36) $\angle BAD = \angle ACB = 90°$
$AC \perp BD$.

From ΔBAD and ΔACB
$\angle BAD = \angle ACB = 90°$
$\angle DBA = \angle ABC$ (common)
$\therefore \Delta ABD \sim \Delta CBA$ (AA criterion)

$$\frac{BD}{AB} = \frac{AD}{AC} = \frac{AB}{BC}$$

$$\frac{BD}{AB} = \frac{AB}{BC}$$

$$AB = BD \cdot BC$$

From $\triangle ACD$ & $\triangle ACB$.

$\angle ACD = \angle ACB = 90^\circ$

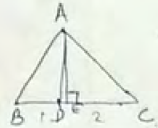
$\angle CAD = \angle ABC = x$

$\angle ADC = \angle CAB = 90 - x$

$\triangle CAD \sim \triangle CBA$ (AAA criterion)

$$\frac{AD}{AB} = \frac{CD}{AC} = \frac{AC}{BC}$$

$$AC^2 = BC \cdot CD$$



37) $\angle A = \angle B = \angle C = 60^\circ$

$BD = \frac{1}{3} BC$ $\quad AB = BC = AC$

$BE = EC = \frac{BC}{2}$

In $\triangle DEA$

$DA^2 = DE^2 + AE^2$ —— (1)

$AE = AD^2 - DE^2$

In $\triangle BEA$

$AB^2 = AE^2 + BE^2$

$AE^2 = AB^2 - BE^2$ —— (2)

From (1) and (2).

$$AD^2 - DE^2 = AB^2 - BE^2$$

$$AD^2 - (BE - BD)^2 = AB^2 - \left(\frac{BC}{2}\right)^2$$

$$AD^2 - \left(\frac{BC}{2} - \frac{BC}{3}\right)^2 = AB^2 - \frac{AB^2}{4}$$

$$AD^2 - \frac{AB^2}{36} = \frac{3AB^2}{4}$$

$$AD^2 = \frac{3AB^2}{4} + \frac{AB^2}{36}$$

$$AD^2 = \frac{28AB^2}{36}$$

$$9AD^2 = 7AB^2 \Rightarrow \text{Hence proved.}$$

38) Coin $r = \frac{1.75}{2} = \frac{175}{200}$ cm $\quad h = 2mm = \frac{2}{10}$ cm

Cuboid $l = 5.5cm = \frac{55}{10}$ cm $\quad b = 10$ cm $\quad h = 3.5cm = \frac{35}{10}$ cm

$$n \times \pi r^2 h = l \times b \times h$$

$$n \times \frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10} = \frac{55}{10} \times 10 \times \frac{35}{10}$$

$$n = \frac{55 \times 35 \times 100 \times 100}{11 \times 175 \times 25} = 400$$

39) No. of cones × (volume of cone + vol. of hemisphere) = vol. of cylinder.

$n \times \left(\frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3\right) = \pi r^2 h.$

$n \times \frac{1}{3}\pi r_1^2 \left[h_1 + 2r_1\right] = \pi r^2 h.$

$n \times \frac{1}{3} \cancel{\pi} \times \frac{\cancel{6}}{\cancel{2}} \times \frac{\cancel{6}^3}{2}\left[12 + \frac{\cancel{6}^3}{\cancel{2}}\right] = \cancel{\pi} \times \frac{\cancel{12}^6}{\cancel{2}} \times \frac{\cancel{12}^6}{\cancel{2}} \times 15$

$18 \times 3 \times n = 6 \times 6 \times 15$

$n = \frac{{}^2\cancel{6} \times \cancel{6} \times \cancel{15}^5}{\cancel{3} \times \cancel{18}} = 10.$

40) Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h.$

Modal class = 40 − 50 $l = 40.$

$h = 10.$ $f_1 = 20$ $f_0 = 12$ $f_2 = 11$

Mode = $40 + \left[\frac{20 - 12}{40 - 12 - 11}\right] \times 10$

$= 40 + \left(\frac{8}{11} \times 10\right)$

$= \underline{\underline{47.3}}$

41) $S_m = S_n.$

$\frac{m}{2}\left[2a + (m-1)d\right] = \frac{n}{2}\left[2a + (n-1)d\right]$

$m\left[2a + md - d\right] = n\left[2a + nd - d\right]$

$2am + m^2 d - md = 2an + n^2 d - nd.$

$2am - 2an + m^2 d - n^2 d + nd - md = 0$

$2am - 2an + d(m^2 - n^2) + d(n - m) = 0.$

$2a(m-n) + d(m^2-n^2)$

$(m-n)[2a + (m+n-1)d] = 0$

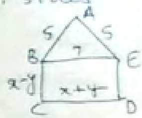
$S_{(m+n)} = \frac{m+n}{2}\left[2a + (m+n-1)d\right]$

$= \cancel{\frac{m+n}{2}\left[2a + md + nd - d\right]}$

$= \frac{m+n}{2} \times 0$

$= \underline{\underline{0}}$

42) Since BE ∥ CD & BC ∥ ED, BC ⊥ CD. opp. sides are equal.



Perimeter = 27 cm.

$x+y = 7$ —— ①

$5 + x - y + x + y + x - y + 5 = 27$

$10 + 2(x-y) + 7 = 27$

$2(x-y) = 10$

$x - y = 5$ —— ②

① − ②.

$2x = 12$

$x = \underline{\underline{6\text{ cm}}}$

$y = \underline{\underline{1\text{ cm}}}.$

43) $\frac{21}{x} + \frac{47}{y} = 110$ $\frac{47}{x} + \frac{21}{y} = 162.$

Let $p = \frac{1}{x}$ $q = \frac{1}{y}$.

$21p + 47q = 110$ — ① $47p + 21q = 162.$ — ②

① + ②.

$21p + 47q = 110$

$47p + 21q = 162$

$68p + 68q = 272.$

$p + q = 4$ —— ③.

① − ②

$21p + 47q = 110$

$47p + 21q = 162$

$-26p + 26q = -52.$

$p - q = 2$ —— ④

③ + ④

$p + q = 4$

$p - q = 2$

$2p = 6$

$p = 3$

$p - q = 2$

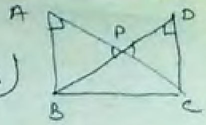
$q = 1$

$p = \frac{1}{x}.$

$x = \frac{1}{p} = \underline{\underline{\frac{1}{3}}}$

$q = \frac{1}{y}$

$\underline{\underline{q = 1}}$

44) From ΔAPB & ΔDPC 

$\angle BAP = \angle CDP = 90^\circ$ (given)

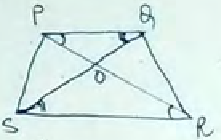
$\angle APB = \angle DPC$ (vertically opp angles)

$\therefore \Delta APB \sim \Delta DPC$ (AA criterion)

$$\frac{AP}{DP} = \frac{BP}{PC} = \frac{AB}{CD}$$

$$\frac{AP}{DP} = \frac{BP}{PC}$$

$AP \times PC = BP \times DP \Rightarrow$ Hence proved.

45) Given $PQ \parallel RS$. 

$PQ = 3RS$.

From ΔPOQ and ΔROS

$\angle OPQ = \angle ORS$ (alternate interior $\angle$s)

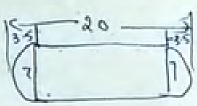
$\angle OQP = \angle OSR$ (" " ")

$\therefore \Delta POQ \sim \Delta ROS$ (AA criterion)

$$\frac{PQ}{RS} = \frac{OQ}{OS} = \frac{OP}{OR}.$$

$$\frac{\Delta POQ}{\Delta ROS} = \left(\frac{PQ}{RS}\right)^2 = \left(\frac{3RS}{RS}\right)^2 = 9$$

i.e. $\Delta POQ : \Delta ROS = 9:1$

46) Vol. of solid = volume of cylinder + (volume of ~~hem~~ sphere) $\times 2$ 

$$= \pi r^2 h_1 + \frac{2}{3}\pi r^3 \times 2.$$

$$= \pi r^2 h_1 + \frac{4}{3}\pi r^3$$

$$= \pi r^2 \left[h_1 + \frac{4}{3} r\right]$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \left[13 + \frac{4}{3} \times \frac{7}{2}\right]$$

$$= \frac{77}{2} \times \left[13 + \frac{14}{3}\right]$$

$$= \frac{77}{2} \times \frac{53}{3}$$

$$= 680.17 \text{ cm}^3$$

47)

Class Interval	f_i	x_i	d_i	u_i	f_iu_i
30-35	14	32.5	-15	-3	-42
35-40	16	37.5	-10	-2	-32
40-45	28	42.5	-5	-1	-28
45-50	23	47.5	0	0	0
50-55	18	52.5	5	1	18
55-60	8	57.5	10	2	16
60-65	3	62.5	15	3	9
	Σf_i 110				$\Sigma f_iu_i = -59.$

$$\text{Mean} = \bar{x} = a + \left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right)h$$

$$= 47.5 + \frac{-59\times5}{110}$$

$$= 44.82.$$

48) $x^2+px+16=0$

For equal roots $b^2-4ac=0$

$(-p)^2-4\times16=0$

$p^2 = 4\times16$

$p = \pm 8.$

For $p=8$

$x^2+8x+16=0$

$x^2+4x+4x+16=0$

$x(x+4)+4(x+4)=0$

$(x+4)^2=0$

$x+4=0$

$x=-4$

For $p=-8$

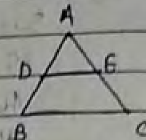
$x^2-8x+16=0$

$(x-4)^2=0$

$x-4=0$

$x=4.$

SECTION D



49) Given $DE \parallel BC$.

From ΔADE and ΔABC.

$\angle DAE = \angle BAC$ (common)

$\angle ADE = \angle ABC$ (corresponding angles)

$\angle AED = \angle ACB$ (" ")

$\therefore \Delta ADE \sim \Delta ABC$. (AAA criterion)

ie $\frac{DE}{BC} = \frac{AD}{DB} = \frac{AE}{EC}$

Since $\frac{AD}{AB} = \frac{AE}{EC}$

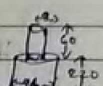
$\frac{AD}{AD+DB} = \frac{AE}{AE+EC}$

or $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$

$\cancel{1} + \frac{DB}{AD} = \cancel{1} + \frac{EC}{AE}$

$\therefore \frac{DB}{AD} = \frac{EC}{AE} \Rightarrow$ Hence proved.

50) Vol of solid = vol. of cylinder 1 + vol. of cylinder 2



$= \pi r_1^2 h_1 + \pi r_2^2 h_2$

$= \pi \left[r_1^2 h_1 + r_2^2 h_2 \right]$

$= \pi \left[8 \times 8 \times 60 + 12 \times 12 \times 220 \right]$

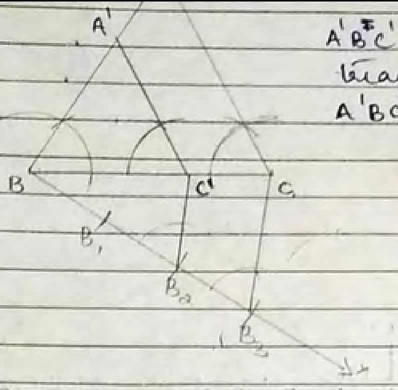
$= \pi \times \left[3840 + 31680 \right]$

$= 111532.8 \text{ cm}^3$

Mass $=$ Vol. $\times 8$ gm.

$= \frac{\text{vol} \times 8}{1000}$ kg

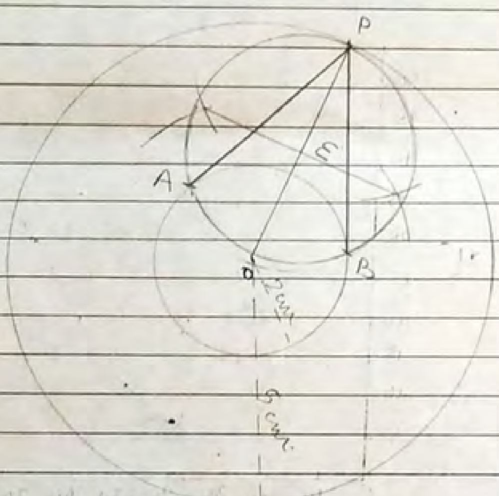
$= \frac{111532.8 \times 8}{1000} = 892.26$ kg



A'B'C' is the required triangle

A'BC' = $\frac{2}{3}$ ABC

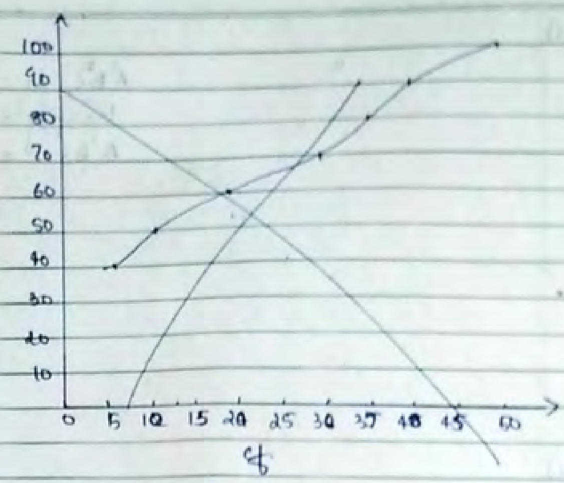
52)

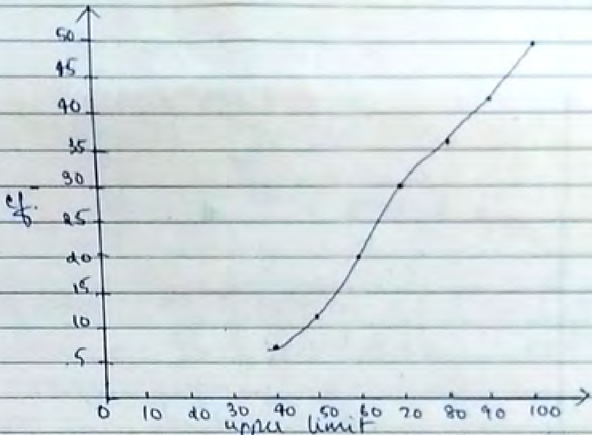


PA and PB are the tangents to circle with centre O and radius 2cm.

53)

Class Interval	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	7	5	8	10	6	6	8
c.f.	7	12	20	30	36	42	50
Upper limit	less than 40	>50	>60	>70	>80	>90	>100





54) AP $\Rightarrow$ $-7, -12, -17, -22, -17, -22 \ldots$

$a_n = a + (n-1)d$

$-82 = -7 + (n-1) \times -5$

$-75 = -5n + 5$

$-80 = -5n$

$n = \underline{\underline{16}}$

$$-100 = -7+(n-1)\times -5$$
$$-93 = -5n+5$$
$$-98 = -5n$$
$$n = \frac{98}{5}$$

100 is not a number of the AP since $n = \frac{98}{5}$ which is not possible. n has to be a natural number.

55) AP 45, 39, 33 ....

$$S_n = 180$$
$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$180 = \frac{n}{2}\left[2\times 45+(n-1)\times -6\right]$$
$$360 = n\left[90-6n+6\right]$$
$$360 = -6n^2+96n$$
$$6n^2-96n+360=0$$
$$n^2-16n+60=0$$
$$n^2-10n-6n+60=0$$
$$n(n-10)-6(n-10)=0$$
$$(n-6)(n-10)=0$$
$$n = 6, 10,$$

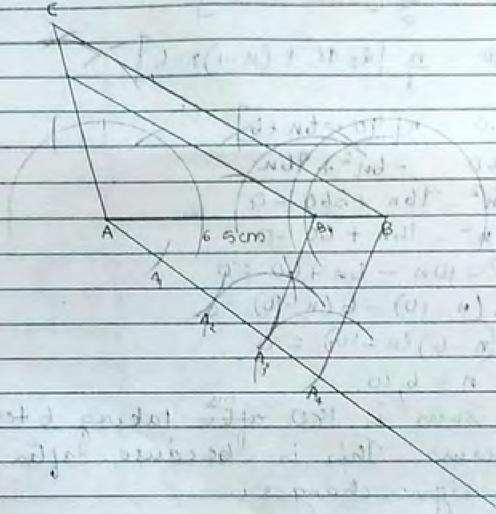
The sum is 180 ~~after~~ on taking 6 terms and 10 terms. This is because after 6 terms the sign changes.

56 Let Arun's marks in Hindi be x and English be y.

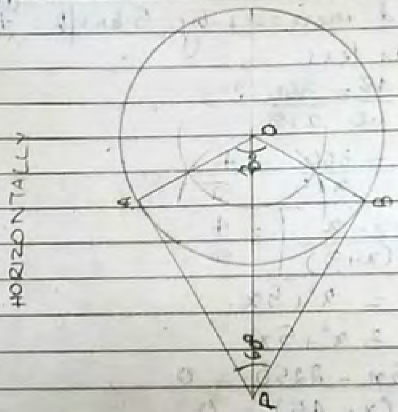
$$x+y = 30. \qquad y = 30-x.$$
$$(x+2)(y-3) = 210$$
$$(x+2)(30-x-3) = 210$$
$$(x+2)(27-x) = 210$$
$$27x-x^2+54-2x = 210$$

$25x - x^2 - 156 = 0$
$x^2 - 25x + 156 = 0$
$x^2 - 12x - 13x + 156 = 0.$
$x(x - 12) - 13(x - 12) = 0$
$(x - 12)(x - 13) = 0$
$x = 12, 13.$
for $x = 12$ $y = 18$
for $x = 13$ $y = 17$

57)



58)



Given $\angle APB = 60°$. Since $\triangle OAP \cong \triangle OBP$ $\angle OPA = \angle OPB = 30$

$\sin 30° = \frac{OA}{OP}$

$\frac{1}{2} = \frac{3}{OP}$

$OP = 6cm$ i.e., distance of external point P from centre O is equal to 6 cm.

57) Let the speed of the train be x km/hr.

Total distance = 360 km.

Time taken = $\frac{360}{x}$.

When speed increases by 5 km/h, time taken is 48 minutes less.

$$\frac{360}{x} - \frac{48}{60} = \frac{360}{x+5}$$

$$\frac{360}{x} - \frac{360}{x+5} = \frac{4}{5}$$

$$360\left[\frac{x+5-x}{x(x+5)}\right] = \frac{4}{5}$$

$$90 \times 25 = x^2+5x$$

$$2250 = x^2+5x$$

$$x^2+5x-2250 = 0$$

$$(x+50)(x-45) = 0$$

$$x = -50, 45$$

Here $x = 45$ km/h, since speed cannot be negative.

60) $\frac{1}{x} - \frac{1}{x-2} = 3.$

Let $\frac{1}{x} = y$

$$\frac{x-2-x}{x(x-2)} = 3$$

$$-2 = 3x^2 - 6x$$

$$3x^2 - 6x + 2 = 0$$

$$\frac{6 \pm \sqrt{6^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$= \frac{6 \pm \sqrt{12}}{6}$$

$$= \frac{6 \pm 2\sqrt{3}}{6}$$

$$= 1 \pm \frac{\sqrt{3}}{3}$$

6b) Vol. of frustum $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3}\pi \times 20[10^2 + 25^2 + 10 \times 25]$

$= \frac{1}{3}\pi \times 20(100 + 625 + 250)$

$= \frac{1}{3}\pi \times 20 \times 975$

$= 20 \times \pi \times 325$

Cost of petrol $= 20 \times \pi \times 325 \times 70$

$= ₹430000$

Surface area $= \pi l(r_1 + r_2)$

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{20^2 + (25 - 10)^2}$

$$= \sqrt{400 + 225}$$

$$= 25 \text{ m}$$

Surface area $= \pi \times 25 \, (10 + 25)$

$$= \frac{22}{\cancel{7}} \times 25 \times \cancel{35}\,5$$

$$= 2750 \text{ m}^2$$

(22) Let h be the height of the pipe.

Vol. of pipe = Vol. of tank

$\pi r^2 h = lbh_1$

$\frac{22}{\cancel{7}} \times \frac{\cancel{7}}{100} \times \frac{7}{100} \times h = 50 \times 44 \times \frac{21}{100}$

$h = \frac{50 \times \overset{2}{\cancel{44}} \times \overset{3}{\cancel{21}} \times 100}{\cancel{22} \times \cancel{7}} = 30000\,m = 30\,km$

Since speed of water is 15 km/h.

Time taken to travel 30 km $= \frac{30}{15} = 2$ hrs

63)

Daily wages	f_i	x_i	d_i	u_i	f_iu_i
100-120	10	110	-60	-3	-30
120-140	15	130	-40	-2	-30
140-160	20	150	-20	-1	-20
160-180	22	170 = a	0	0	0
180-200	18	190	20	1	18
200-220	12	210	40	2	24
220-240	13	230	60	3	39
	$\Sigma f_i = 110$				$\Sigma f_iu_i = 1$

$-80 + 81$

Mean $\bar{x} = a + \left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right) h$

$= 170 + \frac{1 \times 20}{110}$

$= 170.18$

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$

Modal class $= 160 - 180$.

Mode $= 160 + \left(\frac{22 - 20}{44 - 20 - 18}\right) \times 20$

$= 160 + \frac{2 \times 20}{6}$

$= 160 + 6.67$

$= 166.67$

64) Time taken by larger tap $= x$ h

" " " smaller tap $= x + 10$ hr.

Time taken by both taps together is $9\frac{3}{8}$ h $= \frac{75}{8}$ hrs

ie $\frac{1}{x} + \frac{1}{x+10} = \frac{8}{75}$

$$\frac{x+x+10}{x(x+10)} = \frac{8}{75}$$

$$75(2x+10) = 8(x^2+10x)$$

$$150x + 750 = 8x^2 + 80x$$

$$8x^2 - 70x - 750 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{70 \pm \sqrt{70^2 + 4\times 8\times 750}}{2\times 8}$$

$$= \frac{70 \pm \sqrt{4900 + 24000}}{16}$$

$$= \frac{70 \pm 170}{16}$$

$$= \frac{240}{16}, \frac{-100}{16}$$

$$= 15, \frac{-25}{4}$$

Since time cannot be negative, time taken by larger tap = 15 hr.
time taken by smaller tap = 25 hrs

65) Let the speed of the stream be x km/h.
Total distance = 24 km.
Time taken upstream = Time take downstream + 1

$$\frac{24}{18-x} = \frac{24}{18+x} + 1$$

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$24\left[\frac{18+x-18+x}{18^2-x^2}\right] = 1$$

$$48x = 324 - x^2$$

$$x^2 + 48x - 324 = 0$$

$$x^2 + 54x - 6x - 324 = 0$$

$$x(x+54) - 6(x+54) = 0$$

$$(x-6)(x+54) = 0$$

$x = 6, -54$

Since speed cannot be negative, $x = 6$ km/h

66) $a = 20 \quad d = -1 \quad S_n = 200.$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$200 = \frac{n}{2}\left[2 \times 20 - n + 1\right]$$

$$400 = n(41 - n)$$

$$400 = 41n - n^2$$

$$n^2 - 41n + 400 = 0$$

$$n^2 - 25n - 16n + 400 = 0$$

$$n(n-25) - 16(n-25) = 0$$

$$(n-25)(n-16) = 0$$

$$n = 25, 16$$

For $n = 25$

$$a_{25} = 20 - 24$$
$$= -4 \text{ which is not possible}$$

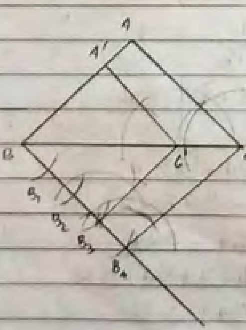
$n = 16$

$$a_{16} = 20 - 15$$
$$= 5$$

ie total no. of rows = 16.

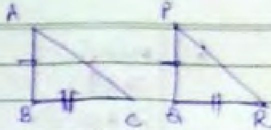
no. of logs in the top row = 5

67)



68) Given

In ΔABC $AC^2 = AB^2 + BC^2$.



To prove.

$\angle B = 90^\circ$

Construction

Draw ΔPQR, right angled at Q, such that

$PQ = AB$ and $QR = BC$.

Proof

In ΔPQR.

$\angle Q = 90^\circ$

According to Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$PR^2 = AB^2 + BC^2$

Since $AC^2 = AB^2 + BC^2$

$PR^2 = AC^2$.

or $PR = AC$ $\longrightarrow$ (1).

From ΔABC & ΔPQR.

$AB = PQ$

$BC = QR$.

$AC = PR$.

i.e. $\Delta ABC \cong \Delta PQR$ (SSS congruency)

$\therefore \angle B = \angle Q = 90^\circ$ (CPCT)

Hence proved.

69)

weight	cf	Interval	f
>38	0	36-38	0
>40	3	38-40	3
>42	5	40-42	2
>44	9	42-44	7
>46	14	44-46	10
>48	28	46-48	24
>50	32	48-50	14
>52	35	50-52	4

$\frac{n}{2} = \frac{35}{2} = 17.5$

weight	cf	interval	f
less than 38	0	36-38	0
lt 40	3	38-40	3
lt 42	5	40-42	2
lt 44	9	42-44	4
lt 46	14	44-46	5
lt 48	28	46-48	14
lt 50	32	48-50	4
lt 52	35	50-52	3

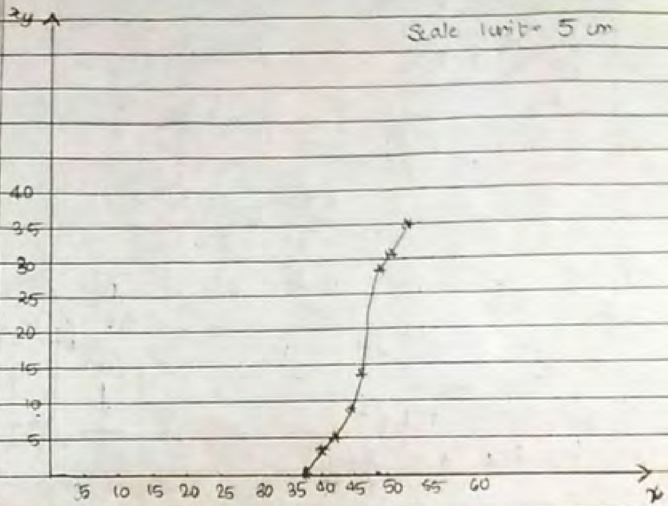
$\frac{n}{2} = \frac{35}{2} = 17.5$

$$\text{Median} = l + \left[\frac{n/2 - cf}{f}\right] \times h$$

$$= 46 + \left[\frac{17.5 - 14}{14}\right] \times 2$$

$$= 46 + \frac{\overset{0.5}{\cancel{3.5}} \times \cancel{2}}{\cancel{14}\,\cancel{7}}$$

$$= \underline{\underline{46.5}}$$



70)

Class Interval	f	cf
0 - 10	5	5
10 - 20	x	$5+x$
20 - 30	20	$25+x$
30 - 40	15	$40+x$
40 - 50	y	$40+x+y$
50 - 60	5	$45+x+y$
	60	

$n = 45+x+y = 60$

$x+y = 15$

$\frac{n}{2} = 30$

Median = 28.5

$\therefore$ median class = 20 - 30

i.e. $l = 20$, $cf = 5+x$. $f = 20$. $h = 10$.

$$\text{Median} = l + \left[\frac{n/2 - cf}{f}\right] \times h.$$

$$28.5 = 20 + \frac{30 - 5 - x}{20_2} \times 10$$

$8.5 = \frac{25 - x}{2}$

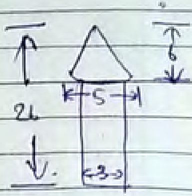
$17 = 25 - x$

$x = \underline{\underline{8}}$

$x + y = 15$

$y = \underline{\underline{7}}$

7) Cylindrical portion to be painted yellow



= CSA of cylinder + one Base

$= 2\pi r h + \pi r^2$

$h = 20\text{cm} \quad r = \frac{3}{2}\text{cm}$

$\text{Area} = \cancel{2}\pi r(2h + r)$

$= \pi \times \frac{3}{2}\left(40 + \frac{3}{2}\right)$

$= \pi \times \frac{3}{2} \times \frac{83}{2}$

$= \underline{\underline{195.465 \text{ cm}^2}}$

Conical portion to be painted orange.

= CSA of cone + (base area of cone − base area of cylinder)

$h_1 = 6\text{cm}. \quad r_1 = \frac{5}{2}\text{cm}.$

$l_1 = \sqrt{h^2 + r_1^2} = \sqrt{36 + \frac{25}{4}}$

$= \sqrt{\frac{144 + 25}{4}} = \sqrt{\frac{169}{4}} = \frac{13}{2} \text{ cm}.$

$\text{Area} = \pi r_1 l_1 + \pi r_1^2 - \pi r^2.$

$$= \pi \left[r_1 l_1 + r_1^2 - r^2\right]$$

$$= \pi \left[\frac{5}{2} \times \frac{13}{2} + \frac{5}{2} \times \frac{5}{2} - \frac{3}{2} \times \frac{3}{2}\right]$$

$$= \pi \left[\frac{65}{4} + \frac{25}{4} - \frac{9}{4}\right]$$

$$= 3.14 \times \frac{81}{4}$$

$$= 63.585 \text{ cm}^2$$