VIDYAJYOTHI

(2019 - 2020)

MATHEMATICS

(Support Material for Teaching & Learning)

CLASS X

District Institute of Education and Training (DIET)

Thiruvananthapuram

Vidyajyothi

Mathematics

(Support Material for Teaching & Learning)

First Edition
September 2018

Third Edition
November 2019

Layout & Cover Design
Kallingal Graphics, Attingal

Concepts & Conceptualisation
District Panchayath, Thiruvananthapuram

Administrative Charge
C.Manojkumar, Deputy Director of Education, Thiruvananthapuram

Academic Charge
T.V.Gopakumar
(Principal, DIET Thiruvananthapuram)

Coordinator
Dr.V.Sulabha, Senior Lecturer, DIET

Subject Charge
B.S.Santhosh, Lecturer, DIET Thiruvananthapuram

Printing
Govt. Press, Thiruvananthapuram

Message

Dear students

Kerala has made many strides in the field of education. The foundation of our success in this field is that we are able to attain academic excellence along with school excellence. Local authorities and people's committees are very supportive for the development of the school. Meaningful interventions by the Kerala Government and the Department of Public Instruction have become critical in this field. The role of teachers in organizing activities according to new perspectives on learning is not a trivial one. The use of ICT has enabled the collection and dissemination of information and thus facilitated learning. All of you are preparing for a very crucial exam. Systematic study is required to approach the exam with confidence and achieve high success. Everyone is here to help you. Vidyajyothi, the study materials prepared by the District Panchayat, Thiruvananthapuram and DIET Thiruvananthapuram will no doubt be an effective tool to ensure your greater success. An updated book which includes the revised lessons is now in your hands. Make use of it, the maximum. Wishing you all the best.

With love

V.K.Madhu

President, District Panchayath,
Thiruvananthapuram

Dear children

The report by NITI Aayog, which states that Kerala is the number one in Education in India is a source of great excitement for the education sector. The perspectives and activities based on secular democratic principles helped us achieve this aim. The General Education Rejuvenation Mission is another exemplary Kerala model. Many of the initiatives proposed by the new National Education Policy under the leadership of Dr. Kasturirangan have been implemented in Kerala. The fact that Kerala is on a par with the educational standard of many developed countries is a visible manifestation of the will power of the Kerala community. You have made many strides in education by self learning, following the guidelines suggested by your teachers who are research oriented in their approach. Now it's time for you to prepare for the public examination. You need not be afraid of exams. Consider your exam as an opportunity to apply the knowledge and skills you have acquired in the classrooms. Remember to take necessary preparations to face the exam well. The Vidyajyothi study materials prepared by the District Panchayat Thiruvananthapuram and DIET Thiruvananthapuram serves as a real guide for you. Make use of the study materials to the extent possible. Wish you all the best.

Wishing you all success

C. Manojkumar
Deputy Director of Education,
Thiruvananthapuram

T.V.Gopakumar
Principal, DIET
Thiruvananthapuram

Members Participated in Workshop

1.	T.Anil	Govt. H.S, Ayilam
2.	K.J. Prakash	Govt. Model Girls HSS, Pattom, Tvpm
3.	T.Vijayakumar	Govt. H.S, Madatharakani
4.	G.Jayakumar	MVHSS, Thundathil
5.	B.S.Santhosh	Govt.HSS, Navaikulam
6.	C.Ajitha	Govt. HSS, Bharathannoor
7.	B.L.Jyothi	Govt.Tribal HS, Meenamkal
8.	S.Sankarlal	Govt. Girls HSS, Attingal
9.	L.Sukumaran	Govt.HSS, Kazhakuttom
10.	S.Shihayas	Govt.HSS, Palayamkunnu
11.	P.S.Krishnakumar	Govt.HSS, Cherunniyoor
12.	T.Sureshkumar	Govt. HSS, Kallara
13.	V.S. Anitha	HM, Govt. DVHSS, Charamangalam
14.	R.Selvaraj	Sr.Lecturer, DIET, Tvpm.
15.	P.Lalykutty	Sr.Lecturer, DIET, Tvpm.

PREFACE

Dear friends

All the progress in the field of education in Kerala is the result of effective changes in the curriculum and activities and approaches undertaken and adopted in accordance with the ongoing changes in our modern world. Examples of these include the idea of organizing learning activities considering each child as a single unit, and awareness that there is always a social environment for learning. Similarly, Kerala has adopted a new humanitarian approach towards culture. The idea to value the cultural background of each student in the class and to provide a perspective that culture of each individual is lofty. These are assessed as stepping stones to development. So, we have taken the right and integrated approach encompassing the cultural diversity of each individual. There have been continuous effort in this sector to dismantle traditional notions of education and create a genuinely civic-minded generation. The District Panchayat, Thiruvananthapuram has implemented various exemplary models which are helpful for achieving this aim. The most important among these is the Vidyajyothi learning materials, prepared for six subjects, which are intended to increase the pass percentage of students in class 10 and help them to face the exams with more confidence. A lot of hard work is behind this venture. We express our sincere gratitude to the Honourable District Panchayat President V.K. Madhu, District Panchayat members, District Panchayat Secretary, Standing Committee Officers for their invaluable suggestions. We also thank the Principal, DIET Thiruvananthapuram, Faculty Members, Deputy Director of Education, Headmasters, Teachers, Teacher Organizations and PTA / SMC members for their wholehearted cooperation.

With love

V. Renjith

Standing Committee Chairman –
Health and Education, District Panchayath,
Thiruvananthapuram

CONTENTS

Unit 1

ARITHMETIC SEQUENCES

Points to Remember

1. A set of numbers written in order as the first, second, third and so on based on a specific rule is called a number sequence.

   eg: Sequence of squares of natural numbers 1, 4, 9, 16, ...

2. The algebraic form of a sequence is the relation between position and term.

   eg: In the sequence of squares of natural numbers, each term is the square of position. If 'n' is the position,

   $x_n = n^2$, is the algebraic form.

3. A sequence got by starting with any number and adding a fixed number repeatedly is called an arithmetic sequence..

   eg. 1. The sequence of multiples of 3 ie, 3,6,9,12, ...

   2. The sequence of natural numbers leave a remainder 1 on divison by 5 1,6, 11, 16, 21 ...

4. In an arithmetic sequence, if we subtract any term from the term immediately after it, we get the same number. This fixed number is called the common difference.

5. In an arithmetic sequence, the difference between any two terms is the product of the position difference of that terms and the common difference.

   OR

   The difference between any two terms of an arithmetic sequence is a multiple of the common difference.

6. If the terms of an arithmetic sequence are natural numbers, we get the same remainder on division by the common difference.

   Eg. In the arithmetic sequence 4, 7, 10, 13... we get 1 as the remainder on division by 3 for any term.

7. In an arithmetic sequence, term difference divided by position difference, gives the common difference.

Eg. In the arithmetic sequence 8, 13, 18, 23... $\frac{23-13}{4-2} = \frac{10}{2} = 5$.

8. The difference of any two terms of an arithmetic sequence divided by the common difference gives the position difference and one more than the position difference is the number of terms.

   Eg. In the arithmetic sequence 3, 7, 11, 15, ..., 99.

   Number of terms = $\frac{99-3}{4}+1 = \frac{96}{4}+1 = 24+1 = 25$.

9. Arithmetic Sequences are got by multiplying natural numbers from 1 by a fixed number and then adding a fixed number.

10. If the first term of an arithmetic sequence is 'f' and common difference is 'd' then $x_n = f+(n-1)d$

    or

    $$x_n = dn+(f-d)$$

11. Algebraic form of an arithmetic sequence is $x_n = an+b$ where 'a' is the common difference and 'a + b' is the first term.

    Eg. In $x_n = 3n+2$. First term is 5 and the common diffence is 3.

12. If the number of terms of an arithmetic sequence is odd, then sum of terms is the product of the number of terms and the middle term.
    Eg. Three consecutive terms of an arithmetic sequence are 6, 9, 12, then sum = 3 × 9 = 27

    Five consecutive terms of an arithmetic sequence are 8, 14, 20, 26, 32,

    then sum = 5 × 20 = 100.

13. In an arithmetic sequence, if the sum of positions of two pairs of terms are equal, then the sums of the pairs of the terms are also equal.
    eg: 7 + 12 = 5 + 14 = 3 + 16 = 1 + 18 = ...

    So in the arithmetic sequence $x_1, x_2, x_3, x_4, \ldots$, we have

    $x_7 + x_{12} = x_5 + x_{14} = x_3 + x_{16} = x_1 + x_{18} = \ldots.$

14. Sum of consecutive natural numbers starting with 1 is half the product of the last number and the next number. Algebraically

    $$1+2+3+\ldots+n = \frac{1}{2}n(n+1)$$

Eg. $1+2+3+...+50=\frac{1}{2}\times 50\times 51=25\times 51=1275$

15.. For the arithmetic sequece $x_n=an+b$, the sum of first n' terms is

$S_n = a\times\frac{n(n+1)}{2}+bn$

16. The sum of consecutive terms of an arithmetic sequence is half the product of the number of terms and the sum of the first and last terms.

Algebraically, $x_1+x_2+x_3+...+x_n=\frac{1}{2}n(x_1+x_n)$

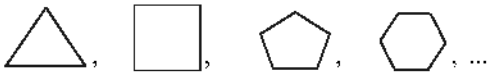
17. Sum of consecutive terms of an arithmetic sequence is in the form $s_n=pn^2+qn$, where 2p is the common difference and p + q is the first term.

18. For two arithmetic sequences with same common difference,

- Difference of terms in the same position are equal.
- Difference of the sum of 'n' terms is n × difference of terms in the same position.

Activity - 1

Write sequences based on the regular Polygons given below. Check whether each of them is an arithmetic sequence.



a. Sequence of number of sides.

b. Sequence of number of angles

c. Sequence of one interior angle.

d. Sequence of sum of interior angles

e. Sequence of one exterior angle.

f. Sequence of sum of exterior angles

g. Sequence of number of diagoals.

a. 3, ☐, ☐, 6 .It is an arithmetic sequence Yes ☐ No ☐

b. ☐, 4, ☐, ☐ ... " " Yes ☐ No ☐

c. ☐, 90°, ☐, ☐ ... " " Yes ☐ No ☐

d. ☐, ☐, 540, ☐ ... " " Yes ☐ No ☐

e. 120, ☐, ☐, 60 ... " " Yes ☐ No ☐

f. 360°, ☐, ☐ , ☐ ... " " Yes ☐ No ☐

g. 0, 2, ☐ , ☐, ... " " Yes ☐ No ☐

Activity - 2

2,6,10,14 ...is an arithmetic sequence

a. What is the 17th term?

b. Write the algebraic form of the sequence.

c. Check whether 751, a term of this sequence.

a. First term f = ☐

Common difference d = ☐ − ☐ = ☐

17th term = dn + ☐ − ☐

= 4 × ☐ + (☐ − ☐)

= ☐

b. Algebraic form is in the form x_n = ☐ n + ☐

x_n = ☐ n − 2

c. Each term of the sequence gives a remainder ☐ when divided by 4

When 751 is divided by 4, the remainder is ☐

Thus 751 is a term in the sequence Yes ☐ / No ☐

Activity - 3

The table given below is based on arithmetic sequences. Fill up the empty columns.

First term	Common Difference	Arithmetic Sequence	10^{th} Term	Algebraic form
4	3			
		3,7,11,15, ...		
2			29	
7				5n + 2
	7			7n + 3
				6n - 1
	5		50	
$\frac{1}{2}$	$\frac{1}{2}$			

Activity - 4

What is the sum of first 20 terms of the arithmetic sequence 6,10,14, ... ?

First Term = ☐, Common difference = ☐

Algebraic form of the arithmetic sequence , x_n = ☐ × n + ☐

Algebraic form of sum of first n terms ,S_n = $\frac{1}{2}$ ☐ n (n + 1) +☐ n

Then, sum of 20 terms = ☐ × ☐ + ☐ × ☐ = ☐

Activity - 5

What is the difference between the sum of the first 25 terms of the airthmetic seqences 2, 5, 8, 11 ... and 10, 13, 16...

(10 + 13 + 16 +...+ 25^{th} term) – (2+5+8+... +25^{th} term)

Difference of terms at the same position are

10 - 2 = 8

13 - 5 = ☐

16 - 8 = ☐ Continuing like this

Difference between 25^{th} terms = ☐

Then difference between sums = 25 × ☐ = ☐

Practice Problems

1. a. Write an arithmetic sequence with common difference 4.
   b. What is the 15^{th} term of this sequence.
   c. Is 145, the difference of any two terms of this sequence? Why?
2. a. Write the algebraic form of the arithmetic sequence 5,8,11...
   b. Write the 8^{th} and 24^{th} term of this sequence.
3. The 8^{th} term of an arithmetic sequence is 25 and 16^{th} term is 49.
   a. What is the common difference?
   b. What is its first term?
   c. Write the algebraic form of this arithmetic sequence.
4. The algebraic form of an arithmetic sequence is $x_n = 7n + 3$.
   a. Write the first 3 terms.
   b. What is the remainder on dividing its terms by 7?
   c. Is 500, a term of this sequence? Justify.
5. Consider arithmetic sequence 6, 11, 16, ...
   a. What is the common difference?
   b. Write the algebraic from of the sequence.
   c. Find the sum of first 20 terms?
6. The sum of 2^{nd} and 30^{th} terms of an arithmetic sequence is 50.
   a. What is the sum of 1^{st} and 31^{st} terms?
   b. What is the sum of 15^{th} and 17^{th} terms?
   c. What is the 16^{th} term?
   d. What is the sum of first 31 terms?
7. Answer the following questions based on the given pattern.

$$1 = 1$$
$$1 + 3 = 4$$
$$1 + 3 + 5 = 9$$

.....

.....

.....

a. Write the next two lines?

b. Find the sum of first 15 odd numbers?

c. Find $\frac{1}{2}+\frac{3}{2}+\frac{5}{2}+\ldots+\frac{29}{2}$?

8. The sum of first 9 terms of an arithmetic sequence is 90.

   a. What is its 5^{th} term?

   b. Write three arithmetic sequences with 90 as the sum of first 9 terms.

9. a. Write the sequence of natural numbers between 100 and 300 which leave a remainder 2 on division by 3.

   b. How many terms are there in this sequence?

   c. Find the sum of terms of this sequence.

10. The algebraic form of the sum of first 'n' terms of an arithmetic sequence is $S_n = 2n^2 + 3n$

    a. Write the first term and common difference.

    b. Write the algebraic form of this sequence.

    c. Find the sum of first 25 terms?

11. The sum of natural numbers from 1 to 100 is 5050. Using this, find the sum of the terms of the arithmetic sequence 3, 6, 9, ... 300.

12. The algebraic form of two arithmetic sequences are $x_n = 6n + 2$ and $x_n = 6n - 2$. What is the difference of the sums of first 25 terms of these sequences.

13. Find the following sums:

    a. 1 + 2 + 3 + ... + 40

    b. 5 + 6 + 7 + ... + 44

    c. 6 +12 + 18 + ... + 240

    d. –1 + 5 + 11 + ... + 233

14. The 7^{th} term of an arithmetic sequence is 17 and 17^{th} term is 7.

    a. What is the common difference?

    b. Find the 24^{th} term?

    c. Find the sum of first 47 terms?

15. The 5^{th} term of an arithmetic sequence is 40 and 31^{st} term is 160.

a. What is the sum of 17^{th} and 19^{th} terms?

b. Find the 18^{th} term?

c. Find the sum of first 35 terms?

d. What is the sum of first 35 terms of the arithmetic sequence with 5^{th} term 43 and 31^{st} term 163?

Answers

1. a. 1, 5, 9, ...; 4, 8, 12, ...; 6, 10,14 ...; are examples.

b. $x_{15} = f + 14d$. Or $x_{15} = d \times 15 + (f - d)$

c. The difference between any two terms of this sequence is a multiple of 4. Since 145 is not a multiple of 4, it is not the difference between any two terms of this sequence.

2. a. $x_n = 3n + 2$

b. $x_8 = 3 \times 8 + 2 = 24 + 2 = 26$

$x_{24} = 3 \times 24 + 2 = 72 + 2 = 74$

3. a. $x_8 + 8d = x_{16}$

$25 + 8d = 49$ OR $x_{16} - x_8 = 49 - 25$

$8d = 49 - 25 = 24$ $8d = 24$

$d = \frac{24}{8} = 3$ $d = \frac{24}{8} = 3$

b. First term $x_1 = x_8 - 7d = 25 - 3 \times 7 = 25 - 21 = 4$

c. $x_n = 3n + 1$

4. a. $x_1 = 7 \times 1 + 3 = 10$

$x_2 = 7 \times 2 + 3 = 17$

$x_3 = 7 \times 3 + 3 = 24$

b. The terms of this sequence leaves a remainder 3 on division by 7.

c. 500 leave a remainder 3 on division by 7. So 500 is a term.

5. a. Common difference = 5

b. Algebraic from, $x_n = dn + (f - d) = 5n + 1$

c. $S_{20} = 5 \times \frac{20 \times 21}{2} + 20 \times 1$

$= 1050 + 20 = 1070$

6. a. $x_1 + x_{31} = 50$

   b. Since $x_2 + x_{30} = 50$

   $x_{15} + x_{17}$ is also 50.

   c. Middle term is half the sum of the pair

   $$x_{16} = \frac{50}{2} = 25$$

   d. Sum of 31 terms $= 31 \times x_{16}$

   $= 31 \times 25$

   $= 775$

7. a. $1 + 3 + 5 + 7 = 16$

   $1 + 3 + 5 + 7 + 9 = 25$

   b. Sum of first 'n' consecutive odd numbers is n^2.

   $\therefore$ Sum of first 15 odd numbers $= 15^2 = 225$

   c. $\frac{1}{2}\ (1 + 3 + 5 + ... + 15) = \frac{1}{2} \times 225 = \frac{225}{2}$

8. a. $9 \times x_5 = 90$

   $$x_5 = \frac{90}{9} = 10$$

   b. 2, 4, 6, 8, 10, 12, 14, 16, 18

   –10, –5, 0, 5, 10, 15, 20, 25, 30

   –2, 1, 4, 7, 10, 13, 16, 19, 22

   That is, in any arithmetic sequence with middle term 10 and any number as common difference the sum of 9 terms is always 90.

9. a. $x_1 = 101$

   d = 3

   Sequence 101, 104, 107, 110, ... 229

   b. Number of terms $= \frac{299 - 101}{3} + 1$

   $= \frac{198}{3} + 1 = 66 + 1 = 67$

c. $s_{67} = \frac{1}{2}n(x_1 + x_{67})$

$= \frac{1}{2} \times 67 \times (101 + 299)$

$= \frac{1}{2} \times 67 \times 400$

$= 13400$

10. a. In the algebraic form of the sum $S_n = pn^2 + qn$ of the arithmetic sequence 2p is the common difference.

$s_n = 2n^2 + 3n$

$\therefore d = 2 \times 2 = 4$

First term $x_1 = 2 + 3 = 5$

b. Algebraic form of the arithmetic sequence is $x_n - 4n + 1$

c. $s_{25} = 2 \times 25^2 + 3 \times 25$

$= 2 \times 625 + 3 \times 25$

$= 1250 + 75$

$= 1325$

11. $1 + 2 + 3 + 4 \ldots + 100 = 5050$

$3 + 6 + 9 + \ldots + 300 = 3(1 + 2 + 3 + \ldots + 100)$

$= 3 \times 5050$

$= 15150$

12. Algebraic form of first arithmetic sequence, $x_n = 6n + 2$

Algebraic form of second arithmetic sequence, $x_n = 6n - 2$

First term of the first sequence = 8

First term of the second sequence = 4

In two arithmetic sequences with same common difference, the difference between the first terms is also the difference between any terms at the same position.

Difference between first terms = 8 – 4 = 4

$\therefore$ Difference of sums = 25 × 4 = 100

13. (a) $1 + 2 + 3 + \ldots\ldots + 40 = \frac{40 \times 41}{2} = 820$

(b) $5 + 6 + 7 + \ldots\ldots + 44 = 820 + 40 \times 4 = 980$

(c) $6 + 12 + 18 + \ldots\ldots + 240 = 6\,(1+2+3+ \ldots\ldots + 40)$

$= 6 \times 820 = 4920$

(d) $-1 + 5 + 11 + \ldots\ldots + 233 = 4920 - 40 \times 7$

$= 4640$

14. (a) Common difference $= \frac{\text{Term difference}}{\text{Position difference}}$

$= \frac{7-17}{17-7}$

$= -1$

(b) 24^{th} term $= x_{17} + 7d$

$= 7 + 7 \times (-1)$

$= 0$

(c) Sum of first 47 terms $= 47 \times 24^{th}$ term

$= 47 \times 0$

$= 0$

15. $x_5 = 40, \; x_{31} = 160$

(a) $x_{17} + x_{19} = x_5 + x_{31}$

$= 40 + 160$

$= 200$

(b) $x_{18} = \frac{200}{2} = 100$

(c) $S_{35} = 35 \times 100$

$= 3500$

(d) $x_5 = 43, \; x_{31} = 163$

Sum of 35- terms $= 3500 = 35 \times 3$

$= 3605$

16. $x_6 = 28,\ x_{11} = 63$

(a) $x_1 = x_6 - 5d$

$= 28 - 35$

$= -7$

(b) Common difference $= \frac{35}{5} = 7$

(c) Algebraic form, $x_n = dn + (f - d)$

$= 7n + (-7 - 7)$

$= 7n - 14$

MORE PRACTICE PROBLEMS

1. Using the alternate terms of the arithmetic sequence 5,8,11,14, ...
   a. Write two sequences
   b. Are these arithmetic sequences?
   c. Write the algebraic form of these sequences.
2. Two arithmetic sequences are given below.
   Sequence I : 2, 8, 14, ...
   Sequence II : 23, 26, 29, ...
   a. Write the algebraic form of both sequences.
   b. Is there any common term in these sequences. If so write such a term.
3. Consider the arithmetic sequence 3, 7, 11, ...
   a. Write its algebraic form.
   b. What is the sum of first 'n' terms?
4. The sum of first 10 terms of an arithmetic sequence is 4 times the sum of first 5 terms. Then how many times the first term is the common difference?
5. The angles of a polygon are in arithmetic sequence such as 172°, 164°, 156°, ...
   a. Write the sequence of exterior angles.
   b. Find the number of sides of this polygon?

6. Consider the following number pattern

1

2 3 4

5 6 7 8 9

10 11 12 13 14 15 16

.....

.....

a. Write the next two lines?

b. Write the sequence of number of numbers in each row.

c. What are the first and last numbers in the 10^{th} line?

d. Find the sum of all the numbers in the first 10 lines?

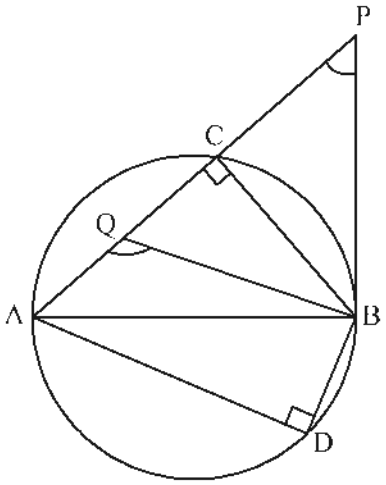
Unit 2

CIRCLES

Points to Remember

◆ If we join the ends of a diameter of a circle to any point on the circle, we get a right angle.

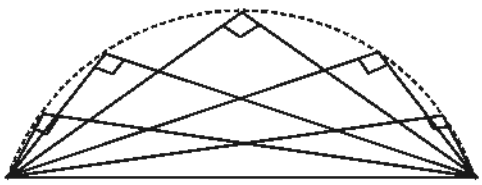
If we join the end points of a diameter of a circle to a point inside the circle, we get an angle more than a right angle, and if the point is outside, the angle is less than a right angle.



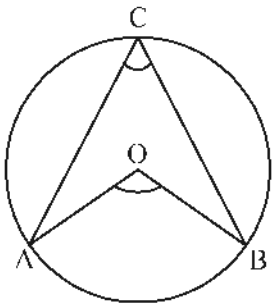
$\angle ACB = 90^{\circ}$, $\angle ADB = 90^{\circ}$

$\angle APB < 90^{\circ}$, $\angle AQB > 90^{\circ}$

◆ All the mutually perpendicular lines drawn from the two end points of a line will meet at the circle with the line as diameter.

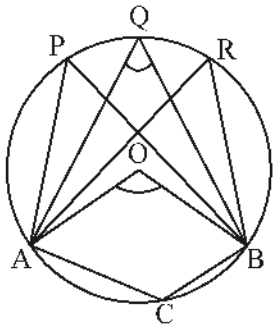


◆ The angle made by any arc of a circle on the alternate arc is half the angle made at the centre.



$$\angle ACB = \frac{1}{2}\angle AOB$$

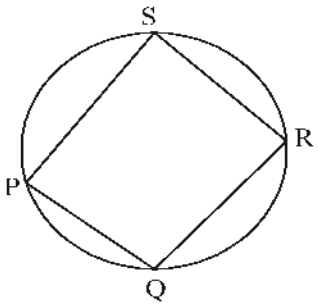
- All the angles made by an arc on the alternate arc are equal and a pair of angles on alternate arcs are supplementary.



$$\angle P = \angle Q = \angle R$$

$$\angle P + \angle C = 180^o, \quad \angle Q + \angle C = 180^o, \quad \angle R + \angle C = 180^o$$

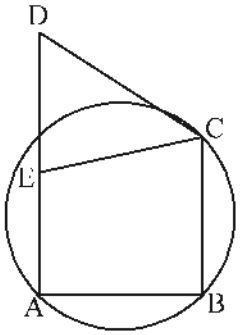
- If all the four vertices of a quadrilateral are on a circle, then its opposite angles are supplementary.



$$\angle P + \angle R = 180^o$$

$$\angle Q + \angle S = 180^o$$

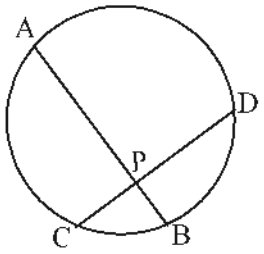
- If one vertex of a quadrilateral is outside the circle drawn through the other three vertices, then the sum of the angles at this vertex and the opposite vertex is less than 180°, if it is inside the circle, the sum is more than 180°.



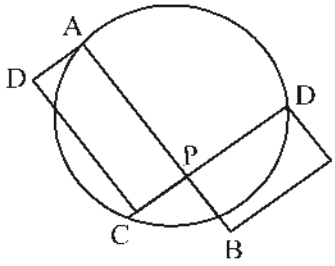
$\angle ABC + \angle D < 180^o$

$\angle ABC + \angle AEC > 180^o$

- If the opposite angles of a quadrilateral are supplementary, we can draw a circle passing through all four of its vertices.
- If two chords of a circle intersect within the circle, then the product of the parts of the two chords are equal. That is the rectangle formed by the parts of the same chord have equal area.

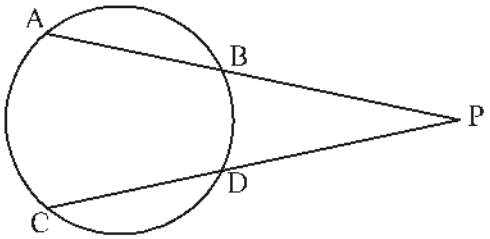


PA × PB = PC × PD



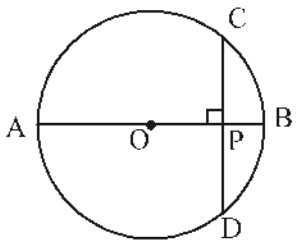
The area of two rectangels are equal

- If the two chords of a circle intersect externally, the product of the distances from the external point to the ends of the chords are equal.

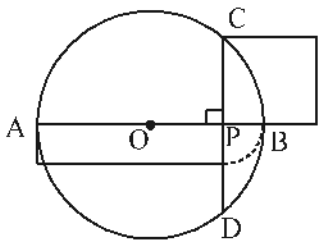


PA × PB = PC × PD

◆ The product of the parts into which a diameter of a circle is cut by a perpendicular chord is equal to the square of half the chord. That is the area of the rectangle formed of parts into which a diameter of a circle is cut by a perpendicular chord is equal to the area of the square formed by half the chord.



$PA \times PB = PC^2$

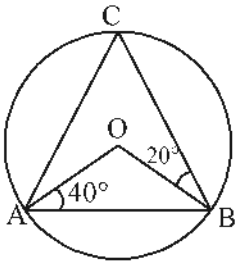


Area of the square is equal to area of the rectangle.

Activity- 1

In the figure A,B,C are points on a circle with centre 'O'. Find the angles of the triangle ABC.

ΔOAB is an isosceles triangle.



$\angle ABO =$ ☐

$\angle AOB =$ ☐

$\angle ACB = \frac{1}{2} \times$ ☐ $=$ ☐

Consider ΔOBC

$\angle OCB =$ ☐, $\angle BOC =$ ☐, $\angle BAC = \frac{1}{2} \times$ ☐, $=$ ☐

$\angle ABC = 180 - ($☐ $+$ ☐$) =$ ☐

Activity- 2

In the figure A, B, C, D are points on the circle. Find the angles of the quadrilateral ABCD and the angles between the diagonals of it.

Since the angles in the same segment are equal.

$\angle BDC =$ ☐

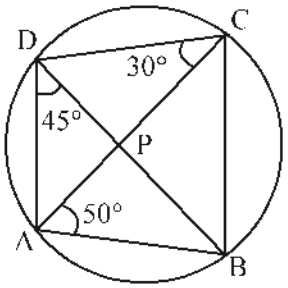
$\angle ACB =$ ☐

$\angle ABD =$ ☐

$\angle BCD = 30^o +$ ☐ = ☐

$\angle BAD = 180^o -$ ☐ = ☐

$\angle ADC = 45^o +$ ☐ = ☐



Since the opposite angles of the cyclic quadrilateral are supplementary,

$\angle ABC = 180^o -$ ☐ = ☐

To find the angles between the diagonals,

Consider ΔCDP

$\angle CPD = 180^o - ($☐ $+ 30^o) =$ ☐

$\angle BPA = \angle CPD =$ ☐

$\angle BPC = 180^o -$ ☐ = ☐

$\angle APD =$ ☐

Activity - 3

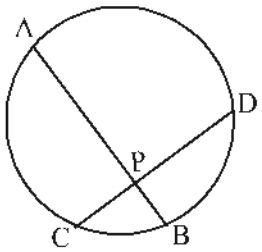
In the figure, chords AB and CD intersect at P. If AB =15 cm, PB = 3cm and CP = 4cm, find PD. Find the area of the rectangle with sides CP and PD.

PA × PB = ☐ × ☐

PA = 15 - ☐ = ☐

☐ × 3 = 4 × PD

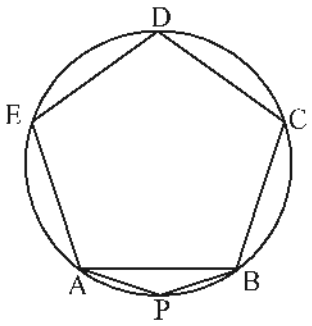
PD = $\frac{\square}{4}$ = ☐



Area of the rectangle = PC × PD

= ☐ × ☐ = ☐

Activity - 4



In the figure, ABCDE is a regular pentagon.
Find $\angle APB$

Consider ΔABC, which is an isosceles triangle.

Since $\angle ABC = 108^\circ$

$\angle BAC = \angle BCA =$ ☐

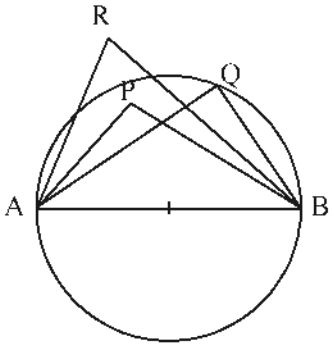
Since the quadrilateral APBC is cyclic

$\angle APB = 180 -$ ☐

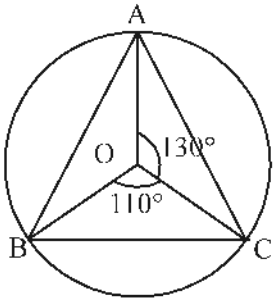
$=$ ☐

PRACTICE PROBLEMS

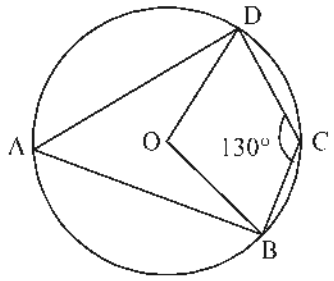
1. In the figure, AB is a diameter of the circle. Which angle has measure 90°? List out the angles that have measures less than 90° and more than 90°.



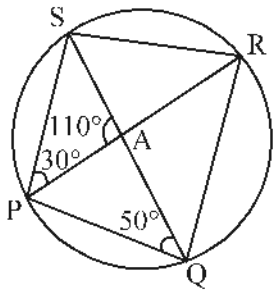
2. In the figure, 'O' is the centre of the circle. Find each angle of the ΔABC.



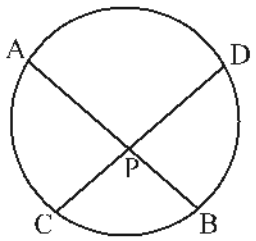
3. In the figure, 'O' is the centre of the circle. $\angle BCD = 130°$. Find $\angle BAD$ and $\angle BOD$.



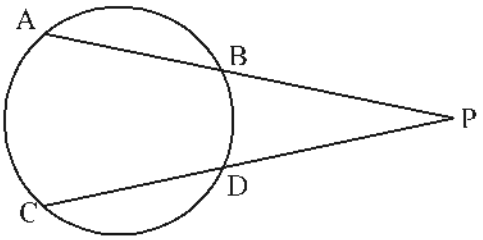
4. In the figure, P, Q, R, S are points on the circle. $\angle PAS = 110°$, $\angle APS = 30°$, $\angle PQA = 50°$. Find each angle of the quadrilateral.



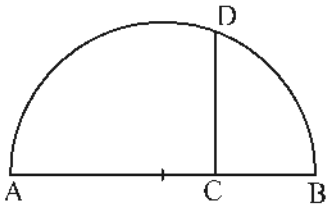
5. In the figure, the chords AB and CD intersect at P. If AP = 8cm, CP = 3cm and CD = 19cm, find AB.



6. The chords AB and CD intersect at a point P outside the circle. PB = 4cm, AB = 6cm, PC = 8cm. Find PD



7. In the figure, AB is the diameter of the semicircle and $AB \perp CD$. If AC = 6cm, and CB = 3 cm. Find CD. How can we draw lines of lengths $\sqrt{14}$ cm, and $\sqrt{20}$ cm.



8. Draw an angle of measure 50°. Mark 25° angle without drawing its bisector.
9. Draw a circle of radius 3cm. Construct a triangle with this circle as circumcircle and two angles 50° and 70°.
10. Draw a circle of radius 3.5cm. Construct a triangle with this circle as circumcircle and two angles 30° and 110°.
11. Construct a square of area 7cm^2.
12. Draw a rectangle of length 6cm and breadth 4cm. Draw a square of equal area.

Answers

1. $\angle AQB = 90°$

   $\angle APB$ has measure more than 90° and $\angle ARB$ has measure less than 90°.

2. $\angle A = \frac{1}{2} \times 110 = 55°$

   $\angle B = \frac{1}{2} \times 130 = 65°$

   $\angle C = 180 - (55 + 65) = 60°$

3. $\angle BAD = 180 - 130$
   $= 50°$

   $\angle BOD = 2 \times 50 = 100°$

4. Since the angles in the same segments are equal.

   $\angle RQS = \angle RPS = 30°$

   $\angle PRS = \angle PQS = 50°$

   $\angle PSQ = 180 - (110 + 30)$
   $= 40°$

   $\therefore \angle PRQ = \angle PSQ = 40°$

   $\angle PQR = \angle PQS + \angle RQS$
   $= 30 + 50 = 80°$

   Since the opposite angles of a cyclic quadrilateral are supplementary,

   $\angle PSR = 180 - 80 = 100°$

   $\angle QRS = \angle PRQ + \angle PRS$

$= 40 + 50 = 90°$

$\angle QPS = 180 - 90 = 90°$

5. AP = 8cm, CP = 3 cm, PD = 19 – 3 = 16 cm.

$AP \times PB = CP \times PD$

$8 \times PB = 3 \times 16$

$$PB = \frac{48}{8} = 6$$

$\therefore AB = 8 + 6 = 14cm$

6. PA = 6 + 4 = 10cm

PB = 4cm, PC = 8 cm

$PA \times PB = PC \times PD$

$10 \times 4 = 8 \times PD$

$$PD = \frac{40}{8} = 5 \text{ cm}$$

7. AC = 6 cm CB = 3 cm

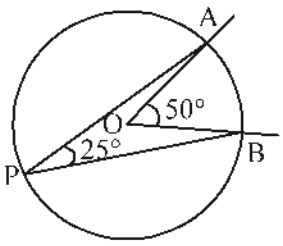
$AC \times CB = CD^2$

$6 \times 3 = CD^2$

$CD = \sqrt{18}$ cm

Mark a point 'C' 7cm away from A and draw a perpendicular at C; then it has length $\sqrt{14}$ cm. Mark a point C, 5 cm from A and draw a perpendicular at C; then it has length $\sqrt{20}$ cm.

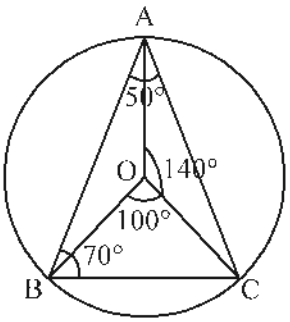
8.



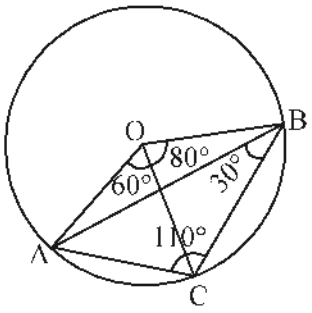
The angle made by any arc of a circle on

the alternate arc is half the angle made at the centre.

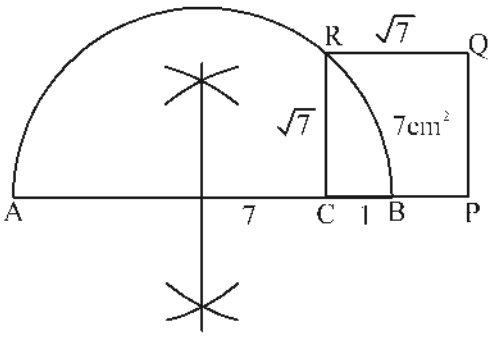
9.



10.

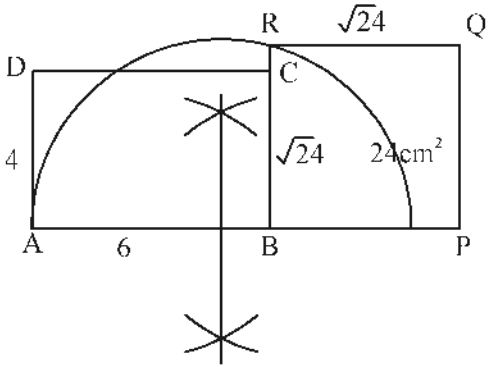


11.



AC = 7cm, CB = 1cm

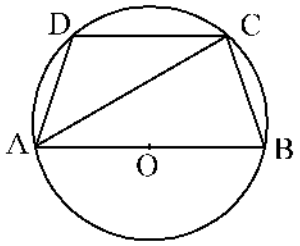
12.



MORE PRACTICE PROBLEMS

1. Draw a triangle with two angles 27½°, 32½° and has circum radius 3cm.

2.

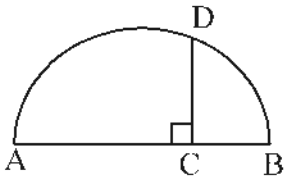


In the figuer, AB is diameter of the circle and CD is parallel to AB. $\angle CAB = 30°$. Find the measures of the following angles.

(a) $\angle ACB$ (b) $\angle DCA$ (c) $\angle CBA$

(d) $\angle CDA$ (e) $\angle DAC$

3.



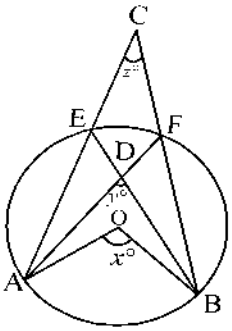
In the figure, AB is the diameter of the semicricle. AC = 6cm, AB = 8 cm.

(a) What is the length of CB ?

(b) Find the length of CD?

(c) Draw an equilateral triangle of perimeter $6\sqrt{3}$ cm.

4.



In the figure 'O' is the centre of the circle.

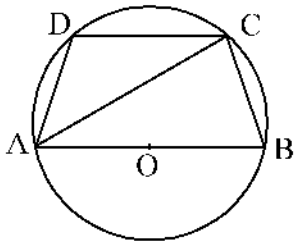
(a) If $\angle AOB = x°$, what is the measure of $\angle AFB$?

(b) What is the measure of $\angle DEC$?

(c) If $\angle ADB = y°$, $\angle ACB = z°$, then prove that $x = y + z$

5. Draw a rectangle of length 7cm and breadth 4cm. Draw another rectangle of equal area to the first and has length 8cm.

2.

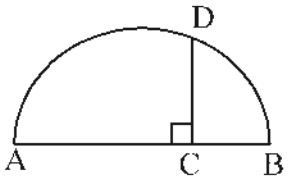


In the figuer, AB is diameter of the circle and CD is parallel to AB. $\angle CAB = 30°$. Find the measures of the following angles.

(a) $\angle ACB$ (b) $\angle DCA$ (c) $\angle CBA$

(d) $\angle CDA$ (e) $\angle DAC$

3.



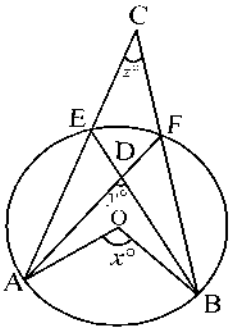
In the figure, AB is the diameter of the semicricle. AC = 6cm, AB = 8 cm.

(a) What is the length of CB ?

(b) Find the length of CD?

(c) Draw an equilateral triangle of perimeter $6\sqrt{3}$ cm.

4.



In the figure 'O' is the centre of the circle.

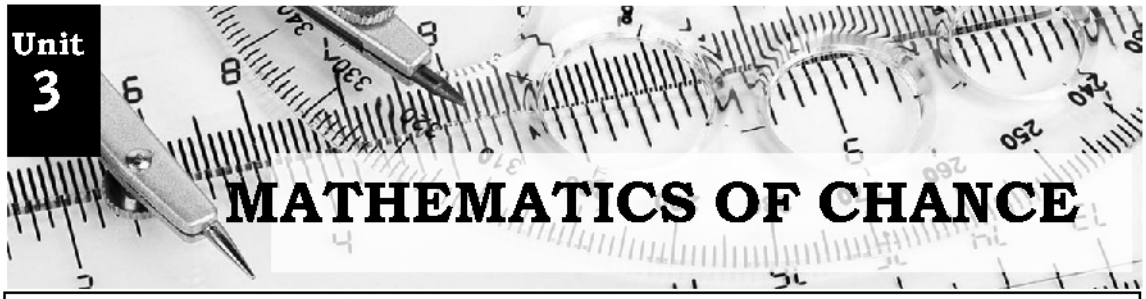
(a) If $\angle AOB = x°$, what is the measure of $\angle AFB$?

(b) What is the measure of $\angle DEC$?

(c) If $\angle ADB = y°$, $\angle ACB = z°$, then prove that $x = y + z$

5. Draw a rectangle of length 7cm and breadth 4cm. Draw another rectangle of equal area to the first and has length 8cm.

Unit 3



MATHEMATICS OF CHANCE

Points to Remember

1. The probability of occuring an event is the number, which indicates how many part of the favourable outcomes of it to the total outcomes.
2. If there are different ways to happen two events, then the number of ways they can occur together is the product of the number of ways they occur seperately.

Activity –1

There are 5 black beads and 4 white beads in a box, another box contains 6 black beads and 7 white beads. One bead is taken out from it without looking. Which box is preferable to get a black bead?

- Total number of beads in the first box = ☐

  Number of black beads in the first box = ☐

  Probability of getting a black bead from the first box = $\frac{\square}{\square}$

- Total number of beads in the second box = ☐

  Number of black beads in the second box = ☐

  Probability of getting a black bead from the second box

  = $\frac{\square}{\square}$

Compairing the two chances

$\frac{5}{9} = \frac{5\times 13}{9\times \square}$ = $\frac{\square}{\square}$

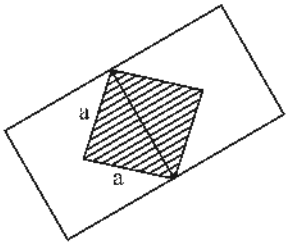
$\frac{6}{13} = \frac{6\times \square}{13\times \square}$ = $\frac{\square}{\square}$

Larger fraction = $\frac{\square}{\square}$

Which box is to be selected : First box ☐ / Second box ☐

Activity –2

In the figure two squares are drawn on the two sides of the diagonal of the small square with length of the diagonal as side. If a dot is put in the figure without looking, what is the probability of it being in the shaded region?



Length of the side of the small square = ☐

Length of the diagonal of the small square = ☐

Area of the small square = ☐ cm^2

Area of the large rectangle = Sum of the area of the two squares with the diagonal as side = ☐ + ☐ = ☐

Probability of the dot put is in the shaded region = $\frac{\square}{\square}$

Activity –3

There are 10 slips with even numbers written on that and 15 slips with odd numbers written on that in a box and 20 slips of even numbers and 30 slips of odd numbers are in another box.

a. If one slip is taken from each box what is the probability of both of them are odd numbers?

b. What is the probability of at least one of them is an odd number?

a. Total number of slips in the first box = ☐ + ☐ = ☐

Total number of slips in the second box = ☐ + ☐ = ☐

If one paper is taken from each box,

total number pairs = ☐ × ☐ = ☐

Number of slips of odd numbers in the first box = ☐

Number of slips of odd numbers in the second box = ☐

Total pairs in which both of them are odd numbers

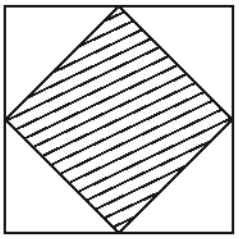
= ☐ × ☐ = ☐

Probability of both of them are odd $= \frac{\square}{1250} = \frac{\square}{\square}$

PRACTICE PROBLEMS

1. One is asked to say a three digit number. What is the probability of all the digits being the same?
2. There are 4 black beads and 5 white beads in a box. 6 black beads and 5 white beads is another box.
   a. A bead is taken out from the first box without looking. What is the probability of it being black ? What about white?
   b. Which box is preferable to get a white bead?
   c. If all the beads are put in a single box and take a bead without looking, what is the probability of it being black?

3. There are white and black beads in a box, with total 16. The probability of getting a black bead from this box is ½ . Then,
   a. How many black beads are there?
   b. How many white beads are there?
   c. What is the probability of getting a white bead?
   d. If one more black bead is put into this box, what is the probability of getting a black bead ?
4. Two dice are rolled together. From the number pairs so get,
   a. What is the probability of both being odd?
   b. What is the probability of both being same?
   c. What is the probability of one being even and another being odd?
5. In the figure a square is drawn by joining the mid-points of another square. If a dot is put inside the larger square, what is the probability of it being inside the smaller square?



Answers

1. Number of three digit numbers = 900
   Number of three digit numbers with same digits = 9
   (111, 222, 333, 444, 555, 666, 777, 888, 999)

   Probability of getting three digit numbers with same digits = $\frac{9}{900} = \frac{1}{100}$

2. a. Probability of getting black bead from the first box = $\frac{4}{9}$

   b. Probability of getting white bead from the first box=- $\frac{5}{9}$

   c. Probability of getting white bead from the second box = $\frac{5}{11}$

   since $\frac{5}{9} > \frac{5}{11}$, first box is preferable to get a white bead.

   d. When all the beads are put in a single box, number of beads = 20

Number of black beads = 10

Probability of getting a black bead = $\frac{10}{20} = \frac{1}{2}$

3. a. Since the probability is ½, number of black beads = $\frac{1}{2} \times 16 = 8$

   b. Number of white beads = 8

   c. Probability of getting white bead = $\frac{8}{16} = \frac{1}{2}$

   d. When one more black bead is put in the box, probability of getting a black bead = $\frac{9}{17}$

4. a. Total number of pairs = 6 × 6 = 36
   Number of pairs with both of them are odd = 3 × 3 = 9

   Probability of both being odd = $\frac{9}{36} = \frac{1}{4}$

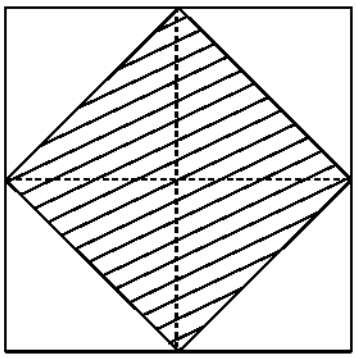
   b. Pairs with same digits are
   (1, 1), (2, 2), (3, 3), (4, 4) (5, 5), (6, 6)

   Probability of both being same digits = $\frac{6}{36} = \frac{1}{6}$.

   c. Total pairs in which the number in the first die is odd and second die is even = 3 × 3 = 9
   Total pairs in which the number from the first die is even and number in the second die is odd = 3 × 3 = 9
   Total pairs in which one is even and the other is odd = 9 + 9 = 18

   Probability of one being even and the other being odd = $\frac{18}{36} = \frac{1}{2}$

5. Area of small square is half the area of the large square.



   ∴ Probability that the dot is inside the small square = $\frac{1}{2}$

MORE PRACTICE PROBLEMS

1. Slips numbered 1 to 100 are put in a box. One is taken out from it without looking.
   a. What is the probability of it being a multiple of 4 ?
   b. What is the probability of it being a multiple of 5 ?
   c. What is the probability of it being a multiple of both 4 and 5?
2. In two digit natural numbers up to 50.
   a. What is the probability of occuring the digit at tens place is greater than the ones place?
   b. What is the probability of occuring the digit at tens place is less than the ones place?
3. Slips numbered odd numbers less than 10 are put in a box and slips numbered natural numbers from 1 to 5 in another box. One slip is taken from each box without looking.
   a. What is the probability of both being odd?
   b. What is the probability of both being even?
4. What is the probability of having 5 sundays in January?

Unit 4

SECOND DEGREE EQUATIONS

Points to Remember

Algebraic form

- Two consecutive natural numbers : $x,\ x+1$
- Two consecutive odd numbers : $x,\ x+2$
- Two consecutive even numbers : $x,\ x+2$
- Two consecutive terms of an arithmetic sequence with common difference 'd' : $x,\ x+d$
- Two numbers with sum 12 : $x,\ 12-x$
- Two numbers with difference 12 : $x,\ x+12$ or $x,\ x-12$

- Method of solving second degree equations of the form $(x-a)^2=b^2$

$(x-a)^2=b^2$

$x-a=\pm b$

$x-a=b$ or $x-a=-b$

$x=b+a$ $x=-b+a$

Example: $(x-3)^2=25$

$x-3=\pm\sqrt{25}$

$x-3=\pm5$

$x-3=5$ or $x-3=-5$

$x=5+3$ $x=-5+3$

$=8$ $=-2$

- To convert $x^2 + 2ax$ into a perfect square, we have to add the square of half the coefficient of x. That is $x^2 + 2ax + a^2 = (x+a)^2$

  $x^2 - 2ax + a^2 = (x-a)^2$

- The general form of a second degree equation is $ax^2 + bx + c = 0,\ a \neq 0$.
- Method of solving a second degree equation by completing the square
  - Make the coefficient of x^2 as 1
  - Take the constant term to the right side of the equal sign.
  - Add the square of half the coefficient of x to both sides.
  - Write the equation in perfect square form
  - Take square root on both sides
  - Find x.
- Example: $2x^2 + 9x + 4 = 0$

  Dividing all the terms by 2

$$x^2 + \frac{9}{2}x + \frac{4}{2} = 0$$

$$x^2 + \frac{9}{2}x + 2 = 0$$

$$x^2 + \frac{9}{2}x = -2$$

Now adding $\left(\frac{9}{4}\right)^2$ to both sides

$$x^2 + \frac{9}{2}x + \left(\frac{9}{4}\right)^2 = -2 + \left(\frac{9}{4}\right)^2$$

$$= -2 + \frac{81}{16}$$

$$= \frac{-2\times16+81}{16}$$

$$= \frac{-32+81}{16}$$

$$\left(x+\frac{9}{4}\right)^2 = \frac{49}{16}$$

$$x+\frac{9}{4} = \pm\sqrt{\frac{49}{16}}$$

$$= \pm\frac{7}{4}$$

$$x+\frac{9}{4} = \frac{7}{4}, \qquad x+\frac{9}{4} = \frac{-7}{4}$$

$$x = \frac{7}{4}-\frac{9}{4}, \qquad x = \frac{-7}{4}-\frac{9}{4}$$

$$= \frac{-2}{4} \qquad = \frac{-16}{4}$$

$$= \frac{-1}{2} \qquad = -4$$

- To get $ax^2 + bx + c = 0$, we must take

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

- A second degree equation can have at most 2 solutions

Activity–1

When each side of a square is increased by 2 cm, the area will become 100 cm^2. What is the length of the side of the original square?

One side of the original square = x

One side of the new square = $x +$ ☐

Area of the new square = $(x +$ ☐$)^2$

$(x +$ ☐$)^2 = 100$

$x +$ ☐ $= \sqrt{100}$

$x +$ ☐ $=$ ☐

$x - $ ☐ - ☐

$-$ ☐

$\therefore$ One side of the original square = ☐ cm

Activity-2

The length of a rectangle is 6 cm more than its breadth and its area is 91 cm^2 . What are the lengths of the sides of the rectangle?

Breadth = x

Length = $x +$ ☐

Area = $x\,(x +$ ☐$)$

$x\,(x +$ ☐$) = 91$

The number to be added to change it into a perfect square – ☐

$x^2 - 6x +$ ☐ $= 91 +$ ☐

$(x -$ ☐$)^2 =$ ☐

x ☐ $-$ ☐

$x - $ ☐ - ☐

$-$ ☐

$\therefore$ Breadth of the rectangle $-$ ☐ cm

Length of the rectangle = ☐ $+ 6$

$-$ ☐ cm

Activity -3

Length of a rectangle is 2cm less than twice its breadth. The length of its diagonal is 5cm. Find the length and breadth of the rectangle.

Let breadth = x,

Then length = ☐

$x^2 + (2x - 2)^2 =$ ☐

$5x^2 -$ ☐ $-$ ☐ $= 0$

a = 5, b = ☐ , c = ☐

$b^2 - 4ac =$ ☐

= ☐

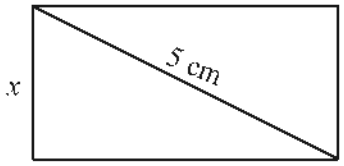
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{8 \pm \sqrt{\square}}{10}$$

= ☐, ☐

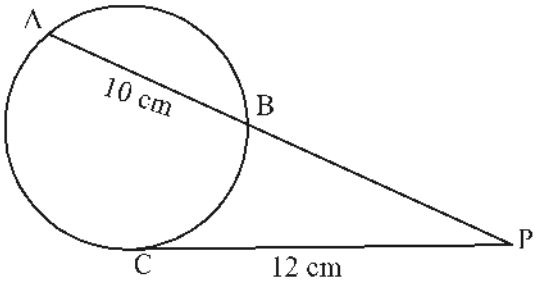
Breadth = ☐

Length = ☐



PRACTICE PROBLEMS

1. The square of the difference of 3 times a number and 1 is 121. What is the number ?
2. One side of rectangle is 2cm longer than the other side and its diagonal is 10cm. What are the lengths of its sides?
3. What should be the lengths of the sides of a rectangle having perimeter 28 cm and area 24cm^2.

4. 

In the figure, the chord AB extended and the tangent at the point C, meet at P. AB = 10 cm, PC = 12 cm Find the length of PB ?

5. The sum of the squares of two consecutive odd numbers is 130. What are the numbers?

6. The sum of the perpendicular sides of a right angled triangle is 14 cm and its area is 24 cm^2. Find the length of its sides?

7. The hypotenuse of a right angled triangle is 3cm more than twice the base and the third side is 1 cm less than the hypotenuse. Find the length of the sides?

8. How many terms of the arithmetic sequence 7,11,15, ... must be added to get 250?

ANSWERS

1. Let the number = x

Three times the number = $3x$

Square of the difference of 3 times the number and 1 is $(3x - 1)^2$

$$(3x-1)^2 = 121$$

$$3x - 1 = \pm\sqrt{121} = \pm 11$$

$$3x - 1 = 11, \quad 3x - 1 = -11$$

$$3x = 11+1 = 12, \quad 3x = -11+1 = -10$$

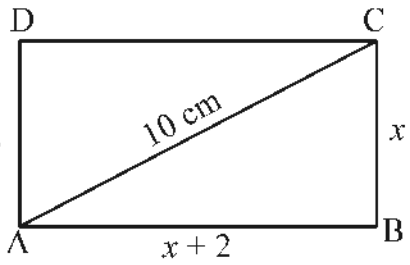
$$x = \frac{12}{3}, \quad x = \frac{-10}{3}$$

$$= 4$$

$\therefore$ The number = 4 or $\frac{-10}{3}$

2. Let breadth = x

Then length = $x + 2$



From the right angled triangle ABC,

$AB^2 + BC^2 = AC^2$

$(x + 2)^2 + x^2 = 10^2$

$x^2 + 2 \times x \times 2 + 2^2 + x^2 = 100$

$x^2 + 4x + 4 + x^2 = 100$

$2x^2 + 4x = 100 - 4$

$2x^2 + 4x = 96$

Dividing all the terms by 2,

$x^2 + 2x = 48$

Adding 1 to both sides

$x^2 + 2x + 1 = 48 + 1$

$(x + 1)^2 = 49$

$x + 1 = \pm\sqrt{49}$

$x + 1 = \pm 7$

$x + 1 = 7, \; x + 1 = -7$

$x = 7 - 1 \quad x = -7 - 1$

$= 6 \qquad = -8$

(But length of a side cannot be negative)

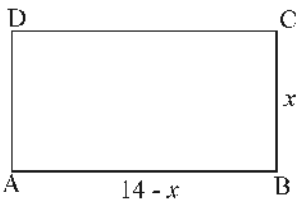
∴ Breadth of the rectangle = 6 cm

Length of the rectangle = 6 + 2 = 8 cm

3. Perimeter of the rectangle = 28

2 (length + breadth) = 28

length + breadth = $\frac{28}{2}$ = 14



If breadth x, then length = $14 - x$

Area = length × breadth

$x(14 - x)$ = 24

$14x - x^2$ = 24

$-x^2 + 14x$ = 24

$x^2 - 14x$ = – 24

Adding 49 to both sides

$x^2 - 14x - 49$ = – 24 + 49

$(x - 7)^2$ = 25

$x - 7$ = $\pm\sqrt{25}$ = ±5

$x - 7 = 5$ $x - 7 = -5$

$x = 5+7$ $x = -5 + 7$

$= 12$ $= 2$

If x =12, then breadth = 12 cm and length = 14 – 12 = 2 cm

If x =2, then breadth = 2 cm and length = 14 – 2 = 12 cm

4. If PB = x then PA = x +10

PA × PB = PC^2

$x(x+10)$ = 12^2

$x^2 + 10x$ = 144

Adding 25 to both sides

$x^2 + 10x + 25$ = 144+25

$(x + 5)^2$ = 169

$x + 5$ = $\pm\sqrt{169}$

$x + 5$ = ±13

$x + 5 = 13$ $x + 5 = -13$

$x + 5 = 13$ $x = 13 - 5 = 8$

$x + 5 = -13$ $x = -13 - 5 = -18$

Length cannot be negative

∴ Length of PB = 8 cm

5. Two consecutive odd numbers = $x, x + 2$

$x^2 + (x + 2)^2$ = 130

$x^2 + x^2 + 2 \times x \times 2 + 2^2 = 130$

$2x^2 + 4x + 4 = 130$

$2x^2 + 4x = 130 - 4 = 126$

Dividing all the terms by 2

$x^2 + 4x = 63$

Adding 1 to both sides

$x^2 + 2x + 1 = 63 + 1$

$(x + 1)^2 = 64$

$x + 1 = \pm\sqrt{64}$

$x + 1 = \pm 8$

$x + 1 = 8$, $x = 8 - 1 = 7$

$x + 1 = -8$, $x = -8 - 1 = -9$

-9 is not a natural number

$\therefore$ The odd numbrs are $= 7, 7 + 2$

ie, $7, 9$

6. Let the perpendicular sides $= x, 14 - x$

Area $= \frac{1}{2}x(14 - x)$

$\frac{1}{2}x(14 - x) = 24$

$x(14 - x) = 2 \times 24 = 48$

$14x - x^2 = 48$

$-x^2 + 14x = 48$

$x^2 - 14x = -48$

Adding 49 to both sides

$x^2 - 14x - 49 = -48 + 49$

$(x - 7)^2 = 1$

$x - 7 = \pm\sqrt{1}$

$x - 7 = 1$, $x - 7 = -1$

$x = 1 + 7$ $x = -1 + 7$

$= 8 \qquad = 6$

If $x = 8$, he perpendicular sides are 8 cm, 6 cm.

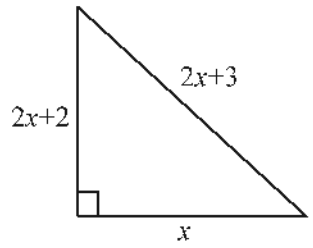
If $x = 6$, the perpendicular sides are 6 cm, 8 cm,

Hypotenuse = $\sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$ cm

7. $x^2 + (2x + 2)^2 = (2x + 3)^2$

$x^2 + (2x)^2 + 2 \times 2x \times 2 + 2^2 = (2x)^2 + 2 \times 2x \times 3 + 3^2$

$x^2 + 4x^2 + 8x + 4 = 4x^2 + 12x + 9$



$x^2 + 8x - 12x = 9 - 4$

$x^2 - 4x = 5$

Adding 4 to both sides.

$x^2 - 4x - 4 = 5 + 4$

$(x - 2)^2 = 9$

$x - 2 = \pm\sqrt{9}$

$= \pm 3$

$x - 2 = 3, \qquad x - 2 = -3$

$x = 3 + 2, \qquad x = -3 + 2$

$= 5 \qquad = -1$

Base $= 5$ cm

Height $= 2 \times 5 + 2 = 12$ cm

Hypotenuse $= 2 \times 5 + 3 = 13$ cm

8.. Let sum of n terms = 250

$$\frac{d}{2}n^2 + \left(f - \frac{d}{2}\right)n = 250$$

$2n^2 + 5n = 250$ (d = 4, f = 7)

$2n^2 + 5n - 250 = 0$

a = 2, b = 5, c = –250

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4\times2\times-250}}{2\times2}$$

$$= \frac{-5 \pm \sqrt{25 + 2000}}{2\times2}$$

$$= \frac{-5 \pm \sqrt{2025}}{4}$$

$$= \frac{-5 \pm 45}{4}$$

$$= \frac{-5+45}{4}, \frac{-5-45}{4}$$

$$= \frac{40}{4}, \frac{-50}{4}$$

$$= 10, \frac{-25}{2}$$

∴ Number of terms = 10

MORE PRACTICE PROBLEMS

1. The sum of the squares of two consecutive even numbers is 244. Which are the numbers?

2. Four times a number, subtracted from the square of the number gives 96. Which is the number?

3. The length of a rectangle is 6 cm longer than its breadth and its area is 40 cm. By taking the breadth as x, write the length in terms of x. Find the length and breadth of the rectangle by forming a second degree equation?

4. The perimeter of a rectangular sheet of paper is 42 cm and the length of its diagonal is 3 cm shorter than twice the breadth. By taking the breadth as x, write the length and diagonal in terms of x. Find the length and breadth of the rectangular sheet by forming a second degree equation?

5. A rectangular garden of sides 10m and 6m has a path all around inside it. The area of the garden excluding the path is $32m^2$. What is the width of the path?

6. a) What is the value of the polynomial $p(x) = 2x^2 + 3x$, when $x = 2$?

   b) For which number x, the value of $p(x) = 2x^2 + 3x$, becomes 2.

Unit 5

TRIGONOMETRY

Things to Remember

- For triangles with the same set of angles, the ratio of the lengths of the sides is the same.
- The angles of a triangle determines the ratio of the sides. The perpendicular sides of a right triangle with angles 45°, 45°, 90° are equal. To find the length of the hypotenuse, multiply the perpendicular side by $\sqrt{2}$.

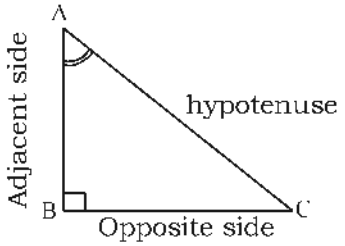
  (Ratio of the sides of this triangle is $1 : 1 : \sqrt{2}$)

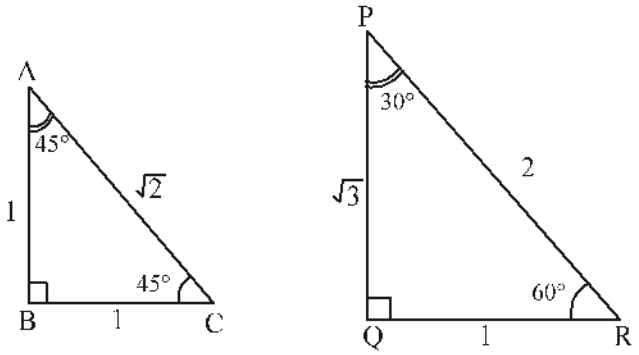
  In a right angled triangle with angles 30°, 60°, 90°, hypotenuse will be two times the length of the side opposite to 30° angle. Also the length of the side opposite to 60° angle will be $\sqrt{3}$ times the length of the side opposite to 30° angle.

  (Ratio of the sides of this triangle is $1 : \sqrt{3} : 2$)
- In all right triangles with the same angles, the number got by dividing the opposite side of an acute angle by the hypotenuse is the same. It is called the sine of the angle, written as 'sin'.
- The number got by dividing the adjacent side of an acute angle (Shorter of the two sides containing the angle) by the hypotenuse is also the same. It is called the cosine of the angle. It is shortened as 'cos'.
- The number got by dividing the opposite side of an angle by the adjacent side will be same number. It is called the tangent of the angle. It is shortened as 'tan'.

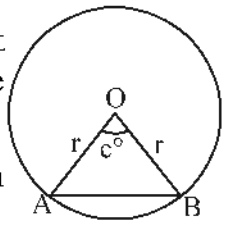
$$\sin A = \frac{BC}{AC} \qquad \cos A = \frac{AB}{AC}$$

$$\tan A = \frac{BC}{AB}$$





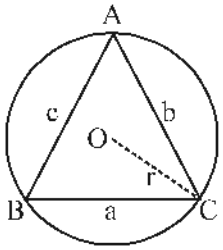
	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$



- In a circle, the length of a chord is double the product of the sine of half the central angle and the radius.

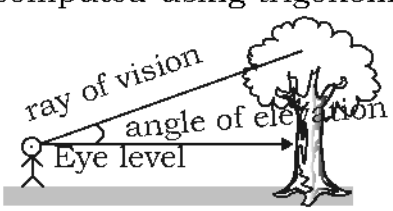
  In a circle of radius 'r', the length of the chord with central angle c is $2r \text{ Sin } \left(\frac{c}{2}\right)$

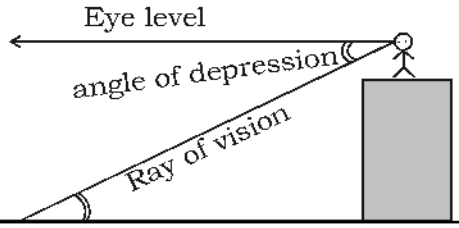
- In any triangle, the ratio of the sides is equal to the ratio of the sines of the angles opposite them.
- The length of the sides of a triangle are the sines of its angles opposite to that side, multiplied by the diameter of its circum-circle. If any angle is greater than the right angle the sine of its supplementary angle should be taken. If the angle is 90° the opposite side is equal to the circum diameter.
- To find the circum diameter, divide the length of one side of a triangle by its Sine of angle opposite to that side.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$

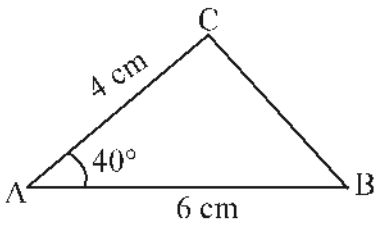
◆ Heights and distances which cannot be directly measured can be computed using trigonometric ratios.





Activity - 1

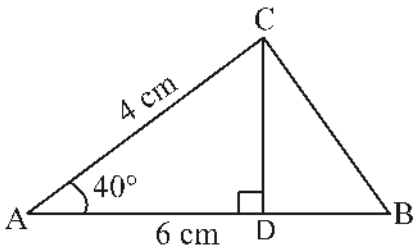
i. Find the area of the triangle given.



Draw CD perpendicular to AB.

Area $= \frac{1}{2} \times \square \times \square$

How to find the length of CD



In the triangle ADC $\angle CAD = \square$

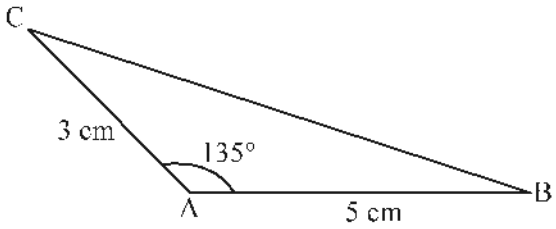
hypotenuse AC = $\square$ cm

In right. triangle ADC, Sin 40 = $\frac{CD}{\square}$

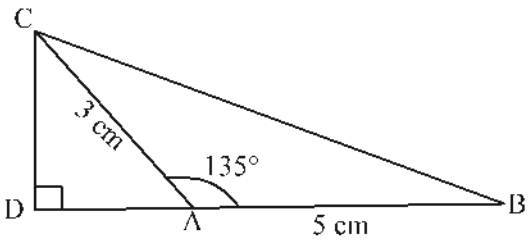
CD = $\square \times \square = \square$ cm

Area of Δ ABC = $\frac{1}{2} \times \square \times \square = \square$ cm^2

ii. Find the area of the triangle.



Extend the line AB backwards.



Draw CD Perpendicular to the extended line AB.

Area = $\frac{1}{2} \times \square \times \square$

How to find the length of CD?

In right angled triangle CDA, $\angle CAD = \square - \square = \square$

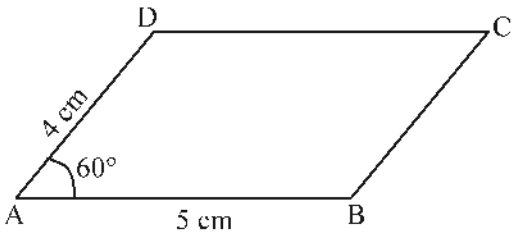
Hypotenuse AC = $\square$ cm.

In right angled triangle CDA, Sin 45 = $\frac{\square}{\square}$

$\therefore$ CD = $\square \times \square = \square$ cm.

Area of ΔABC = $\frac{1}{2} \times \square \times \square = \square$ cm^2.

Activity - 2

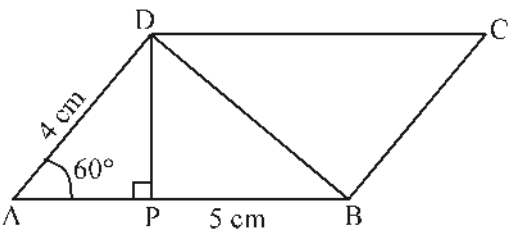


In the parallelogram ABCD, AB = 5 cm, AD = 4 cm, $\angle A = 60°$

(a) Find the perpendicular distance from D to AB?

(b) Find the area of ABCD?

(c) Find the length of the diagonal BD?



Angles of triangle APD are 30°, 60°, 90°

PD = $2\sqrt{3}$ cm

Area of ABCD = ☐ × ☐ = ☐ cm^2

AP = ☐

BP = ☐ – ☐ = ☐

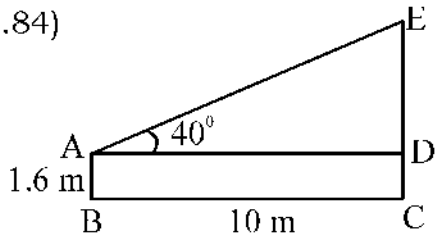
BD^2 = ☐^{2} + ☐^{2} = ☐

BD = ☐ cm

Activity - 3

i. A boy 1.6m tall observes the top of a tree which is 10m away from him, at an angle of elevation 40° . Find the height of the tree.

(sin 40 = 0.64, cos 40 = 0.77, tan 40 = 0.84)



Draw a rough figure.

AD = ☐ m

CD = ☐ m

In right angled triangle ADE , ∠DAE = ☐

tan 40 = ☐/☐

DE = ☐ × ☐ = ☐ m

Height of the tree, CE = ☐ + ☐ = ☐ m

ii. A man looks down from the top of a light house 35 meters high and sees a ship at an angle of depression 35°. How far is the ship from the foot of the light house? (tan 35 = 0.7)

Draw a rough figure.

∠ADC = ☐

∠CAD = ☐ –☐ = ☐

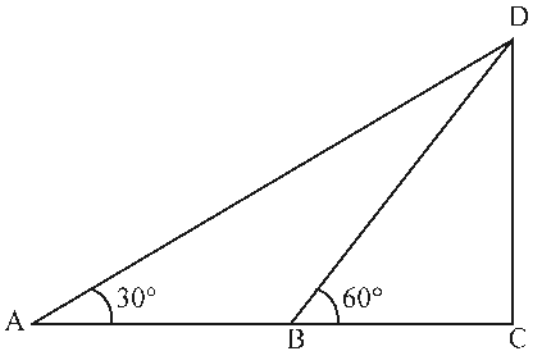
In right angled triangle ACD, tan 35 = ☐/☐

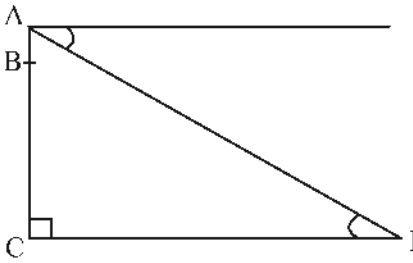
CD = ☐ ÷ ☐ = ☐

Distance from the foot of the light house to the ship = ☐ m

Activity - 4

A man saw the top of a tree at an elevation of 30°. Moving 10 meters towards the tree, he sees it at an elevation of 60°. Find the hight of the tree?





In ΔABC

$\angle ABD =$ ☐

$\angle ADB =$ ☐

Since AB = 10m, BD = ☐ m

In ΔBCD $\angle BDC =$ ☐

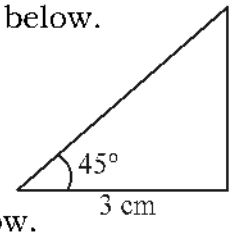
Since BD = 10m, BC = ☐ m

$\therefore$ CD = ☐ m

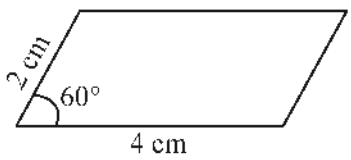
$\therefore$ Height of the tree = ☐ m

PRACTICE PROBLEMS

1. Compute the perimeter of the triangle given below.



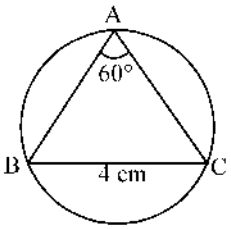
2. Find the area of the parallelogram given below.



3. The length of two sides of a triangle are 4cm and 6cm. The angle between these sides is 55°. Compute the area of the triangle.

   (sin 55 = 0.82, cos 55 = 0.57)

4. A triangle and its circumcircle is drawn in the figure. Find the diameter.



5. Hypotenuse of a right angled triangle is 8cm. One of its angle is 40°. Find the length of other two sides.

   (sin 40 = 0.64, cos 40 = 0.77)

6. A rod leans against a wall and it makes 40° angle with the ground.

   i. Draw a rough figure based on the question

   ii. Find the height of the wall

   iii. How far is the foot of the rod from the wall?

   (sin 40 = 0.64, cos 40 = 0.77, tan 40 = 0.84)

7. A man 1.6m tall observes the top of a building at an elevation of 35° from a point 12 metres away from the building. What is the height of the building?

   (sin 35 = 0.57, cos 35 = 0.82, tan 35° = 0.70)

8. A boy 1.5m tall looks down from the top of a light house 18.5m high and sees a ship at an angle of depression 40°. How far is the ship from the foot of the light house.

   (sin 50 = 0.76, cos 50 = 0.64, tan 50 = 1.19)

ANSWERS

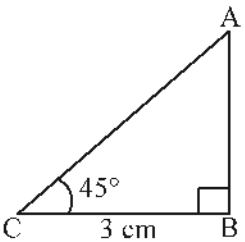
1. In right angled triangle ABC

   BC = 3 cm

   AB = 3 cm

   AC = $3\sqrt{2}$ cm

   Perimeter = $3 + 3 + 3\sqrt{2}$



$= 6 + 3\sqrt{2}$

$= 10.24$ cm

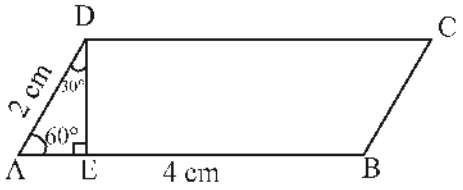
2. In right angled triangle, the angles are $30°, 60°, 90°$

Ratio of the sides $1 : \sqrt{3} : 2$

$DE = \sqrt{3}$ cm

Area $= AB \times DE = 4\sqrt{3}$

$= 6.93$ cm

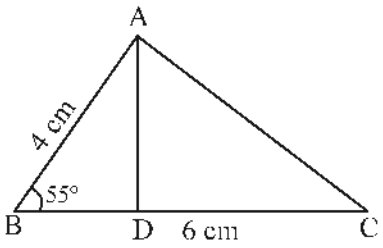


3. In right angled triangle ADB

$\sin 55 = \frac{AD}{AB}$

$AD = 4 \times \sin 55 = 3.28$ cm

Area $= \frac{1}{2} \times BC \times AD = 9.84$ cm^2.

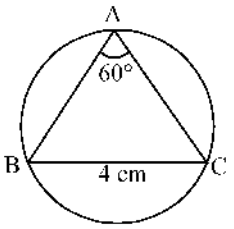


4. Let the circumradius be 'r'

$BC = 2r \sin 60$ (Length of the chord $= 2r \sin\left(\frac{C}{2}\right)$, central angle $= c°$)

$$d = \frac{BC}{\sin 60°} = \frac{4}{(\sqrt{3}/2)} = 4 \times \frac{2}{\sqrt{3}}$$

$$= \frac{8}{\sqrt{3}} \text{ cm}$$

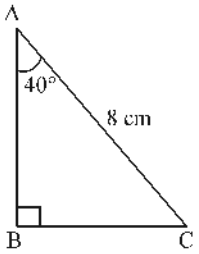


5. In right angled triangle ABC

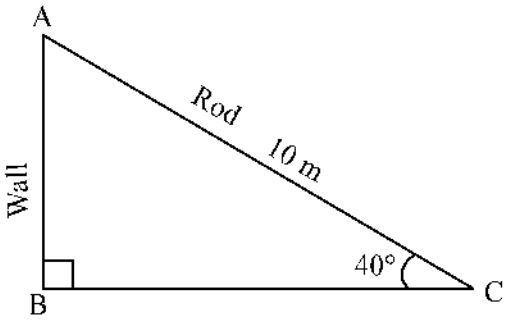
$\sin 40 = \frac{BC}{AC}$

$BC = 8 \times \sin 40 = 5.12$ cm

$AB = 8 \times \cos 40 = 6.16$ cm.



6.



Height of the wall, AB = 10 × sin40° = 10 × 0.64 = 6.4m

Distance between the wall and the foot of the road, BC = 10 × cos 40°

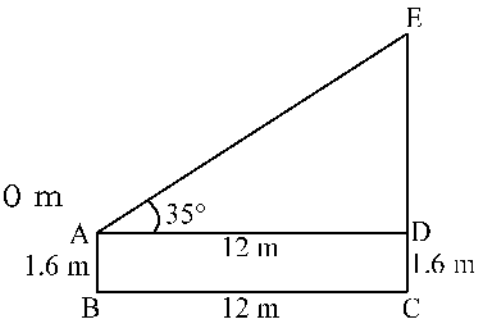
= 10 × 0.77 = 7.7 m

7. In right angled triangle ADE

$$\tan 35 = \frac{DE}{AD}$$

DE = 12 × tan 35 = 8.40 m

Height of the building = 8.4 + 1.6 = 10 m



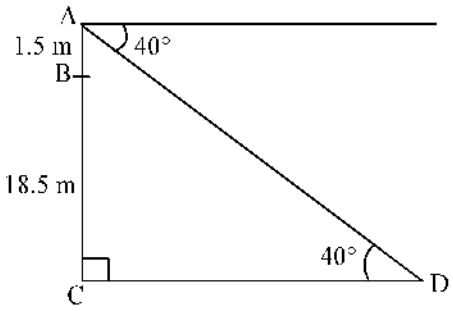
8. In right angled triangle ACD

AC = 18.5 + 1.5 = 20 m

$\angle CAD = 50°$

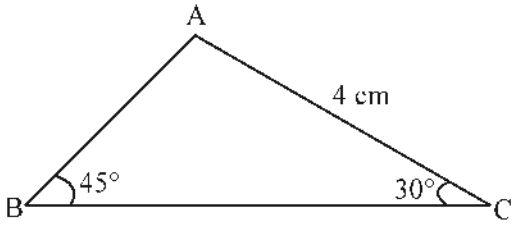
$$\tan 50 = \frac{CD}{AC}$$

CD = 20 × tan 50 = 25.8 m



MORE PRACTICE PROBLEMS

1. In right angled triangle ABC, $\angle B = 45°$, $\angle C = 30°$. AC = 4 cm. Find the perimeter.



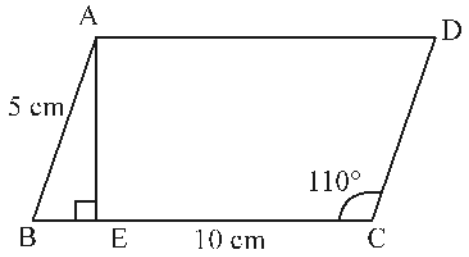
2. In the figure, ABCD is a parallelogram. AB = 5cm, BC = 10cm, $\angle C = 110°$. AE is drawn perpendicular to BC.Compute

   i. Measure of $\angle B$

   ii. Length of AE

   iii. Area of parallelogram ABCD

   (sin 70 = 0.94, cos 70 = 0.34, tan 70 = 2.75)



3. Sides of a rhombus are 4cm each and one of its angles is 70° . Find the area?

   (sin 70 = 0.94, cos 70 = 0.34, tan 70 = 2.75)

4. In the figure triangle ABC and its circumcircle is drawn.

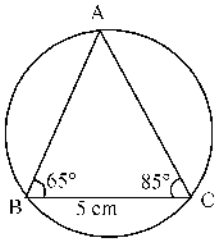
   If BC = 5cm, $\angle B = 65°$, $\angle C = 85°$. Find

   i. Measure of $\angle A$?

   ii. Circum diameter

   iii. Length of the other two sides of the triangle.

   (sin 65 = 0.91, sin75 = 0.99)



5. A boy 1.5m tall sees the top of a tree at an angle of elevation 40°. When he walks 10m towards the tree, sees the top of the tree at an angle of elevation 80°. Find the height of the tree.

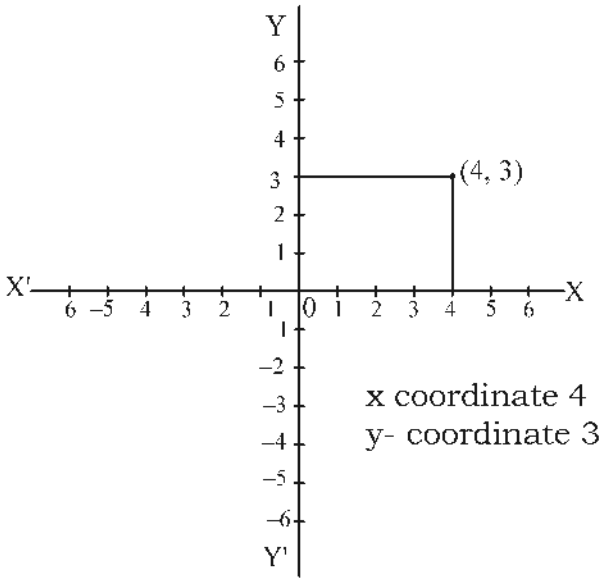
   (tan40 = 0.84, tan 80 = 5.67).

Unit 6

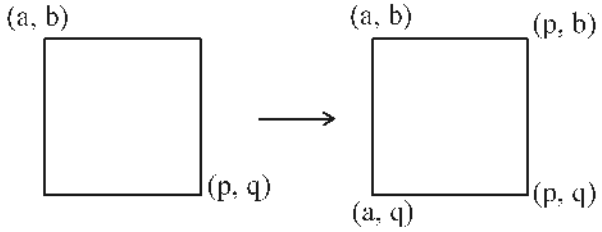
COORDINATES

Points to Remember

- Two measures are needed to locate a point in a plane. To locate a point in a plane, imagine two mutually perpendicular lines called axes of coordinates. The horizontal line is called the x-axis and vertical line is called the y axis
- The distance of a point from the x-axis is called its y coordinate and the distance from y axis is called its x co-ordinate

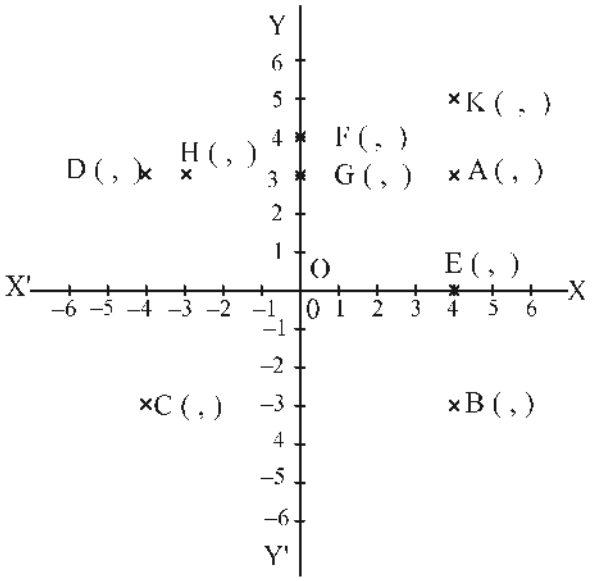


- Points on the x axis have their y coordinate 0
  points on the y axis have their x coordinate 0
- y coordinates of all points on a line parallel to x axis are equal
- x coordinates of all points on a line parallel to y axis are equal
- If the sides of a rectangle are parallel to the axes, from the coordinates of one pair of opposite, vertices the coordinates of other pair of opposite vertices can be found.



- (x_1, y_1), (x_2, y_1) are two points on a line parallel to x axis. The distances between them is $|x_1 - x_2|$
- (x_1, y_1), (x_1, y_2) are two points on a line parallel to y axis. The distance between them is $|y_1 - y_2|$
- The distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- The distance of the point (x, y) from origin is $\sqrt{x^2 + y^2}$

Activity- 1



a. Write the coordinates of A,B,C,D

b. Join A,B,C&D in the same order. what figure you will get

c. Write the coordinates of E,O & F

d. What type of triangle you will get by joining E,O & F

e. What is special about the line through A,B,E & K

f. Write the coordinates of G,H

g. What is special about the line through G,H,A & D

Activity- 2

Consider a circle with centre at origin and radius 5 units

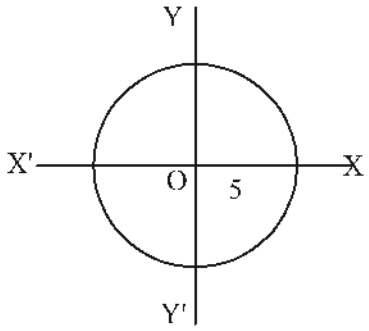
a. Write the coordinates of the points of intersection of the circle and the axes

b. Check whether the points (3,5), (4,3), (-2,3) are inside the circle ,on the circle or outside the circle

c. Write the coordinates of ten points on this circle

a. Point of intersection of x axis ad the circle is

(☐, ☐), (☐, ☐)



points of intersection of y axis and the circle is

(☐, ☐), (☐, ☐)

b. Distance between (0, 0) and (3, 5) $= \sqrt{3^2+5^2} = \sqrt{34}$

$\sqrt{34}$ is greater than the radius 5 so (3,5) is outside the circle

Distance between (0, 0), (4, 3) = ☐

∴ (4, 3) is ☐ the circle.

Distance between (0,0) & (-2,3) ☐

∴ (–2, 3) is ☐ circle

c. (4, 3), (–4, 3), (3, 4), (5, 0) are on the circle. Write six more points such that sum of the squares of the coordinates is 25

ie = (□, □), (□, □), (□, □), (□, □), (□, □), (□, □)

Activity- 3

a. Draw the co-ordinate axes and mark the points A(2, 3), B(7, 5), C(9, 8), and D(4, 6)

b. Find the lengths of AB, BC, CD, AD

Length of AB = distance between A &B

$AB = \sqrt{(\square - \square)^2 + (\square - \square)^2} = \square$

Similarly BC = □, CD = □, AD = □

c. Find the length of AC, BD

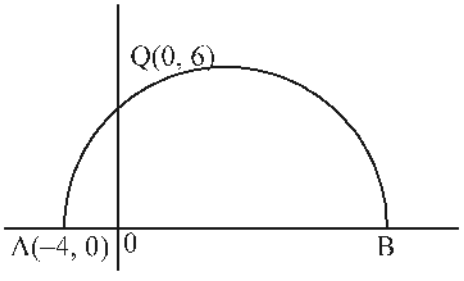
AC = □, BD = □

d. What is the appropriate name of the quadrilateral obtained by joining A, B, C and D - □

e. Find the perimeter of quadrilateral ABCD □

PRACTICE PROBLEMS

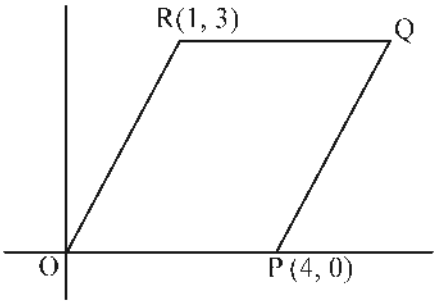
1. Coordinates of some points are given
(4, 0), (0, –4), (–4, 0), (0, 4)
   a. Which of the above points are on the x axis
   b. Which of the above points are on the y axis
   c. Mark the points by drawing the coordinate axes
   d. Name the figure obtained by joining these points
2. In figure, the semicircle with diameter AB=13 cm passes the point Q.



a. If A (–4, 0), Q (0, 6), Find the lengths of OA, OQ, OB

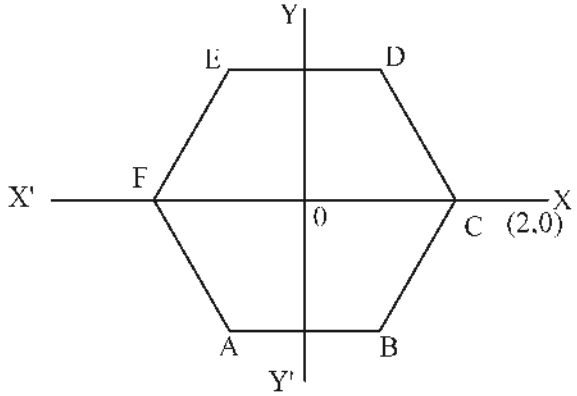
b. Write the coordinates of B

3. In the figure, OPQR is a parallelogram. Find the coordinates of Q



4. Sides of a rectangle are parallel to the axes. If two vertices are (3,2) & (7,-4), Find the coordinates of the other two verticles

5. ABCDEF is a regular hexagon. If the coordinates of C are (2,0) find the coordinates of other vertices

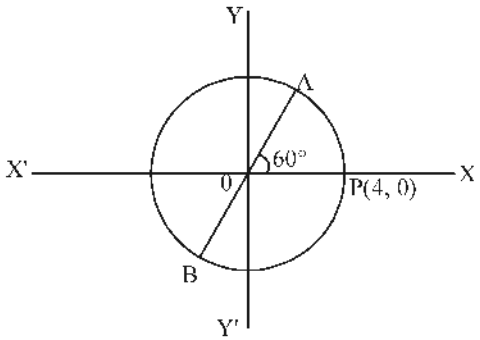


6. What is the speciality of the triangle obtained by joining (2, 1), (3, 4), (–3, 6)

7. If the figure, centre of the circle is at origin

a. Find the coordinates of A & B

b. Find the length of AB



ANSWERS

1. a. Points on the x axis are (–4, 0), (4, 0)

   b. Points on the y axis are (0, 4) (0, –4)

   c. Figure

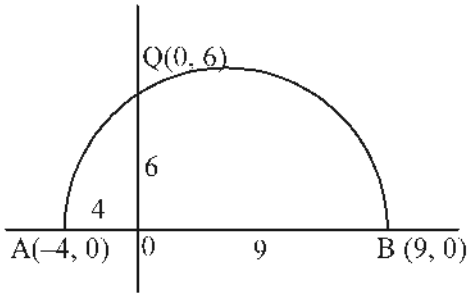
   d. Square

2. OA = 4 unit

   OQ = 6 unit

   OB = 13 – 4 = 9 unit

   Coordinates of B (9, 0)



3. OM = 1 unit

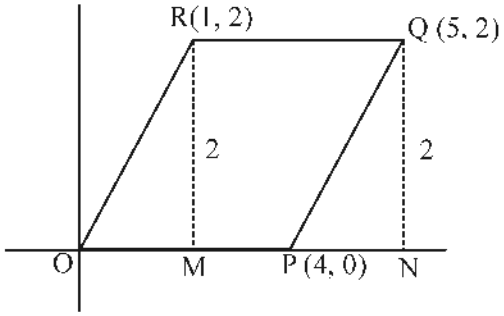
   MR = 2 units

   OP = 4 units

   QN = 2 units

   ON = OP + PN = 4 + 1 = 5 units

   Coordinates of Q are (5, 2)



4. OP = 3 units

   PA = 2 units

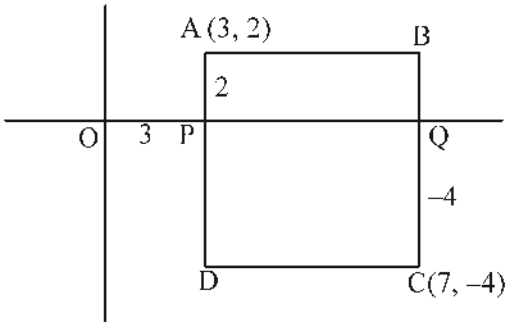
   QB = PA = 2 units

   OQ = 7 units

   QC = PD = 4 units

   Coordinates of B (7,2)

   Coordinates of D (3,-4)



5. ΔDOC is an equilateral triangle

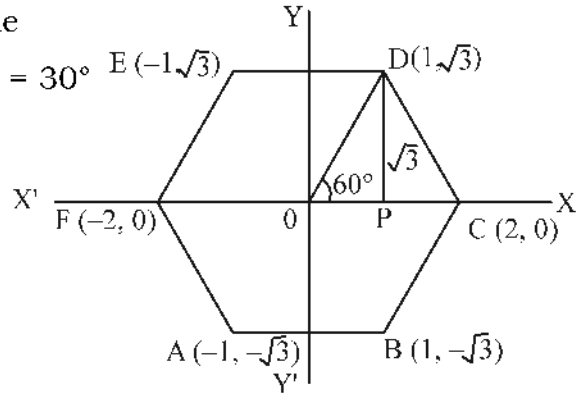
   $\angle$DOC = 60°, $\angle$P = 90°, $\angle$ODP = 30°

   OD = 2 units

   OP = 1 unit (1: $\sqrt{3}$: 2)

   PD = $\sqrt{3}$ units

   Coordinates of D (1, $\sqrt{3}$)



Coordinates of E $(-1, \sqrt{3})$

Coordinates of F $(-2, 0)$

Coordinates of A $(-1, -\sqrt{3})$

Coordinates of B $(+1, -\sqrt{3})$

6. A (2, 1), B (3, 4), C(–3, 6)

$AB = \sqrt{(3-2)^2 + (4-1)^2}$

$= \sqrt{1+9} = \sqrt{10}$

$AC = \sqrt{(-3-2)^2 + (6-1)^2}$

$= \sqrt{25+25}$

$= \sqrt{50}$

$BC = \sqrt{(-3-3)^2 + (6-4)^2}$

$= \sqrt{36+4} = \sqrt{40}$

$AB^2 + BC^2 = 10 + 40 = 50 = AC^2$

$\therefore$ ΔABC is a right angled triangle with $\angle B = 90°$

7. a. OP = 4 units

OA = 4 units

In right angled triangle ANO

$\angle A = 30°$

$\angle AON = 60°$ ON : AN : OA = $1 : \sqrt{3} : 2$

$\angle N = 90°$ OA = 4 units

ON = 2 units

AN = $2\sqrt{3}$ units

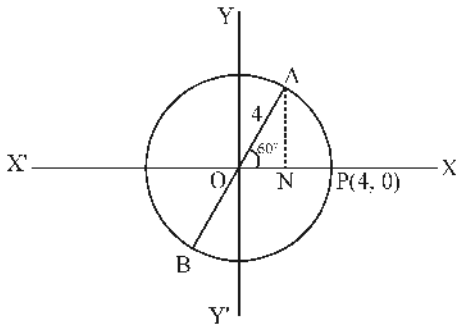
Coordinates of A $(2, 2\sqrt{3})$

Coordinate of B $(-2, -2\sqrt{3})$

b. AB = OA + OB

= 4 + 4

= 8 units



MORE PRACTICE QUESTIONS

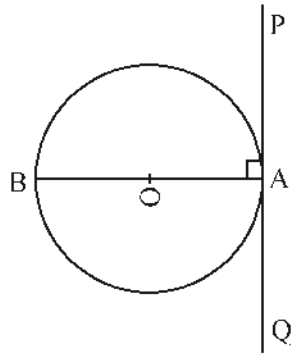
1. Among the points (5, 3), (–4, 6), (–1, 3), (–4, 1), (1, 5), (3, 6)
   a. Write two points on a line parallel to x axis
   b. Write two points on a line parallel to y axis
2. In ΔABC , A(2, 0), B(8, 0). AC = 5 units If the area of ΔABC is 12cm^2. Draw a rough figure of ΔABC and find the coordinates of C.
3. A circle with centre (2, 3) and radius 5 unit crosses the x axis at A & B.
   a. Find the coordinates of A, B
   b. Find the length of the chord AB
4. Which type of triangle is obtained by joining the points (8, 2), (5, –3) & (0, 0)

Unit 7

TANGENTS

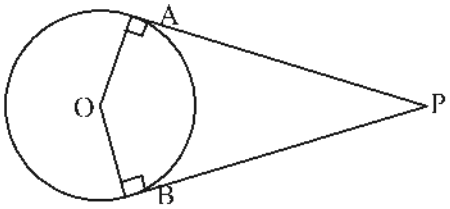
Points to Remember

- A tangent at a point to a circle is perpendicular to the diameter through that point



$AB \perp PQ$

- A quadrilateral formed by joining the centre of the cirlce, two points on that cirlce, and the point of intersection of two tangents at these points is cyclic.

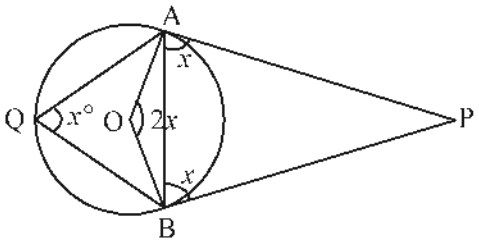


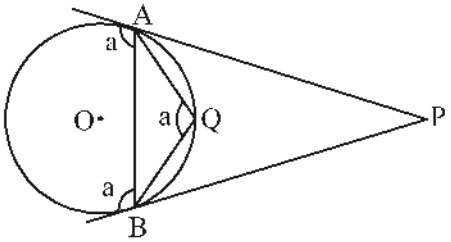
The quadrilateral PAOB is a cyclic quadrilateral

$\angle AOB = 180 - \angle P$

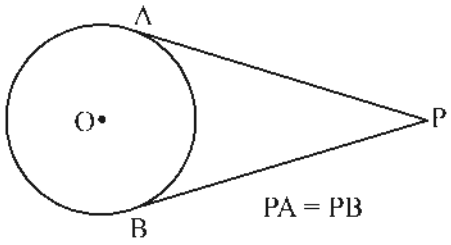
- An angle formed by two radii of a circle through two points and the angle formed by the two tangents through these points are supplementary.
- The angle made by a chord and the tangent at its end point is half the the central angle of the chord. Besides the angle made by a chord and a tangent at its one end point is same as the angle

made in the alternate segment.



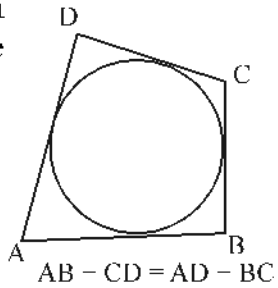


- Two tangents can be drawn from an exterior point of a circle. These two tangents will be equal in length.

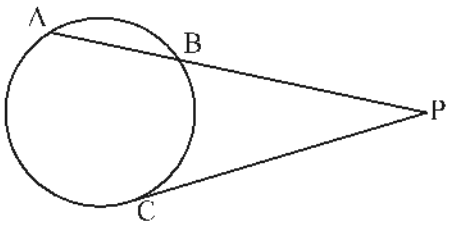


- The sum of the opposite sides of a quadrilateral formed by the tangents at four points of a circle are equal.

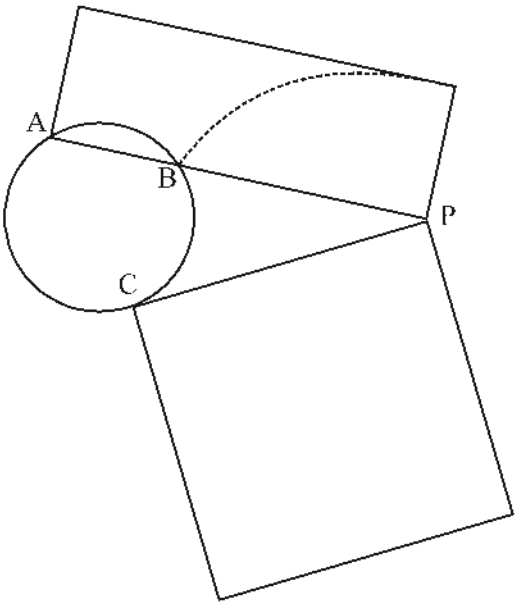
  Conversely, if the sum of the opposite sides of a quadrilateral are equal, then we can draw a circle with the four sides as tangents to the circle.



- In the figure, the tangent at C and the chord AB extended meet at P Then $PA \times PB = PC^2$

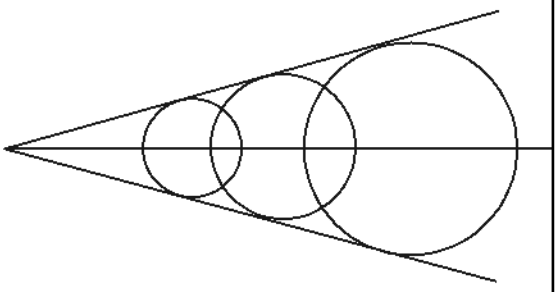


The area of the rectangle formed by PA and PB as sides is equal to the area of a square formed by PC as side.

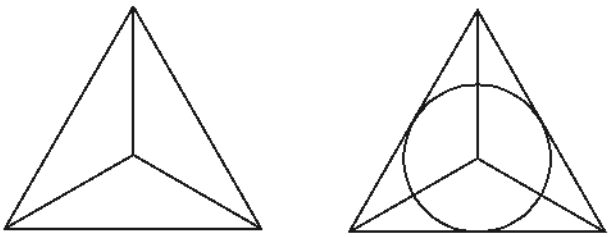


Area of the rectangle is equal to the area of the square

- The centre of a circle which touches two intersecting lines is in the bisector of the angle formed by these lines.



- In any triangle, the angle bisectors meet at a point. We can draw a circle with this point as centre and touching the three sides of the triangle. This circle is called the incircle of the triangle. The perpendicular distance from this centre to a side is the radius of the in circle.



- The radius of the in circle of a triangle is the quotient obtained when the area of the triangle is divided by its semi perimeter.

$$r = \frac{A}{s}$$

A - Area of the triangle

s- Semi perimeter of the triangle

- The length of the tangents from each vertex of a triangle to its incircle is equal to the difference of the semiperimeter of the triangle and the opposite side of that vertex.

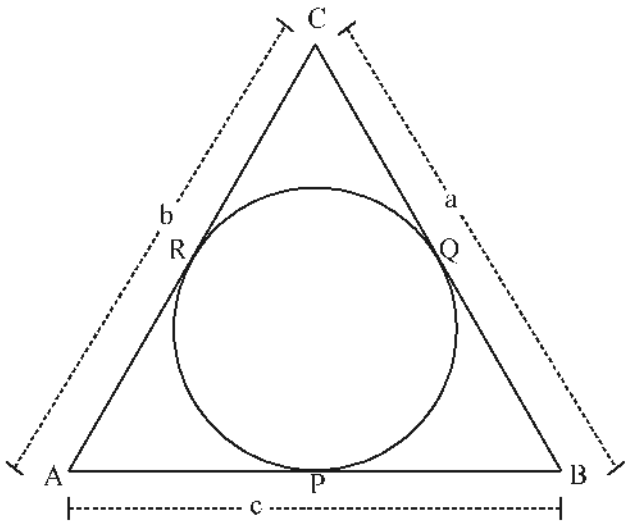
AP = AR = s – a

BP = BQ = s – b

CQ = CR = s – c

a, b, c are the sides of the triangle.

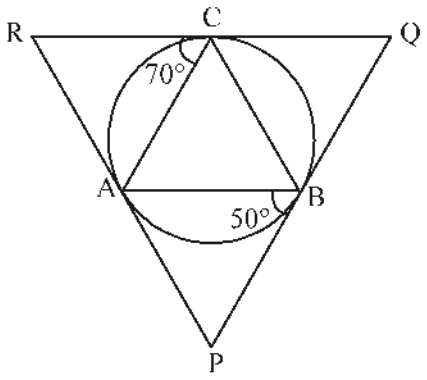
$s = \frac{a+b+c}{2}$ the semi perimeter of the triangle



Activity –1

In the figure, the circumcircle of triangle ABC is drawn. PQR is a triangle formed by the tangents at A,B and C. Find each angle of ΔABC and ΔPQR

ΔACR, ΔABP and ΔBCQ are isosceles triangles.



$\angle BAP =$ ☐

$\angle P = 180 - (☐ + ☐) =$ ☐

$\angle CAR =$ ☐

$\angle R = 180 - (☐ + ☐) =$ ☐

$\angle Q = 180 - (\angle P + \angle R) = 180 - (☐ + ☐)$

$=$ ☐

Since the angle made by a chord and a tangent at its one end point is same as the angle made in the alternate segment,

$\angle C =$ ☐, $\angle B =$ ☐

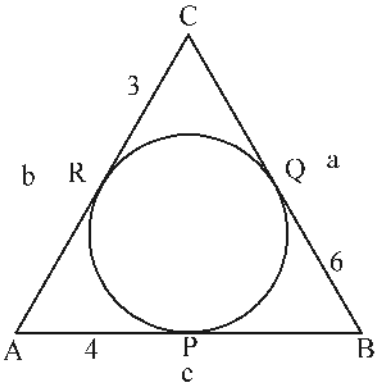
$\angle A = 180 - (☐ + ☐)$

$=$ ☐

Activity –2

In the figure AB, BC and AC are the tangents at P, Q and R. Given AP = 4 cm, BQ = 6 cm, CR = 3 cm. Find the perimeter of the triangle ABC.

Since the two tangents drawn from an exterior point to a circle are equal,



AP = AR, BP = BQ, CR = CQ

AB = AP + ☐ = 4 + ☐ = ☐ = c

BC = BC + ☐ = 6 + ☐ = ☐ = a

AC = CR + ☐ = 3 + ☐ = ☐ = b

Perimeter = ☐ + ☐ + ☐ = ☐

Activity – 3

In the figure, the tangent at C and the chord AB extended meet at P. AB = 5cm, PC = 6 cm, Find PB

$PA \times PB = PC^2$

If PB = x,

PA = 5 + ☐

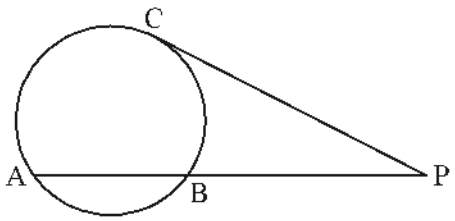
x(5 + ☐) = ☐^{2}

x (5 + ☐) = ☐

$5x$ + ☐^{2} = ☐

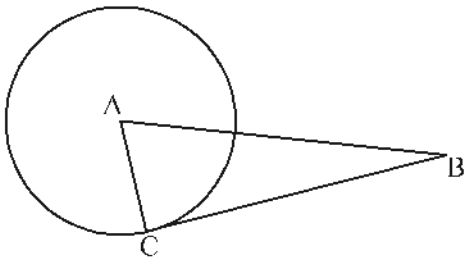
☐^{2} + $5x$ – ☐ = 0

x = ☐, PB = ☐

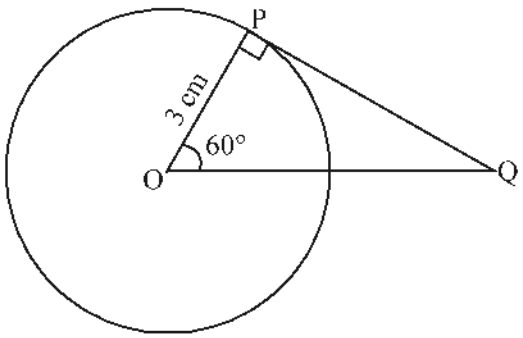


PRACTICE PROBLEMS

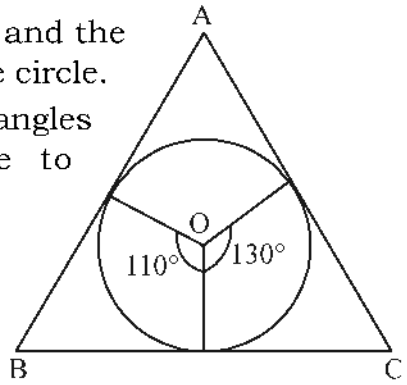
1. In the figure, BC is a tangent to the circle. The radius of the circle is 5cm and AB = 13cm. Find the length of the tangent.



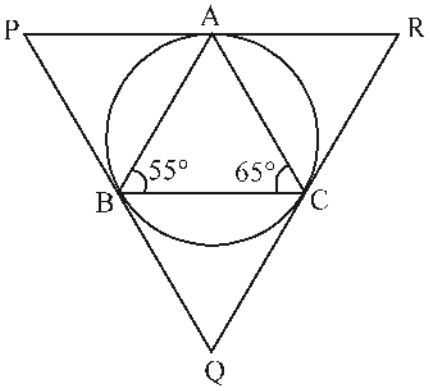
2. In the figure, 'O' is the centre of the circle. Find the length of the tangent PQ.



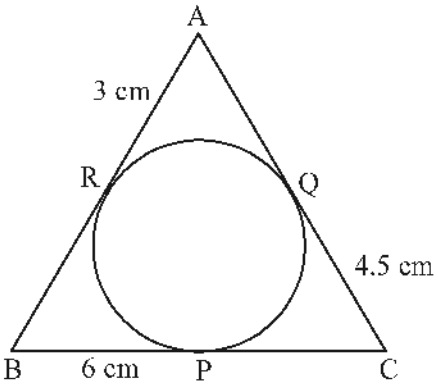
3. In the figure, 'O' is the centre of the circle and the sides of the triangle are the tangents to the circle. Find each angle of ΔABC. What will be the angles marked at the centre of the circle to become the triangle as an equilateral one.



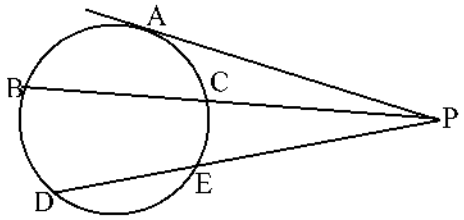
4. In the figure, the circumcircle of the triangle ABC is the incircle of ΔPQR. In ΔABC, $\angle B = 55°$ and $\angle C = 65°$. Find the angles of ΔPQR.



5. In the figure, ABC is a triangle formed by the tangets to the circle at P,Q and R. BP = 6 cm, CQ = 4.5cm, AR = 3 cm. Find the perimeter of ΔABC.



6. In the picture, PA is a tangent of the circle. The chords BC and DE intersect at P. DE = 5cm, PE = 4cm



a) What is the length of PD?

b) Find the length of PA?

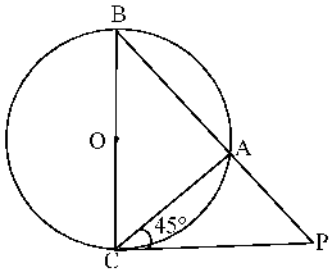
c) What is PC × PB?

7. In the picture, BC is a diameter of the circle with centre O. PC is a tangent. AC = 4cm, $\angle ACP = 45^{\circ}$

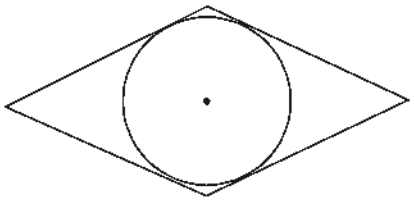
a) What is the measure of $\angle B$?

b) Write the measure of $\angle P$

c) Find PC, BC and PB?



8. In the figure, the sides of the rhombus are the tangents to the circle. Draw a picture like this with diameter of the circle is 5cm and one angle of the rhombus 50^{0}



9. Draw a circle with radius 2cm. Draw a triangle with two angles 50° and 70° and the sides of the triangle touching the circle.

10. Draw a circle with radius 3cm. Mark a point 7cm away from the centre of the circle. Draw two tangents from this point to the circle and measure its lengths.

11. Draw a rectangle of side 7 cm and area equal to that of a square of side 4 cm.

12. Draw a triangle of sides 7cm, 6cm and 5cm . Draw its incircle and measure its radius.

ANSWERS

1. ΔABC is a right angled triangle.

   Hypotenus = 13 cm

   length of one perpendicular side = 5 cm

   length of the tangent $= \sqrt{13^2 - 5^2} = 12$cm

2. $\angle O = 60°$

   The tangent to a circle and the radius containing the point of tangency are perpendicular

   $\therefore \angle P = 90°$.

   So the angles of ΔPOQ are 30°, 60°, 90°

   Side opposite to angle 60^0 is $\sqrt{3}$ times the side opposite to angle 30^0.

   $\therefore$ PQ = $3\sqrt{3}$ cm.

3. An angle formed by two radii of a circle through two points and the angle formed by the two tangents through these points are supplementary.

   $\therefore \angle B = 180° - 110° = 70°$

   $\angle C = 180° - 130° = 50°$

   The third angle of ΔABC, $\angle A = 180° - (70° + 50°)$

   $= 180° - 120°$

   $= 60°$

   To become the triangle as an equilateral one, each angle must be 60^0. For that the angle made at the centre of the circle will be the supplementary angle 120^0.

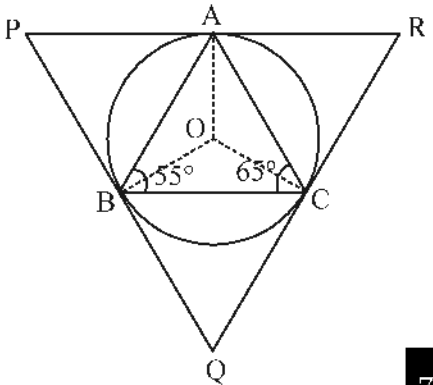
4. $\angle AOC = 2 \times 55° = 110°$

   $\angle R = 180° - 110° = 70°$

   $\angle AOB = 2 \times 65° = 130°$

   $\angle P = 180° - 130° = 50°$

   $\angle Q = 180° - (70° + 50°) = 60°$



5. Since the two tangents from a point to the circle are equal in length,

   BR = 6cm

   CP = 4.5 cm

   AQ = 3 cm

   Perimeter of the triangle = 2(6 + 4.5 + 3)

   = 2 × 13.5

   = 27 cm

6. (a) PD = 4 + 5 = 9cm

   (b) PA^2 = PE ×PD

   = 4 ×9 = 36

   PA = $\sqrt{36}$ = 6cm

   (c) PC × PB = PA^2 = 36

7. a) $\angle B = ACP = 45^0$

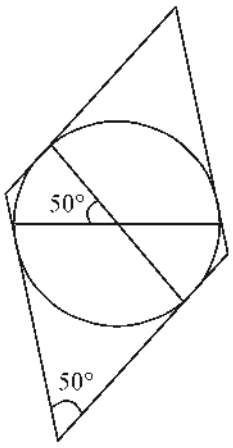
   b) $\angle P = 90 - 45 = 450$

   c) PC = $4\sqrt{2}$ cm

   BC = $4\sqrt{2}$ cm

   PB = $4\sqrt{2} \times \sqrt{2}$ = 8cm

8. The quadrilateral formed by the tangents at the ends of two diameters of this circle is a rhombus. The angles of this rhombus are the angles between the diameters itself.



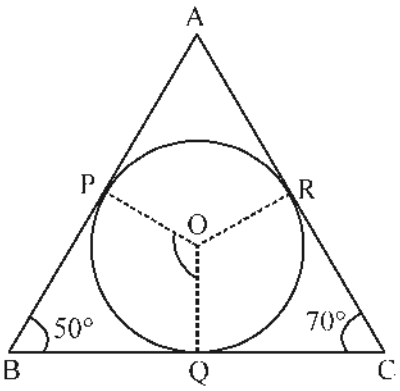
9. Let $\angle B = 50°$, then

$\angle POQ = 180° - 50° = 130°$ Similarly if

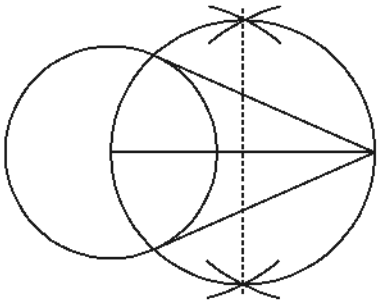
$\angle C = 70°$, then

$\angle QOR = 180° - 70° = 110°$

First of all draw a circle with radius 2cm. Then mark three points on the circle by making the angles 110° and 130° at the centre of the circle. Drawing the tangents at these points will make the required triangle.

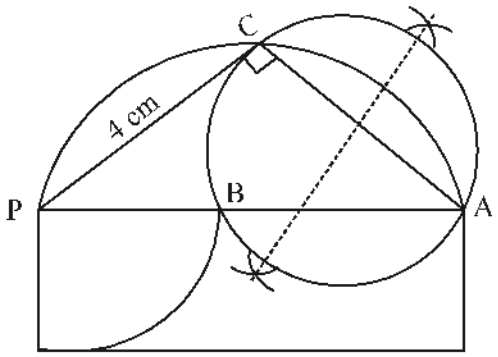


10.

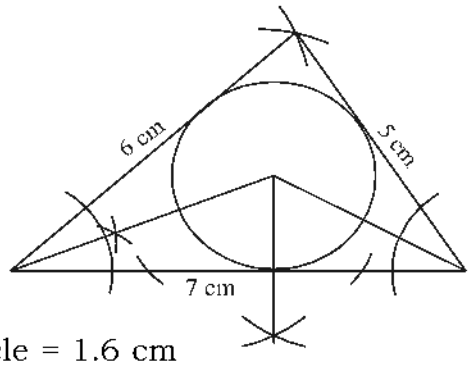


The length of the tangent = 6.5 cm

11. Draw a right angled triangle with hypotenuse 7 cm and 4 cm. Then draw a circle with the next side of the traingle as the diameter of the cirlce. Now area a of the rectangle with sides PA and PB is equal to the area of the square of side PC.



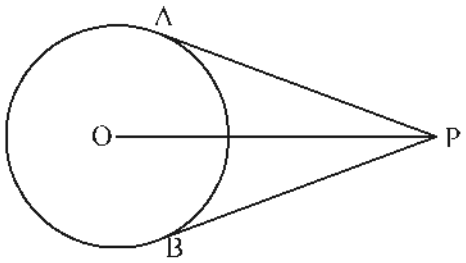
12.



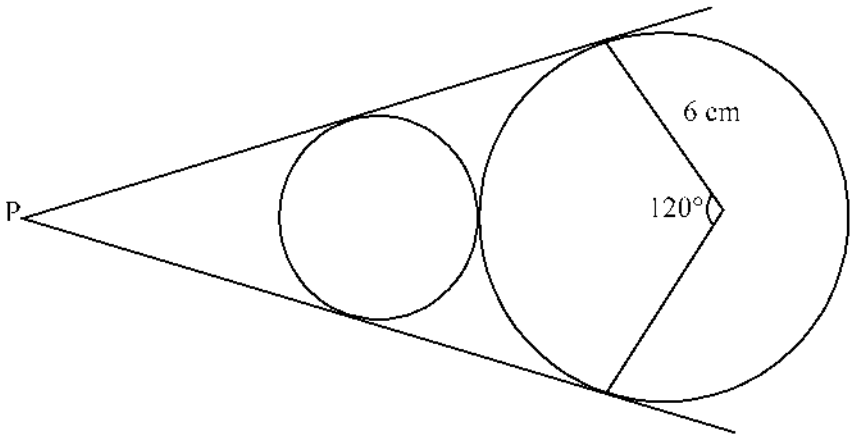
Radius of the incircle = 1.6 cm

MORE PRACTICE QUESTIONS

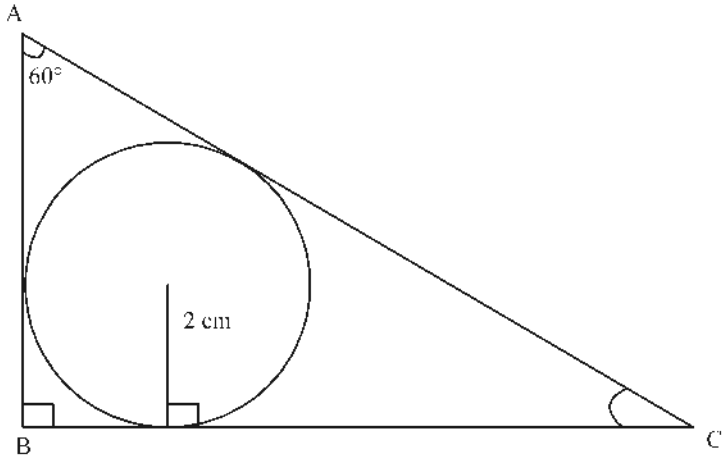
1. In the figure, PA and PB are the tangents from P to the circle with centre 'O'. If the lenght of OP is same as the diameter of the circle, find ∠APB.



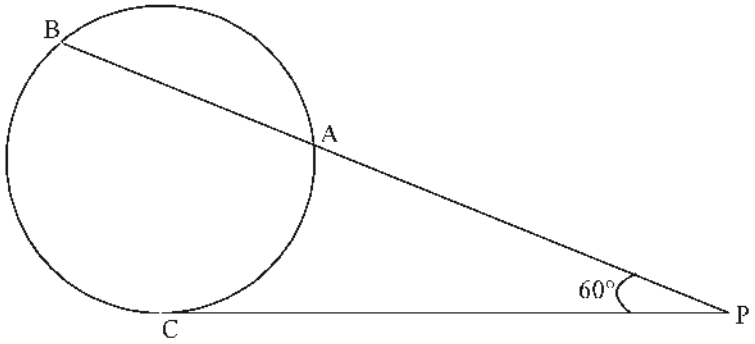
2. Two common tangents are drawn to the circles. Find the radius of the small circle.



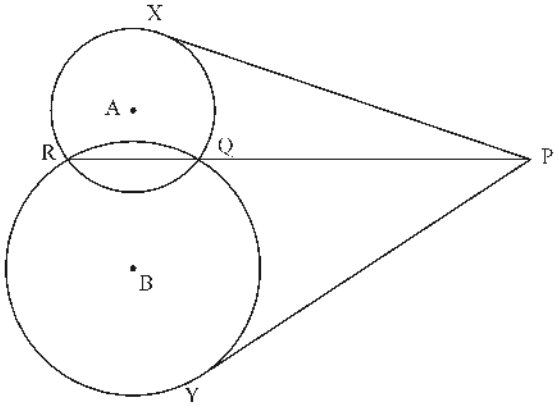
3. In the figure, the radius of the circle is 2cm, $\angle B = 90°$, $\angle A = 60°$. Find the sides of triangle ABC.



4. In the figure, PC is a tangent to the circle. If PA = 16cm and AB = 9 cm, find PC. Calculate the perpandicular distance from C to PB. Also find the area of the ΔPCB.



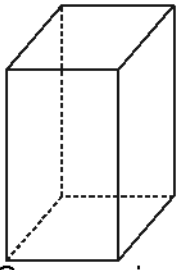
5. In the figure, the circles centered at A and B intersect at Q and R. PX is a tangent to the circle centered at A and PY is a tangent to the circle centered at B. Prove that PX = PY.



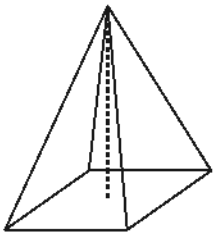
Unit 8

SOLIDS

Points to Remember

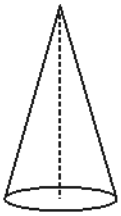


Square prism

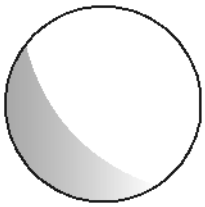


Square pyramid

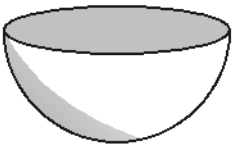
Cylinder



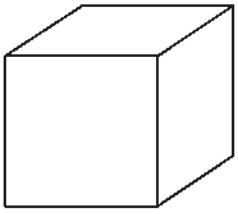
Cone



Sphere

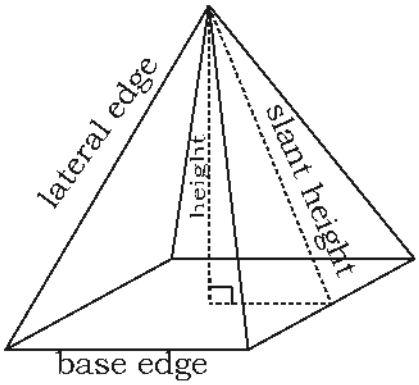


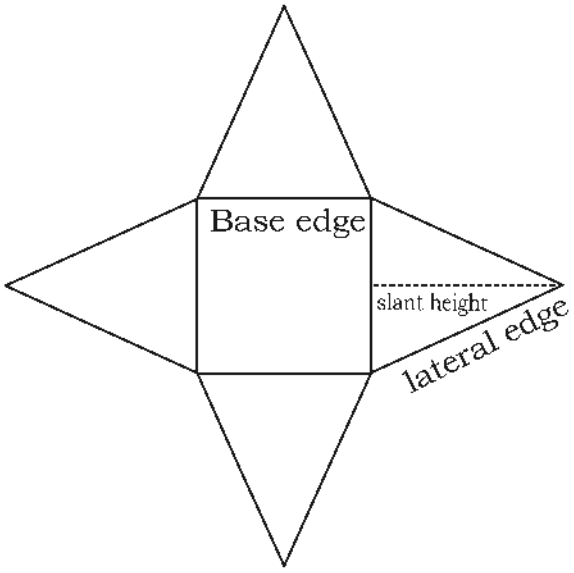
Hemisphere



Cube

Square Pyramid





Total surface Area = Base area + Lateral Surface Area

= Area of square + Area of four triangle

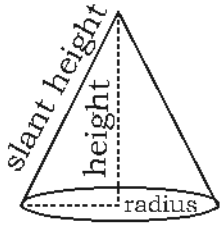
= $(\text{Base edge})^2 + 4 \times \frac{1}{2} \times$ base edge × slant height

Total Surface Area = $(\text{Base edge})^2 + 2 \times$ base edge × slant height

Volume = $\frac{1}{3}$ × base area × height

= $\frac{1}{3} \times (\text{base edge})^2 \times$ height

Cone



Total surface Area of cone

= Base Area + Curved Surface Area

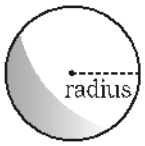
= π × (radius)^2 + π × radius × slant height

Volume of Cone

= $\frac{1}{3}$ × base area × height

= $\frac{1}{3}$ × π (radius)^2 × height

Sphere

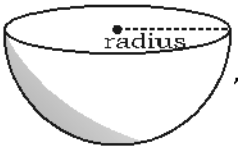


Surface Area of sphere

= 4 π × (radius)^2

Volume = $\frac{4}{3}\pi$ × (radius)^3

Hemi Sphere

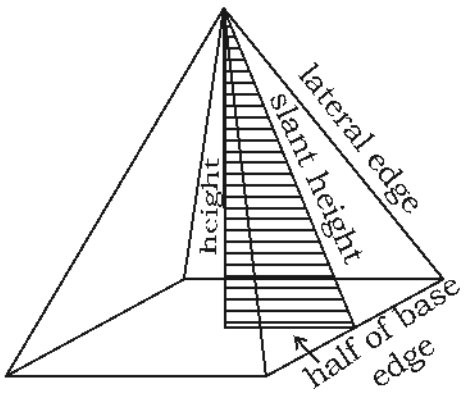


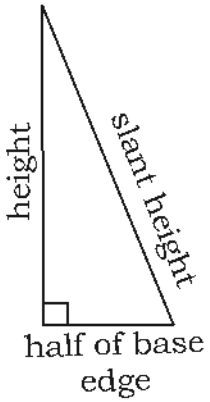
Total surface area of hemisphere

= 3 π × (radius)^2

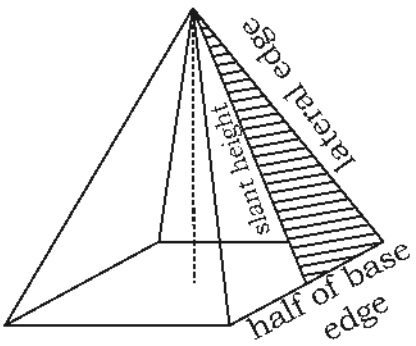
Volume = $\frac{2}{3}\pi$ × (radius)^3

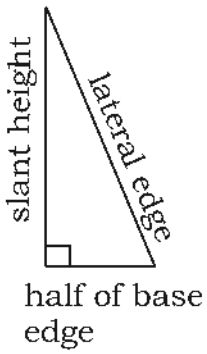
Right triangles in Square pyramid





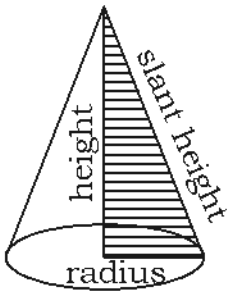
(slant height)^2 = (height)^2 + (half of base edge)^2

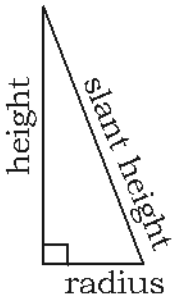




$(\text{lateral edge})^2 = (\text{slant height})^2 + (\text{half of base edge})^2$

Right triangles in Cone





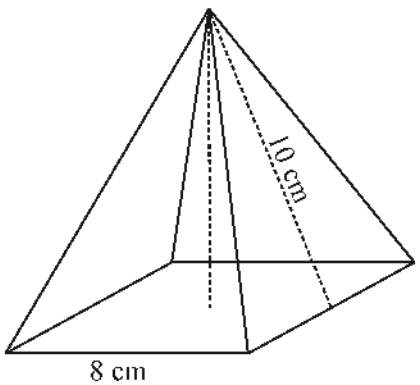
$(\text{Slant height})^2 = (\text{height})^2 + (\text{radius})^2$

Activity –1

Find the total surface area of the given square pyramid

length of base edge = ☐

Slant height = ☐



Total surface area = Base area + lateral surface area

Base area = Area of square

= $(\square)^2$

= $\square$

Lateral surface area = Area of four triangles

= $4 \times \frac{1}{2} \times \square \times \square$

= $\square$

Total aurface area

= Base area + lateral surface area

= $\square$ + $\square$

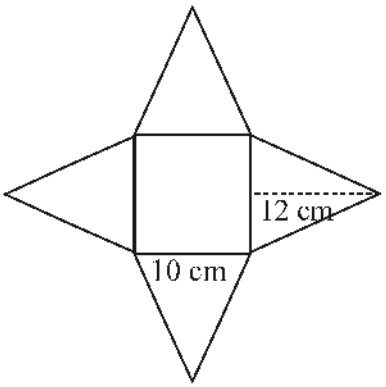
= $\square$ cm^2.

Activity –2

Find the total surface area of the square pyramid made by folding the figure below

length of base edge = $\square$

slant height = . $\square$



Base area = Area of square

= $\square \times \square$

= $\square$

Lateral surface area

= 4 × Area of one triangle

= $4 \times \frac{1}{2} \times \square \times \square$

= $4 \times \square$

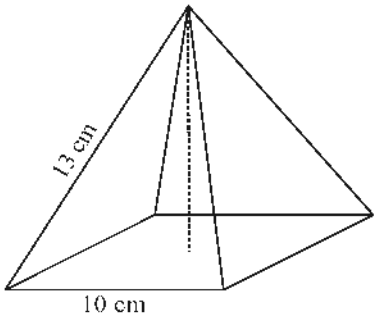
= $\square$

Total SurfaceArea = ☐ + ☐

= ☐ cm^2

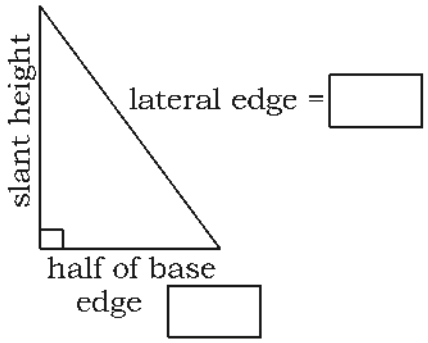
Activity –3

Find the total Surface Area to the given square pyramid



length of base edge = ☐

length of lateral edge = ☐



(Slant height)^2 = $(\square)^2 - (\square)^2$

= ☐

Slant height = ☐

Base Area = Area of square

= (base edge)^2

= $(\square)^2$

= ☐

Lateral Surface Area = Area of four triagles

= $4 \times \frac{1}{2} \times$ base edge $\times$ Slant height

$= 4 \times \frac{1}{2} \times \square \times \square$

$= \square$

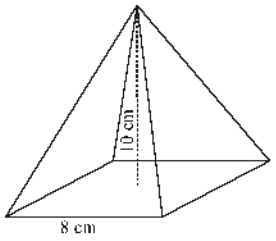
Total Surface Area = Base Area + Lateral Surface area

$= \square + \square$

$= \square$

Activity – 4

Find the volume of the square pyramid given



length of base edge = $\square$

height = $\square$

Volume = $\frac{1}{3}$ × Base Area × $\square$

$= \frac{1}{3} \times \square \times \square$

$= \square$ cm^3

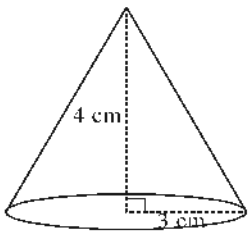
Activity –5

Find the total surface area and volume of the given cone

(Slant height)^2 = (radius)^2 + (height)^2

$= \square + \square$

$= \square$



Base area = Area of circle

= π × (radius)^2

= π × (☐)^2

= ☐

Lateral Surface area = π × radius × Slant height

= π × ☐ × ☐

= ☐

Total Surface area = Base area + lateral surface area

= ☐ + ☐

= ☐ cm^2

Volume = $\frac{1}{3}$ × base area × height

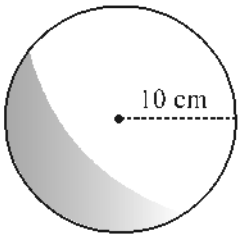
= $\frac{1}{3}$ × ☐ × ☐

= ☐ cm^3.

Activity – 6

Find the Surface area and Volume of the sphere given

Radius = ☐



Surface Area = 4π × (Radius)^2

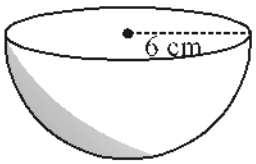
= ☐

Volume = $\frac{4}{3}\pi$ (Radius)^3 = ☐

Activity – 7

Find the surface area and volume of the given hemisphere

Radius = ☐



Total surface area = $3\pi \times (\text{radius})^2$

$= 3\pi \times \square^2$

$= \square$ cm^2

Volume = $\frac{2}{3}\pi \times (\text{radius})^3$

$= 3\pi \times \square^3$

$= \square$ cm^3

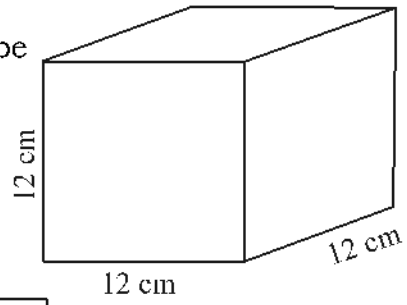
Activity –8

Find the total surface area and volume of the largest sphere that can be carved out of the given cube

diameter of the sphere = Side of the cube

= ☐

radius of the sphere = ☐



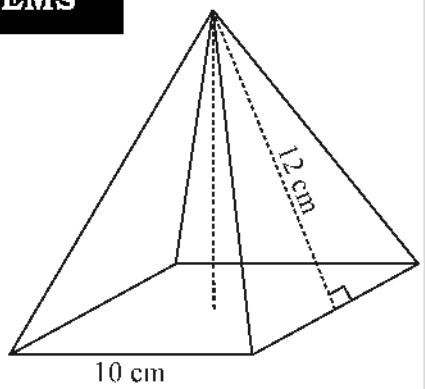
Total surface area = $4\pi \times (\text{radius})^2$ = ☐

Volume = $\frac{4}{3}\pi (\text{radius})^3$

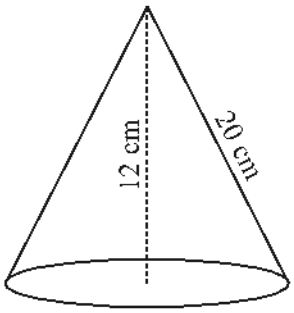
= ☐ cm^3

PRACTICE PROBLEMS

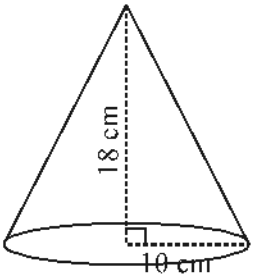
1. For the given square pyramid, find
   a. length of base edge
   b. slant height
   c. total surface area



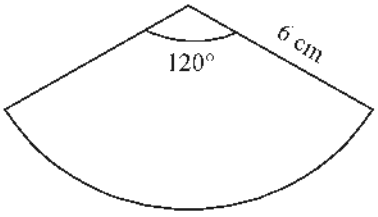
2. Base edge of a square pyramid is of 40 cm and its slant height is 25 cm. Find its
   a. height
   b. total surface area
   c. volume
3. For the given cone, find
   a. height and slant height
   b. radius
   c. total surface area



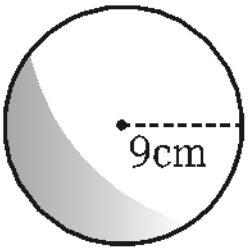
4 For the given cone, find
   a. radius and height
   b. volume



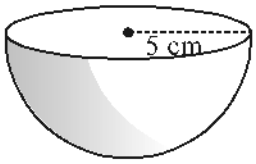
5. Find the radius of the cone made by using the sector given below



6. Radius of a sphere is 9cm. Find
   a. total surface area
   b. volume



7. The total surface area of a sphere is 100 π cm^2. Find its
   a. radius
   b. volume
8. Find the total surface area of the given hemisphere



9. Find to volume of a hemisphere of radius 12cm.
10. Radius of two spheres are in the ratio 2:3
    a. Find the ratio of their surface areas.
    b. Find the ratio of their volumes

ANSWERS

1. a. length of base edge = 10 cm
   b. slant height = 12 cm
   c. Total surface area = Area of square + Area of 4 triangles

$$= 10^2 + 4 \times \frac{1}{2} \times 10 \times 12$$

$$= 100 + 240$$

$$= 340\text{cm}^2.$$

2. a. (height)^2 = (Slant height)^2 – (half of base edge)^2

$$= 25^2 - 20^2$$

$$= 625 - 400$$

$$= 225$$

height = $\sqrt{225}$

= 15cm

   b. Total surface area =Area of square + Area of 4 triangles

$$= 40^2 + 4 \times \frac{1}{2} \times 40 \times 25$$

$$= 1600 + 2000$$

$$= 3600\text{cm}^2.$$

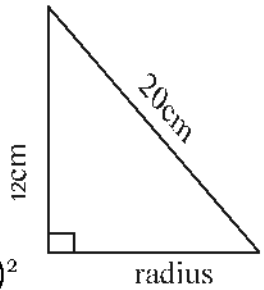
c. Volume $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 40^2 \times 15$

$= \frac{1}{3} \times 1600 \times 15 = 8000 cm^3$

3. a. height $= 12$ cm

slant height $= 20$ cm



b. $(\text{radius})^2 = (\text{slant height})^2 - (\text{height})^2$

$= 20^2 - 12^2$.

$= 400 - 144$

$= 256$

radius $= \sqrt{256} = 16$cm

c. Total surface area = Base area + Lateral surface area

$= \pi \times (\text{radius})^2 + \pi \times \text{radius} \times \text{slant height}$

$= \pi \times 16^2 + \pi \times 16 \times 20$

$= 256\pi + 320\pi$

$= 576\pi \text{ cm}^2$

4. a. radius = 10 cm, slant height = 18 cm

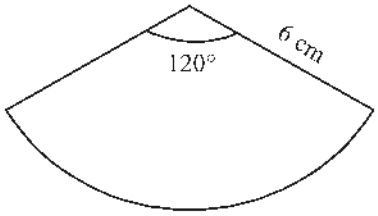
b. Volume $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times \pi \times (\text{radius})^2 \times \text{height}$

$= \frac{1}{3} \times \pi \times 10^2 \times 18$

$= \mathbf{600\ \pi\ cm^3}$

5.



radius of the sector = 6 cm

central angle = 120°

If r is the radius of cone, then

$$\frac{\text{radius of cone}}{\text{radius of sector}} = \frac{\text{Central angle of sector}}{360}$$

$$\frac{r}{6} = \frac{120}{360}$$

$$\frac{r}{6} = \frac{1}{3}$$

$$\therefore\ 3r = 6$$

$$r = \frac{6}{3} = 2 \text{ cm}$$

6 a. Total Surface area $= 4\pi \times (\text{radius})^2$

$= 4\pi \times 9^2$

$= 324\pi \text{ cm}^2$

b. Volume $= \frac{4}{3}\pi \times (\text{radius})^3$

$= \frac{4}{3}\pi \times 9^3$

$= 972\pi \text{ cm}^3$

7 a. Total surface area $= 100\ \pi$

$4\pi \times (\text{radius})^2 = 100\ \pi$

$(\text{radius})^2 = \frac{100\pi}{4\pi} = 25$

radius $= \sqrt{25}$

$= 5$ cm

b. Volume $= \frac{4}{3}\pi \times (\text{radius})^3$

$= \frac{4}{3}\pi \times 5^3$

$$= \frac{4}{3}\pi \times 125$$

$$= \frac{500\pi}{3} \text{ cm}^3$$

8 radius = 5cm

Total surface area of hemisphere

$$= 3\pi \times (\text{radius})^2$$

$$= 3\pi \times 5^2$$

$$= 3\pi \times 25$$

$$= 75\pi \text{ cm}^2$$

9 radius = 12cm

Volume of hemisphere

$$= \frac{2}{3}\pi \times (\text{radius})^3$$

$$= \frac{2}{3}\pi \times 12^3$$

$$= \frac{2}{3}\pi \times 12 \times 12 \times 12$$

$$= 1152\pi \text{ cm}^3$$

10. Let the radius of first sphere = r_1

radius of second sphere = r_2

ratio of radii = 2 : 3

ie $r_1 : r_2 = 2 : 3$

$$\frac{r_1}{r_2} = \frac{2}{3}$$

a. $$\frac{\text{surface area of first sphere}}{\text{surface area of second sphere}} = \frac{4\pi r_1^2}{4\pi r_2^2}$$

$$= \frac{r_1^2}{r_2^2}$$

$$= \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{2}{3}\right)^2$$

$$= \frac{4}{9}$$

ratio of surface area = 4 : 9

b. $$\frac{\text{Volume of first sphere}}{\text{Volume of second sphere}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$$

$$= \frac{r_1^3}{r_2^3}$$

$$= \left(\frac{r_1}{r_2}\right)^3$$

$$= \left(\frac{2}{3}\right)^3$$

$$= \left(\frac{2^3}{3^3}\right)$$

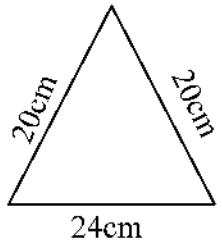
$$= \frac{8}{27}$$

ratio of volume = 8 : 27

MORE PRACTICE PROBLEMS

1. Find the total surface area of a square pyramid having length of base edge 7 cm and slant height 12 cm.
2. Base perimeter of a square pyramid is 40 cm and slant height is 13 cm. Find its
   a. height
   b. volume
3. Radius of a cone is 4cm and its slant height is 13 cm. Find its
   a. height
   b. total Surface area
4. Radius of a cone is 6 cm and its slant hight is 10 cm. Find
   a. its height
   b. its volume
5. Find the total surface area and volume of a sphere of radius 2cm.
6. A water tank is in the shape of a hemisphere of diameter 6m. How many cubic meters of water it can hold.

7. If we make a square pyramid with the following trangle as lateral faces, then what is the height of the pyramid? What would be the height, if base is 3cm instead of 24cm.



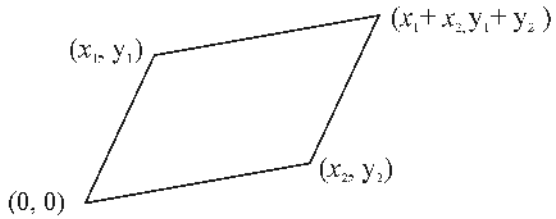
8. A cone is made by rolling up a sector of radius 12cm and central angle 120°.
   a) What is the slant height of the cone?
   b) Find the base radius of the cone?
   c) Find its curved surface area?
9. What is the central angle of the sector needed to make a cone of base radius 6cm and slant height 24cm?
10. The shape of a vessel is a hemisphere attached to one end of a cylinder. Common diameter is 12cm and height is 14cm.
    a) What is the height of the cylinder?
    b) Find the capacity of the vessel in litres?
11. A metal sphere of radius 8cm is melted and recast into a cylinder of radius 6cm
    a) What was the volume of the sphere?
    b) Find the height of the cylinder?
12. Calculate the volume of the biggest sphere that can be carved out of a cone of base diameter and slant height 18cm each?

Unit 9

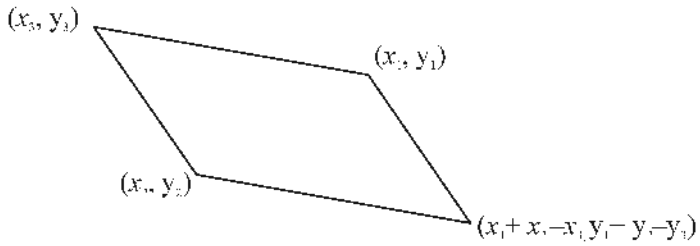
GEOMETRY AND ALGEBRA

Points to Remember

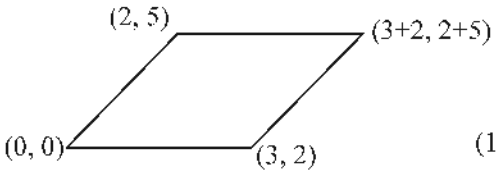
- The fourth vertex of the parallelogram with two adjacent sides are the lines formed by joining the origin with the points $(x_1, y_1)$and (x_2, y_2) is $(x_1 + x_2,\ y_1 + y_2)$.

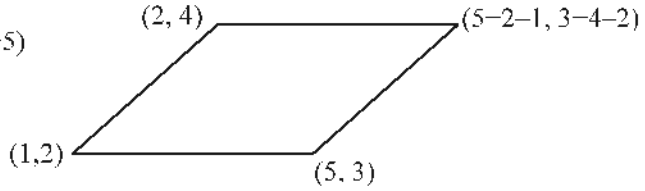


- The fourth vertex of the paralleogram with two adjacent sides are the lines formed by joining the points (x_1, y_1) and (x_2, y_2) with (x_3, y_3) is $(x_1 + x_2 - x_3, y_1 + y_2 - y_3)$



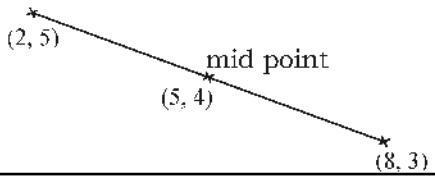
Example:





- The mid point of the line joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Eg:



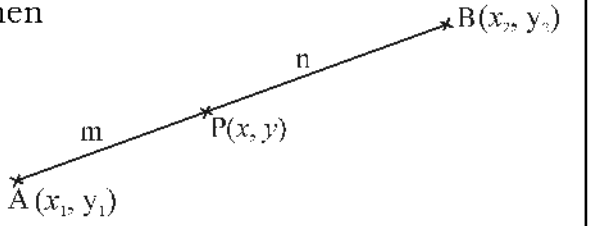
- The centroid of the triangle with vertices (x_1, y_1) (x_2, y_2) and (x_3, y_3) is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

  Eg: The centroid of the triangle with vertices (2, 1), (5, 3) and (8, 2) $\left(\frac{2+5+8}{3}, \frac{1+3+2}{3}\right) = (5, 2)$

- If the point P(x, y) divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio, m : n then

$$x = x_1 + \frac{m}{m+n}(x_2 - x_1)$$

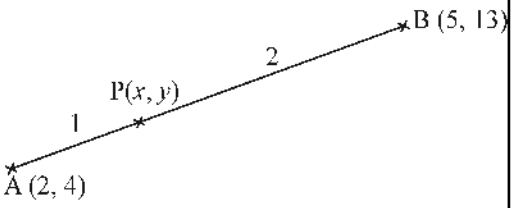
$$y = y_1 + \frac{m}{m+n}(y_2 - y_1)$$



If the point P(x, y) divide the line AB in the ratio 1:2, then

$$x = 2 + \frac{1}{3} \times (5-2) = 2 + \frac{1}{3} \times 3 = 2 + 1 = 3$$

$$y = 4 + \frac{1}{3}(13-4) = 4 + \frac{1}{3} \times 9 = 4 + 3 = 7$$



- In any line which is not parallel to the axes, the change in y is proportional to the change in x. In this case, the proportionality constant is the slope of this line.
- Slope of the line joining (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$

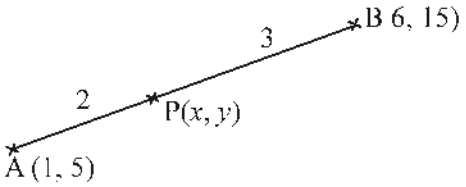
  Eg: Slope of the line joining (2, 3) and (8, 6) is $\frac{6-3}{8-2} = \frac{3}{6} = \frac{1}{2}$
- The constant relation between the x-coordinate and y-coordinate of any point on a line is called its equation.
- The equation of the line joining (x_1, y_1) and (x_2, y_2) is $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
- The constant relation between the x-coordinate and y-coordinate of any point on a circle is called its equation.
- The equation of the circle with centre as the origin and radius 'r' is $x^2 + y^2 = r^2$
- The equation of the circle with centre (x_1, y_1) and radius 'r' is $(x - x_1)^2 + (y - y_1)^2 = r^2$.

Activity- 1

If A (1, 5) and B(6, 15) then find the coordinates of the point which divide the line AB in the ratio 2:3

Let P (x, y) be the point dividing the line AB in the ratio 2:3

AP = $\frac{\square}{\square}$ part of AB.



x- coordinate of A = □

Difference between the x- coordinates of A and B

= □ – □ = □

x- coordinate of P $= 1 + \frac{2}{5} \times 5$

= □ + □

= □

y-coordinate of A = □

Difference between the y-co-ordinates of A and B

= □ – □ = □

y-coordinate of P $= 5 + \frac{2}{5} \times 10$

= □ + □

= □

∴ Co-ordinates of P = (□, □)

Activity- 2

Check whether the points A (2, 3), B(4, 4) and C(8, 6) lie on a line

x - co-ordinate of A = □

x-co-ordinate of B = □

Difference between the x - coordinates of A and B = □ – 2

= □

y-coordinate of A = ☐

y-coordinate of B = ☐

Difference between the y-coordinates of A and B = ☐ – ☐

= ☐

$\therefore$ Slope of the line AB $= \frac{1}{2}$

Difference between the x-coordinates of B and C = ☐ – ☐

= ☐

Difference between the y-coordinates of B and C = ☐ – ☐

= ☐

$\therefore$ Slope of the line BC = $\frac{\square}{\square} = \frac{\square}{\square}$

Slopes of the lines AB and BC are equal ☐/not equal ☐

$\therefore$ The points A, B and C lie on a line ☐/not lie on a line ☐

Activity- 3

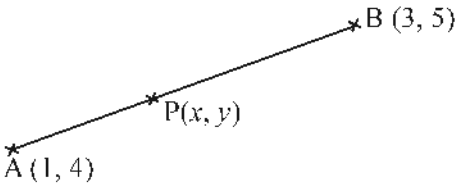
Find the equation of the line joining the points (1, 4) and (3, 5)

Let A (1, 4) and B (3, 5)

When we move from A to B

the x-co-ordinate is increased by ☐

y-co-ordinate is increased by ☐



When x-coordinate is increased by ☐ the y-co-ordinate is increased by ☐

The rate of increase in y with the increase in x, ie, the slope of this line = $\frac{\square}{\square}$

Now consider the points A and P,

the x-co-ordinate is increased by x – ☐

y-co-ordinate is increased by y – ☐

Since the rate of increase in y-coordinate with the increase in x-coordinate is always same at everywhere in a line

$$\frac{y-\square}{x-\square}=\frac{\square}{\square}$$

From this we get $\square(x-\square)=\square(y-\square)$

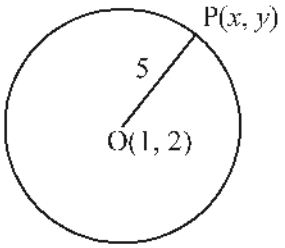
Simplifying this, we get $x-\square y+\square=0$

This is the equation of the line

Activity- 4

Find the equation of the circle with centre (1,2) and radius 5 unit

Let P(x, y) is a point on this circle.



Difference between the x-co-ordinates of O and P = $x-\square$

Difference between the y-co-ordinates of O and P = $y-\square$

length of the line OP = 5 unit

$$\sqrt{(x-\square)^2+(y-\square)^2}=5$$

$$(x-\square)^2+(y-\square)^2=25$$

$x^2-\square x+\square+y^2-\square y+\square=25$

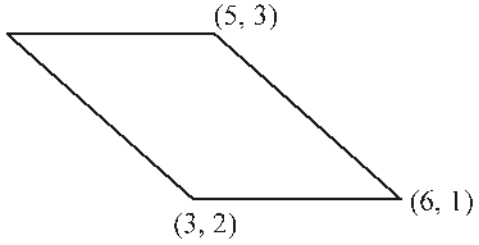
Simplify this we get

$x^2+y^2-\square x-\square y-\square=0$

This is the equation of this circle

PRACTICE QUESTIONS

1. Find the co-ordinates of the fourth vertex of the following parallelogram



2. Find the co-ordinates of the centroid of the triangle with vertices (-2, 4), (4,5) and (2,2).
3. If the co-ordinates of A and B are (2,5)and (8,15) respectively, then
   a. Find the co-ordinates of the midpoint of AB.
   b. Find the co-ordinates of the point P on this line such that AP: PB = 3:2
4. Prove that the points (2,3), (4,6) and (8,12) are on the same line.
5. Find the equation of the line joining (4,1) and (3, 7). Write the equation of the line parallel to this line and passes through (2,3)
6. a. Find the equation of the line passes through (1, 3) with slope 2.
   b. Check whether the point (3,7) is on this line.
7. The equation of a line is 2x-3y+6=0
   a. Find the coordinates of the points of intersection of this line with the axes.
   b. Also find the slope of this line
8. The equations of two lines are 2x +y – 6 = 0 and 2x +y +4 = 0
   a. Find the coordinates of any two points on each of these lines
   b. Prove that these two lines are parallel.
9. Write the equation of the circle with centre as the origin and radius 5 unit.
10. Form the equation of the circle with centre as (2,1) and the radius 4 unit.

ANSWERS

1. Co-ordinates of fourth vertex = (3+5 – 6, 2 + 3 – 1) = (2, 4)

2. Centroid is $\left(\frac{-2+4+2}{3}, \frac{4+5+2}{3}\right)$ $= \left(\frac{4}{3}, \frac{11}{3}\right)$

3. a. Co-ordinates of the midpoint = $\left(\frac{2+8}{2}, \frac{5+15}{2}\right)$
   = (5, 10)

   b. x - coordinate of P = $2+\frac{3}{5}(8-2)$
   $= 2+\frac{3}{5}\times 6$

$$= 2+\frac{18}{5}$$

$$= 2+3\frac{3}{5}$$

$$= 5\frac{3}{5}$$

y co-ordinate of P $= 5+\frac{3}{5}(15-5) = 5+\frac{3}{5}\times 10$

$$= 5 + 6$$

$$= 11$$

$\therefore$ Coordinates of P $= \left(5\frac{3}{5}, 11\right)$

4. Slope of the line joining (2, 3) and (4, 6)

$$= \frac{6-3}{4-2}$$

$$= \frac{3}{2}$$

Slope of the line joining (4, 6) and (8, 12)

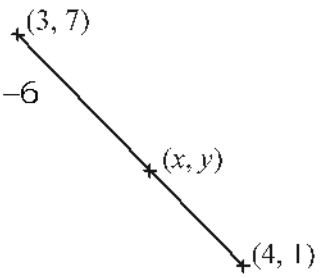
$$= \frac{12-6}{8-4}$$

$$= \frac{6}{4} = \frac{3}{2}$$

Since the slopes are equal, the three points lie on the same line

5. Slope of the line joining (4,1) and (3,7) =

$$= \frac{7-1}{3-4} = -6$$



If (x, y) is a point on this line

$$\frac{y-1}{x-4} = -6$$

$-6(x-4) = y-1$

$-6x+24 = y-1$

$6x+y-25 = 0$

which is the equation of this line

Slope of a line parallel to this line = –6

If (2, 3) is a point on this line

$\frac{y-3}{x-2} = -6$

$-6x+12 = y-3$

$6x+y-15 = 0,$

which is the equation of the parallel line

6. a. Let (x, y) is a point on the line passes through (1, 3) and with slope 2.

$\frac{y-3}{x-1} = 2$

$2x-2 = y-3$

$2x-y+1 = 0$

b. Consider the point (3,7)

$2x-y+1 \Rightarrow 2\times 3-7+1 = 6-7+1 = 0$

$\therefore$ point (3,7) is on this line.

7. Equation of the line is $2x-3y+6=0$

a. Let this line intersect the x-axis at $(x, 0)$

Then $2x-3\times 0+6 = 0$

$\therefore 2x+6 = 0$

$\therefore x = -3$

$\therefore$ The point of intersection of this line with the x-axis is (–3, 0)

Let this line intersect the y-axis at (0, y)

Then $2\times 0-3y+6=0$

$\therefore -3y+6 = 0$

$\therefore y = 2$

$\therefore$ The point of intersection of this line with the y-axis is (0,2)

b. Slope of the line $= \frac{2-0}{0-(-3)} = \frac{2}{3}$

8 a. In the equation $2x + y - 6 = 0$,

when $x = 0$, $y - 6 = 0$

$\therefore \quad y = 6$

when $y = 0$, $2x - 6 = 0$

$\therefore \quad x = 3$

$\therefore$ Two points on this line are (0, 6) and (3, 0)

In the equation $2x + y + 4 = 0$

when $x = 0$, $y + 4 = 0$

$\therefore \quad y = -4$

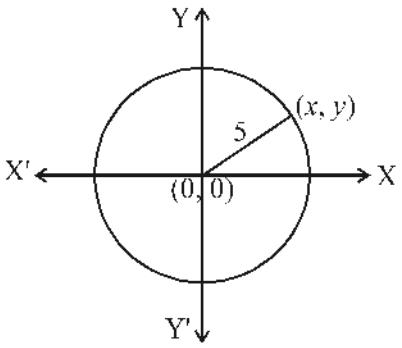
$y = 0$, $2x + 4 = 0$

$\therefore \quad x = -2$

$\therefore$ Two points on this line are (0, –4) and (–2, 0)

b. In the equations $2x + y - 6 = 0$ and $2x + y + 4 = 0$, the difference is only in the constant term. So these two lines are parallel.

9.



If (x,y) is a point on this circle, then the equation of the circle is

$x^2 + y^2 = 5^2$

ie. $x^2 + y^2 = 25$

10. If (x, y) is a point on this circle, then the distance between (x, y) and (2, 1) is 4 unit.

$$\sqrt{(x-2)^2+(y-1)^2} = 4$$

$$(x-2)^2+(y-1)^2 = 16$$

$$x^2-4x+4+y^2-2y+1 = 16$$

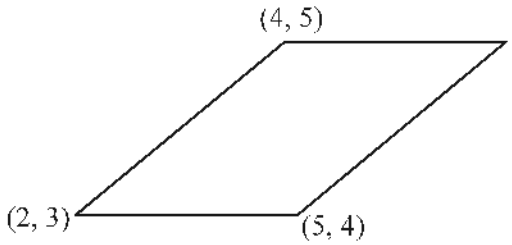
$$x^2+y^2-4x-2y+5-16 = 0$$

$$x^2+y^2-4x-2y-11 = 0$$

which is the equation of this circle.

MORE PRACTICE PROBLEMS

1. Write the co-ordinates of the fourth vertex of the parallelogram in the figure



2. The vertices of ΔABC are A(4, 1), B(2, 7) and C(6, 7). If P,Q and R are the mid points of the sides AB, BC and AC respectively, then prove that ΔPQR is isosceles.

3. P is a point on the line joining A(5, 4) and B (12, –10)such that AP:PB = 4 : 3. Find the co-ordinates of P.

4. Can the line joining (2, 1) and (8, 5) passes through the point (6, 7)? Can this line passes through (5, 3)?

5. Form the equaiton of the line joining (1, 1) and (–1, 2). Find the co-ordinates of the point of intersection of this line with the x-axis.

6. Find the equation of the line passes through (3, 5) with slope $\frac{1}{3}$. Write the equation of another line parallel to this line.

7. Write the equation of a line parallel to the line $2x + 3y - 5 = 0$ and passes through (3, 5).

8. The equations of two lines are $x + 2y - 5 = 0$ and $3x - y - 1 = 0$

   a. Find the co-ordinates of the point of intersection of these lines.

   b. Find the co-ordinates of another point on each of these lines.

9. Form the equation of the circle with centre as the origin and radius 10 unit. Write the co-ordinates of 8 points on this line.

10. Find the equation of the circle with (2, 3) as centre and radius 3unit.

Unit 10

POLYNOMIALS

Points to Remember

- If $p(x) = q(x) \times r(x)$ then the polynomials $q(x)$, $r(x)$ are factors of $p(x)$

  Example: $p(x) = x^2 - 4 = (x + 2)(x - 2)$

  $\therefore$ $(x + 2)$ and $(x - 2)$ are factors of $x^2 - 4$

- If $p(a) = 0$ then $(x - a)$ is a factor of the polynomial $p(x)$

  Example: If $p(5) = 0$ then $(x - 5)$ is a factor of $p(x)$.

- If $p(x)$ can be written as $p(x) = (x - a_1)(x - a_2) \ldots (x - a_n)$ then the numbers a_1, a_2, ... a_n are solutions of the equation $p(x) = 0$

  Example: If $p(x) = (x - 1)(x - 2)(x - 3)$ then 1, 2, 3 are solutions of the equation $p(x) = 0$

Activity- 1

Is $(x - 2)$ a factor of the polynomial $p(x) = x^3 - 6x^2 + 10x - 5$.

$p(x) = x^3 - 6x^2 + 10x - 5$

$p(2) = \square^3 - 6 \times \square^2 + 10 \times \square - 5$

$= \square - 6 \times \square + \square - 5$

$= \square - \square + \square - 5$

$= \square$

$\therefore (x - 2)$ is a factor of $p(x)$ yes $\square$/ No $\square$

Activity - 2

Write $x^2 - 7x + 2$ as the product of two first degree polynomials.

$x^2 - 7x + 12 = (x - a)(x - b)$

$x^2 - 7x + 12 = x^2 - (a + b)x + ab$

$a + b = \square$, $ab = \square$

$a = \square$

$b = \square$

$\therefore\ x^2 - 7x + 12 = (x - \square)(x - \square)$

Activity - 3

Write $6x^2 - 5x + 1$ as the product of two first degree polynomials.

$p(x) = 6x^2 - 5x + 1$

Consider the second degree equation $6x^2 - 5x + 1 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 6 \times 1}}{2 \times 6}$$

$$= \frac{\square \pm \sqrt{\square - \square}}{\square}$$

$$= \frac{\square + \square}{\square}, \frac{\square - \square}{\square}$$

$= \square, \square$

From this $p\left(\frac{1}{2}\right) = 0$, $p(\square) = 0$

$\therefore \left(x - \frac{1}{2}\right), \left(x - \frac{1}{3}\right)$ are factors of $p(x)$

$6x^2 - 5x + 1 = 6(x - \square)(x - \square)$

$= (2x - 1)(3x - 1)$

Activity -4

If $(x-3)$ is a factor of the polynomial $x^3 - 2x^2 - kx + 6$, then what is the value of k ?

$p(x) = x^3 - 2x^2 - kx + 6$

since $(x-3)$ is a factor $p(x)$, $p(3) = \square$

$\square^3 - 2 \times \square^2 - k \times \square + 6 = 0$

$\square - 2 \times \square - \square + 6 = 0$

$\square - \square - 3k + 6 = 0$

$\square - 3k + 6 = 0$

$\square - 3k = 0$

$3k = \square$

$k = \frac{\square}{3}$

$= \square$

PRACTICE PROBLEMS

1. Consider the polynomial $p(x) = x^2 - 5x + 6$

   Is $(x-2)$ a factor of $p(x)$? why ?

2. Consider the polynomial $p(x) = x^3 - 2x^2 + x - 4$

   a. Which number subtracted from $p(x)$ gives a polynomial with $(x-2)$ as a factor

   b. Which number added to $p(x)$ gives a polynomial with $(x-3)$ as a factor

3. Write $x^2 - 3x + 2$ as the product of two first degree polynomials

4. If $(x-2)$ is a factor of the polynomial $3x^3 - 2x^2 + kx - 6$,then what is the value of k ?

5. a. Check whether $(x+1)$ is a factor of the polynomial $p(x) = 6x^3 + 3x^2$

   b. Which first degree polynomial added to $p(x)$ gives a polynomial with $(x^2 - 1)$ as a factor ?

6. If $(x-1)$ and $(x-2)$ are factors of the polynomial $x^3 - 6x^2 + ax + b$, find the values of a and b .

7. a. If $b = a + c$, then prove that $(x + 1)$ is a factor of the polynomial $ax^2 + bx + c$

   b. Write a second degree polynomial with $(x + 1)$ as a factor.

8. If $(x - 1)$ is a factor of the polynomial $ax^2 + bx + c$ then what is the value of $a + b + c$?

9. Prove that the polynomial $x^2 + 3x + 3$ cannot be written as the product of two first degree polynomials.

10. a. Write a third degree polynomial and check whether $(x-1)$ is a factor of this polynomial.

ANSWERS

1. $p(x) = x^2 - 5x + 6$

$p(2) = 2^2 - 5 \times 2 + 6$

$= 4 - 10 + 6$

$= 0$

b. Since $p(2) = 0$, $(x - 2)$ is a factor of $p(x)$

2. a. $p(x) = x^3 - 2x^2 + x - 4$

$p(2) = 2^3 - 2 \times 2^2 + 2 - 4$

$= 8 - 8 + 2 - 4$

$= -2$

For $(x - 2)$ to be a factor of $p(x)$, $p(2)$ must be zero.

$\therefore$ -2 is to be added.

b. $p(3) = 3^3 - 2 \times 3^2 + 3 - 4$

$= 27 - 18 + 3 - 4$

$= 8$

For $(x - 3)$ to be a factor of $p(x)$, $p(3)$ must be zero.

$\therefore$ -8 is to be added.

3. $p(x) = x^2 - 3x + 2$

Let $x^2 - 3x + 2 = (x - a)(x - b)$

$\therefore$ $x^2 - 3x + 2 = x^2 - (a + b)x + ab$

$\therefore$ $a + b = 3$

$ab = 2$

$a = 2$

$b = 1$

$\therefore\ x^2 - 3x + 2 = (x - 2)(x - 1)$

4. $p(x) = 3x^3 - 2x^2 + kx - 6$

Since $(x - 2)$ is a factor of $p(x)$, we have $p(2) = 0$

$p(2) = 3 \times 2^3 - 2 \times 2^2 + k \times 2 - 6 = 0$

$3 \times 8 - 2 \times 4 + 2k - 6 = 0$

$\therefore\ 24 - 8 + 2k - 6 = 0$

$\therefore\ 10 + 2k = 0$

$2k = -10$

$$k = \frac{-10}{2} = -5$$

5. a. $p(x) = 6x^3 - 3x^2$

$p(-1) = 6 \times (-1)^3 + 3 \times (-1)^2$

$= 6 \times -1 + 3 \times 1$

$= -6 + 3 = -3 \neq 0$

$p(-1) \neq 0$

$\therefore\ (x + 1)$ is not a factor of $p(x)$

b. Let $ax + b$ be the first degree polynomial to be added.

$q(x) = p(x) + ax + b$

$= 6x^3 + 3x^2 + ax + b$

$x^2 - 1 = (x + 1)(x - 1)$

so $(x + 1)$ and $(x - 1)$ are factors of $q(x)$

$\therefore\ q(1) = 0$ and $q(-1) = 0$

$\therefore q(1) = 6 \times 1^3 + 3 \times 1^2 + a \times 1 + b = 0$

$\therefore\ 6 + 3 + a + b = 0$

$9 + a + b = 0$

$a + b = -9$ ——— (1)

$\therefore q(-1) = 6 \times (-1)^3 + 3 \times (-1)^2 + a \times (-1) + b = 0$

$\therefore\ 6 \times -1 + 3 \times 1 - a + b = 0$

$$-6 + 3 - a + b = 0$$

$$\therefore \quad -3 - a + b = 0$$

$$\therefore \quad -a + b = 3 \text{ ——— (2)}$$

(1) + (2), $2b = -6$

$$\therefore b = \frac{-6}{2} = -3$$

From (1) $a = -9 - b$

$= -9 - (-3)$

$= -9 + 3 = -6$

$\therefore$ Polynomial to be added is $ax + b$

$= -6x - 3$

6. Since $(x - 1)$ and $(x - 2)$ are factors of the polynomiel $p(x) = x^3 - 6x^2 + ax + b$, we have $p(1) = p(2) = 0$

$p(1) = 1^3 - 6 \times 1^2 + a \times 1 + b = 0$

$\therefore 1 - 6 + a + b = 0$

$-5 + a + b = 0$

$a + b = 5$ ——— (1)

$p(2) = 2^3 - 6 \times 2^2 + a \times 2 + b = 0$

$\therefore 8 - 6 \times 4 + 2a + b = 0$

$8 - 24 + 2a + b = 0$

$-16 + 2a + b = 0$

$\therefore \quad 2a + b = 16$ ——— (2)

(2) - (1), $a = 11$

From (1), $b = 5 - 11 = -6$

$\therefore \quad a = 11, b = -6$

7. a. $p(x) - ax^2 + bx + c$

$p(-1) = a \times (-1)^2 + b \times -1 + c$

$= a - b + c$

Since b = a + c

$p(-1) = a - (a + c) + c$

$= a - a - c + c = 0$

Since $p(-1) = 0$, $(x + 1)$ is a factor of $p(x)$

b. $p(x) = 2x^2 + 5x + 3$ $(\because\ 5 = 2 + 3)$

8. $p(x) = ax^2 + bx + c$

Since $(x - 1)$ is a factor of $p(x)$, we have $p(1) = 0$

$p(1) = a \times 1^2 + b \times 1 + c = 0$

$a + b + c = 0$

That is the value of a + b + c is 0.

9. Consider the second degree equation $x^2 + 3x + 3 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 3}}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{9 - 12}}{2}$$

$$= \frac{-3 \pm \sqrt{-3}}{2}$$

No such number x exists. Therefore $x^2 + 3x + 3$ cannot be written as the product of two frist degree polynomials.

10. a. $p(x)$ is a third degree polynomial

Find p(1)

If p(1) = 0, then $(x - 1)$ is a factor of $p(x)$.

If $p(1) \neq 0$, then $(x - 1)$ is not a factor of $p(x)$

MORE PRACTICE PROBLEMS

1. Write the polynomial $p(x) = x^2 - 5x + 12$ as the product of two first degree polynomials.
2. If $(x + 2)$ is a factor of the polynomial $p(x) = x^2 + 7x + k$, then what is the value of k ?
3. If $(x - 2)$ and $(x + 1)$ are factors of the polynomial $p(x) = x^3 + 3x^2 + ax + b$, then find the values of a and b.
4. Which number added to the polynomial $p(x) = x^2 + 6x + 8$ gives a polynomial with $(x - 2)$ as a factor?
5. Write a ploynomial $p(x)$ such that $p(1) = 0$ and $p(-2) = 0$

Unit 11

STATISTICS

Points to Remember

- Mean $\Rightarrow$ $\dfrac{\text{Sum of the quantities}}{\text{Number of quantities}}$
- Median $\Rightarrow$ The middle most observation when the observations are arranged either in icreasing or decreasing order.
- If the number of observations is 'n', then the middle most observation is
  a. $\left(\frac{n+1}{2}\right)^{th}$ observation, if 'n' is odd,
  b. $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations, if 'n' is even .

Activity -1

The scores got by 10 students in a class test are as follows. Find the mean of the scores 13, 11, 20, 18, 20, 25, 21, 15, 10, 17

$13 + 11 + 20 + 18 + 20 + 25 + 21 + 15 + 10 + 17 = \square$

$$\text{Mean} = \frac{\text{Sum of the quantities}}{\text{Number of quantities}} = \frac{\square}{\square}$$

$$= 17$$

Activity -2

The weights in kg of 13 people are as follows. Find the median weight.

66, 56, 83, 29, 43, 58, 53, 70, 50, 30, 45, 89, 57

When the weights are arranged in order

29, 30, 43, 45, 50, 53, 56, 57, 58, 66, 70, 83, 89

Median $\Rightarrow$ Middle most quantity, when the quantities are arranged in order

Middle most quantity = $\square$

$\therefore$ Median = $\square$

Activity -3

Find the median of the following quantities.

5, 7, 9, 5, 5, 9, 4, 6, 3, 5, 4, 5

When the quantities are arranged in order

$\square, \square, \square, \square, \square, \square, \square, \square, \square, \square, \square, \square$

Median $\Rightarrow$ Middle most quantity, when the quantities are arranged in order.

Quantities in the middle $\square, \square$

$$\therefore \text{Median} = \frac{\square + \square}{2}$$

$$= \square$$

Activity -4

The monthly income of some people in a locality are given in the following table. Find the median of the monthly incomes.

Monthly income	Number of people
2500	6
3000	8
3500	12
4000	20
4500	16
5000	6

Monthly income	Number of peoples
☐	6
Upto 3000	☐
☐	26
Upto 4000	☐
☐	62
Upto 5000	☐

Total number = ☐

34^{th} and 35^{th} are the middle two quantities. Monthly income of peoples from 27^{th} to 46^{th} is ☐

$\therefore$ Monthly income of 34^{th} person = ☐

Monthly income of 35^{th} person = ☐

$$\therefore \text{Median monthly income} = \frac{\square + \square}{2}$$

$$= \square$$

Activity -5

The following table gives the age of workers in a company.Find the median age.

Age	Number of workers
35 – 40	9
40 – 45	11
45 – 50	5
50 – 55	8
55 – 60	7
60 – 65	5

Age	Number of workers
below 40	☐
☐	20
below 50	☐
☐	33
below 60	☐
☐	45

Total number of workers = ☐

When the workers are arranged according to their ages, the middle most worker is in the 23rd position.

The ages of 21st worker to 25th worker are in between ☐ and ☐

Number of workers from 21st to 25th = ☐

When we divide the 5 unit difference between 45 and 50 into 12 equal parts, each part $= \frac{5}{5} = 1$

Age of 21st worker is in the middle of 45 and 46

$$= 45\frac{1}{2}$$

$23 - 21 =$ ☐

$\therefore$ Age of 23rd worker $= 45\frac{1}{2} + 2 \times 1$

$$= 45\frac{1}{2} + \square$$

$$= 47\frac{1}{2}$$

$\therefore$ $\therefore$ Median age $= 47\frac{1}{2}$

PRACTICE PROBLEMS

1. Find the mean of the following quantities

   610, 625, 618, 625, 621, 615, 610, 620

2. The following are the scores got by some pupils in an examination. Find the median score.

   66, 33, 56, 20, 13, 56, 53, 70, 50, 30, 56, 45, 56

3. The weights in kg of some people in a locality are given.Find the median weight.

   66, 56, 83, 29, 43, 58, 53, 70, 50, 30, 45, 89

4. The following table gives the daily wages of some workers in a factory. Find the median daily wage

Daily wages (Rs)	**Number of workers**
400	3
500	7
600	8
700	6
800	5

5. The heights in centimetre of some students in a class are given in the following table. Find the median height.

Height of students (cm)	**Number of students**
135	5
140	8
145	12
150	11
155	5
160	4

6. The following table gives the ages of workers in a firm. Find the median age.

Age	Number of workers
25 - 30	4
30 - 35	12
35 - 40	9
40 - 45	10
45 - 50	17
50 - 55	7

Answers

1. Mean $= \frac{\text{Sum of the quantities}}{\text{Number of quantities}}$

$$= \frac{610+625+618+625+621+615+610+620}{8}$$

$$= \frac{4944}{8} = 618$$

2. When the quantities are arranged in order

13, 20, 30, 33, 45, 50, 53, 56, 56, 56, 56, 66, 70

Median = Middle most quantity, when the quantities are arranged in order

$\therefore$ Middle most quantity = 53

Median = 53

3. Middle two quantities, when the qunaitites are arranged in order = 53 and 56

$\therefore$ Median weight $= \frac{53+56}{2} = 54.5$ kg

4.

Daily wage (Rs.)	Number of workers
Upto 400	3
Upto 500	10
Upto 600	18
Upto 700	24
Upto 800	29

Total workers = 29

Middle most quantity = 15^{th} quantity

15^{th} quantity = 600

$\therefore$ Midean = 600

5.

Height	Number of students
upto 135	5
upto 140	13
upto 145	25
upto 150	36
upto 155	41
upto 160	45

Number of quantities = 45

Middle most quantity = 23^{rd} quantity

Heights of the students from 14^{th} to 25^{th} = 145

$\therefore$ Height of 23^{rd} student = 145

$\therefore$ Median height = 145

6.

Age	Number of workers
below 30	4
below 35	16
below 40	25
below 45	35
below 50	52
below 55	59

Total number of workers = 59

When the workers are arranged according to their ages, the 30^{th} worker is in the middle.

Ages of 26^{th} to 35^{th} workers are in between 40 and 45.

Number of workers from 26^{th} to 35^{th} is 10

When we divide the 5 unit difference betwee40 and 45 into 10 equal parts, each part $= \frac{5}{10} = \frac{1}{2}$

Age of 26^{th} worker is in the middle of 40 and $40\frac{1}{2}$ $= 40\frac{1}{4}$

30 – 26 = 4

$\therefore$ Age of 30th worker $= 40\frac{1}{4} + 4 \times \frac{1}{2}$

$= 40\frac{1}{4} + 2$

$= 42\frac{1}{4}$

MORE PRACTICE PROBLEMS

1. The following are the daily wages of peoples working in a company. Find the mean and median of the daily wages.

   400, 500, 600, 700, 800, 1000

2. The following table gives the monthly income of 30 families in a locality. Find the median monthly income.

Monthly income (Rs)	Number of families
4000	4
4500	6
5000	2
5500	3
6000	5
6500	7
7000	3

3. The following table gives the weight of students in a class. Find the median weight.

Weight (kg)	Number of students
30 - 35	8
35 - 40	7
40 - 45	10
45 - 50	13
50 - 55	4

4. The following table gives the scores got by students in a class for mathematics exam.

Score	Number of students
0 - 10	2
10 - 20	5
20 - 30	10
30 - 40	6
40 - 50	4
50 - 60	11
60 - 70	5
70 - 80	2

Find the median score in the class.

ಅಥವಾ